Problema con potencial anharmónico

Variacional

In[162]:=
```
x[t_] := a * Sin[w * t];  (* trayectoria propuesta ente t0=0 y t1=π/w,
va de x0=0 a x1=0 . Extremos fijos!*)
T = m * D[x[t], t]^2 / 2; (*energía cinética*)
V = k * x[t]^2 / 2 + β * x[t]^4 / 4; (*potencial*)
S = Integrate[T - V, {t, 0, π / w}] (*cálculo de la acción S *)
Solve[D[S, a] == 0, w]
(* hacemos la acción S estacionaria ante variaciones de la amplitud a *)
```

Out[162]= $-\frac{a^2 \pi (8 k - 8 m w^2 + 3 a^2 \beta)}{32 w}$

Out[163]= $\left\{\left\{w \to -\frac{\sqrt{4 k + 3 a^2 \beta}}{2 \sqrt{m}}\right\}, \left\{w \to \frac{\sqrt{4 k + 3 a^2 \beta}}{2 \sqrt{m}}\right\}\right\}$

$\tau a = 2 \pi \Big/ \left(\frac{\sqrt{4 k + 3 a^2 \beta}}{2 \sqrt{m}}\right)$; (* período aproximado para amplitud a *)

Exacto

```
V = k x^2 / 2 + β x^4 / 4;
Ener = k * A^2 / 2 + β A^4 / 4; (* Energía *)
τ0 = Integrate[1 / Sqrt[Ener - k x^2 / 2 - β x^4 / 4], {x, 0, A}]
 (* un cuarto de período exacto, falta factor √m/2 *)
```

Out[172]= $Aborted

In[168]:= $\tau = \frac{4 \sqrt{2} \sqrt{m} \sqrt{1 - \frac{k}{2 k + A^2 \beta}}\ \text{EllipticK}\left[-1 + \frac{2 k}{2 k + A^2 \beta}\right]}{\sqrt{(k + A^2 \beta)}}$;

Comparación exacto aproximado (variacional)

```
Series[τ, {A, 0, 7}]  (* Serie de Taylor en A  a orden 7 del exacto *)
Series[τa, {a, 0, 7}] (* Serie de Taylor en a  a orden 7 del variacional *)
```

Out[169]= $\frac{2\sqrt{m}\,\pi}{\sqrt{k}} - \frac{3\left(\sqrt{m}\,\pi\,\beta\right)A^2}{4\,k^{3/2}} + \frac{57\sqrt{m}\,\pi\,\beta^2 A^4}{128\,k^{5/2}} - \frac{315\left(\sqrt{m}\,\pi\,\beta^3\right)A^6}{1024\,k^{7/2}} + O[A]^8$

Out[170]= $\frac{2\sqrt{m}\,\pi}{\sqrt{k}} - \frac{3\left(\sqrt{m}\,\pi\,\beta\right)a^2}{4\,k^{3/2}} + \frac{27\sqrt{m}\,\pi\,\beta^2 a^4}{64\,k^{5/2}} - \frac{135\left(\sqrt{m}\,\pi\,\beta^3\right)a^6}{512\,k^{7/2}} + O[a]^8$

Pendulo

In[82]:=
```
θ[t_] := a * Sin[w * t]
T = m * l^2 * D[θ[t], t]^2 / 2
V = m * g * l * Cos[θ[t]]
S = Integrate[T - V, {t, 0, π / w}]
Solve[D[S, a] == 0, w]
```

Out[83]= $\frac{1}{2} a^2\, l^2\, m\, w^2\, \mathrm{Cos}[t\,w]^2$

Out[84]= g l m Cos[a Sin[t w]]

Out[85]= ConditionalExpression$\left[\frac{1}{4}\, l\, m \left(a^2\, l\, \pi\, w - \frac{4\, g\, \pi\, \mathrm{BesselJ}[0, \mathrm{Abs}[a]]}{w}\right),\ a \in \mathrm{Reals}\ \&\&\ \frac{1}{w} \in \mathrm{Reals}\right]$

Solve::nsmet : This system cannot be solved with the methods available to Solve. ≫

Out[86]= Solve$\left[\mathrm{ConditionalExpression}\left[\frac{1}{4}\, l\, m \left(2\, a\, l\, \pi\, w + \frac{4\, g\, \pi\, \mathrm{BesselJ}[1, \mathrm{Abs}[a]]\, \mathrm{Abs}'[a]}{w}\right) == 0,\ a \in \mathrm{Reals}\ \&\&\ \frac{1}{w} \in \mathrm{Reals}\right],\ w\right]$

In[175]:= $\mathbf{S = \frac{1}{4}\, l\, m \left(a^2\, l\, \pi\, w - \frac{4\, g\, \pi\, BesselJ[0, a]}{w}\right);}$

```
Solve[D[S, a] == 0, w]
```

Out[176]= $\left\{\left\{w \to -\frac{i\sqrt{2}\,\sqrt{g}\,\sqrt{\mathrm{BesselJ}[1, a]}}{\sqrt{a}\,\sqrt{l}}\right\},\ \left\{w \to \frac{i\sqrt{2}\,\sqrt{g}\,\sqrt{\mathrm{BesselJ}[1, a]}}{\sqrt{a}\,\sqrt{l}}\right\}\right\}$

In[178]:= $\text{Series}\left[\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{\text{BesselJ}[1,a]}}{\sqrt{a}\,\sqrt{l}},\ \{a, 0, 7\}\right]$

Out[178]= $\frac{\sqrt{g}}{\sqrt{l}} - \frac{\sqrt{g}\,a^2}{16\sqrt{l}} + \frac{\sqrt{g}\,a^4}{1536\sqrt{l}} - \frac{\sqrt{g}\,a^6}{73728\sqrt{l}} + O[a]^{15/2}$