Fundamental PB and CT

$$[Q_j, Q_k]_{q,p} = \frac{\partial Q_j}{\partial q_i}\frac{\partial Q_k}{\partial p_i} - \frac{\partial Q_j}{\partial p_i}\frac{\partial Q_k}{\partial q_i} = -\frac{\partial Q_j}{\partial q_i}\frac{\partial q_i}{\partial P_k} - \frac{\partial Q_j}{\partial p_i}\frac{\partial p_i}{\partial P_k} = -\frac{\partial Q_j}{\partial P_k} = 0$$

$$[P_j, P_k]_{q,p} = \frac{\partial P_j}{\partial q_i}\frac{\partial P_k}{\partial p_i} - \frac{\partial P_j}{\partial p_i}\frac{\partial P_k}{\partial q_i} = \frac{\partial P_j}{\partial q_i}\frac{\partial q_i}{\partial Q_k} + \frac{\partial P_j}{\partial p_i}\frac{\partial p_i}{\partial Q_k} = \frac{\partial P_j}{\partial Q_k} = 0$$

$$[Q_j, P_k]_{q,p} = \frac{\partial Q_j}{\partial q_i}\frac{\partial P_k}{\partial p_i} - \frac{\partial Q_j}{\partial p_i}\frac{\partial P_k}{\partial q_i} = \frac{\partial Q_j}{\partial q_i}\frac{\partial q_i}{\partial Q_k} + \frac{\partial Q_j}{\partial p_i}\frac{\partial p_i}{\partial Q_k} = \frac{\partial Q_j}{\partial Q_k} = \delta_{jk}$$

$$[P_j, Q_k]_{q,p} = -[Q_k, P_j] = -\delta_{jk}$$

Used Direct Conditions here

- Fundamental Poisson Brackets are invariant under CT

Poisson Bracket and CT

- What happens to a Poisson Bracket under CT?
  - For a time-independent CT

$$[u,v]_{Q,P} \equiv \frac{\partial u}{\partial Q_i}\frac{\partial v}{\partial P_i} - \frac{\partial u}{\partial P_i}\frac{\partial v}{\partial Q_i}$$

$$= \left(\frac{\partial u}{\partial q_j}\frac{\partial q_j}{\partial Q_i} + \frac{\partial u}{\partial p_j}\frac{\partial p_j}{\partial Q_i}\right)\left(\frac{\partial v}{\partial q_k}\frac{\partial q_k}{\partial P_i} + \frac{\partial v}{\partial p_k}\frac{\partial p_k}{\partial P_i}\right) - \left(\frac{\partial u}{\partial q_j}\frac{\partial q_j}{\partial P_i} + \frac{\partial u}{\partial p_j}\frac{\partial p_j}{\partial P_i}\right)\left(\frac{\partial v}{\partial q_k}\frac{\partial q_k}{\partial Q_i} + \frac{\partial v}{\partial p_k}\frac{\partial p_k}{\partial Q_i}\right)$$

$$= \frac{\partial u}{\partial q_j}\frac{\partial v}{\partial q_k}[q_j, q_k]_{Q,P} + \frac{\partial u}{\partial q_j}\frac{\partial v}{\partial p_k}[q_j, p_k]_{Q,P} + \frac{\partial u}{\partial p_j}\frac{\partial v}{\partial q_k}[p_j, q_k]_{Q,P} + \frac{\partial u}{\partial p_j}\frac{\partial v}{\partial p_k}[p_j, p_k]_{Q,P}$$

$$= \frac{\partial u}{\partial q_j}\frac{\partial v}{\partial p_k}\delta_{jk} - \frac{\partial u}{\partial p_j}\frac{\partial v}{\partial q_k}\delta_{jk}$$

$$= [u,v]_{q,p}$$

Poisson Brackets are invariant under CT