Problema P. Osc.

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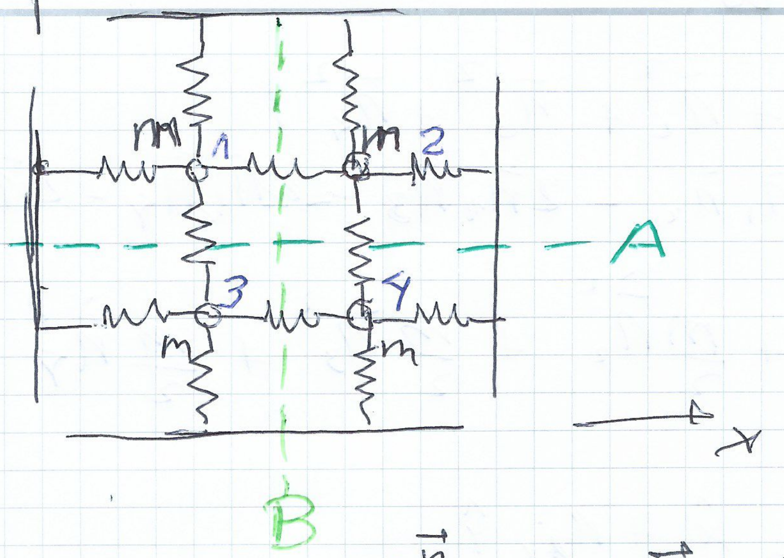
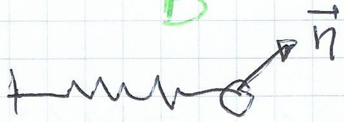
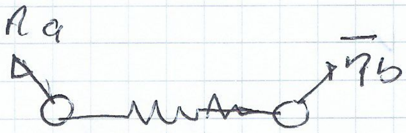


$l_0 = 0$

todos K iguales



$$\vec{l} = \vec{d} + \vec{n}$$
$$l^2 = d^2 + 2\vec{d}\cdot\vec{n} + n^2$$



$$\vec{l} = \bar{d} + \bar{n}_b - \bar{n}_a$$
$$l^2 = d^2 + 2\bar{d}\cdot(\bar{n}_b - \bar{n}_a) + n_b^2 - 2\bar{n}_b\cdot\bar{n}_a + n_a^2$$

$$V = \frac{K}{2}\Big[(d\hat{x} + \vec{n}_1)^2 + (d\hat{y} + \vec{n}_1)^2 + (-d\hat{x} + \bar{n}_2)^2 + (-d\hat{y} + \bar{n}_2)^2 + (-\bar{n}_1 + d\hat{x} + \vec{n}_2)^2 + (-\bar{n}_1 - d\hat{y} + \bar{n}_3)^2 + (-\vec{n}_2 - d\hat{y} + \vec{n}_4)^2 + (d\hat{x} + \bar{n}_3)^2 + (+d\hat{y} + \bar{n}_3)^2 + (-\vec{n}_3 + d\hat{x} + \vec{n}_4)^2 + (-d\hat{x} + \vec{n}_4)^2 + (d\hat{y} + \vec{n}_4)^2\Big]$$

Los términos lineales se anulan pues se parte del equilibrio.

A menos de la constante V_0 el potencial es:

$$V = \frac{1}{2}\Big[4k\left(\bar{r}_1^{\,2} + \bar{r}_2^{\,2} + \bar{r}_3^{\,2} + \bar{r}_4^{\,2}\right) - 2\bar{r}_1\bar{r}_2 - 2\bar{r}_1\bar{r}_3 - 2\bar{r}_2\bar{r}_4 - 2\bar{r}_3\bar{r}_4\Big]$$

$$T = \frac{m}{2}\dot{\bar{r}}_1^{\,2} + \frac{m}{2}\dot{\bar{r}}_2^{\,2} + \frac{m}{2}\dot{\bar{r}}_3^{\,2} + \frac{m}{2}\dot{\bar{r}}_4^{\,2}$$

$$\bar{r}_i = r_{ih}\,\hat{x} + r_{iv}\,\hat{y}$$

La parte vertical se desacopla de la horizontal.

$$\mathcal{L} = \mathcal{L}_h + \mathcal{L}_v$$

Los dos problemas son idénticos.

$$\mathcal{L}_h = \frac{1}{2}\left(m\dot{r}_{1h}^2 + m\dot{r}_{2h}^2 + m\dot{r}_{3h}^2 + m\dot{r}_{4h}^2\right) - \frac{1}{2}\big(4k r_{1h}^2 + 4k r_{2h}^2 + 4k r_{3h}^2 + 4k r_{4h}^2 - 2k r_{1h} r_{2h} - 2k r_{3h} r_{4h} - 2k r_{1h} r_{3h} - 2 r_{2h} r_{4h}\big)$$

$$T_h = \begin{pmatrix} m & & & \\ & m & & \\ & & m & \\ & & & m \end{pmatrix}$$

$$V_h = \begin{pmatrix} 4k & -k & -k & 0 \\ -k & 4k & 0 & -k \\ -k & 0 & 4k & -k \\ 0 & -k & -k & 4k \end{pmatrix}$$

Por simetrías:

$$\vec{a} = \begin{pmatrix} 1 \\ \alpha \\ \beta \\ \gamma \end{pmatrix}$$

- **Simetría plano B:**

$$\bar{a}_{:B} = \begin{pmatrix} 1 \\ \alpha \\ \beta \\ \gamma \end{pmatrix} \qquad S_B \bar{a}_B = \begin{pmatrix} -\alpha \\ -1 \\ -\gamma \\ -\beta \end{pmatrix}$$

$$\bar{a}_{:B} = \pm S_B a_{:B}$$

- Simetrico c/r plano B:

$$\vec{a}_{S_B} = + S_B \bar{a}_{S_B}$$

igualando:

$$1 = -\alpha$$
$$\alpha = -1$$
$$\beta = -\gamma$$
$$\gamma = -\beta.$$

$$\bar{a}_{S_B} = \begin{pmatrix} 1 \\ -1 \\ \beta \\ -\beta \end{pmatrix}$$

- Simetría plan A

$$S_A \bar{a}_{S_B} = \begin{pmatrix} \beta \\ -\beta \\ 1 \\ -1 \end{pmatrix} = \pm \begin{pmatrix} 1 \\ -1 \\ \beta \\ -\beta \end{pmatrix}$$

$\cancel{\bar{a}_{S_A S_B}} =$

Modo Simétrico c/r S_A y S_B

$$\beta = 1$$

$$\bar{a}_1 = \bar{a}_{S_A S_B} = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} \qquad \nabla \bar{a}_{S_A S_B} = 4k \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} = m\omega^2 \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

$$\therefore \boxed{\omega_1^2 = 4k/m}$$

Modo Simétrico c/r S_B y antisimétrico c/r S_A

$$\beta = -1$$

$$\bar{a}_2 = \bar{a}_{S_A A_B} = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\nabla \vec{a}_{S_A A_B} = 6k \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = m\omega^2 \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\therefore \boxed{\omega_2^2 = \frac{6k}{m}}$$

Antisimétrico c/r S_B: $\bar{a}_{S_B} = -S_B \vec{a}_{S_B}$

$1 = \alpha$
$\alpha = 1$
$\beta = \gamma$
$\gamma = B$

$$\vec{a}_{A_B} = \begin{pmatrix} 1 \\ 1 \\ B \\ B \end{pmatrix}$$

- Simetría c/r $\cancel{S_A}$

$$S_A \bar{a}_{AB} = \pm \bar{a}_{AB}$$

$$\begin{pmatrix} \beta \\ \beta \\ 1 \\ 1 \end{pmatrix} = \pm \begin{pmatrix} 1 \\ 1 \\ \beta \\ \beta \end{pmatrix}$$

- Modo antisimetrico c/r S_B, simetr. c/r S_A

$$S_A \bar{a}_{AB} = + \bar{a}_{AB}$$

$$\begin{matrix} \beta = 1 \\ \beta = 1 \\ 1 = \beta \\ 1 = \beta \end{matrix} \Rightarrow \bar{a}_{S_A A_B} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \vec{a}_3$$

$$V \bar{a}_{S_A A_B} = 2k \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = m\omega^2 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\therefore \boxed{\omega_3^2 = \frac{2k}{m}}$$

- Modo antisimétrico c/r S_B, antisim. c/r S_A

$$S_A \bar{a}_{AB} = - \bar{a}_{AB}$$

$$\begin{matrix} \beta = -1 \\ \beta = -1 \\ 1 = -\beta \\ 1 = -\beta \end{matrix} \qquad \beta = -1$$

$$\Rightarrow \quad \vec{a}_{A_A A_B} = \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} = \vec{q}_4$$

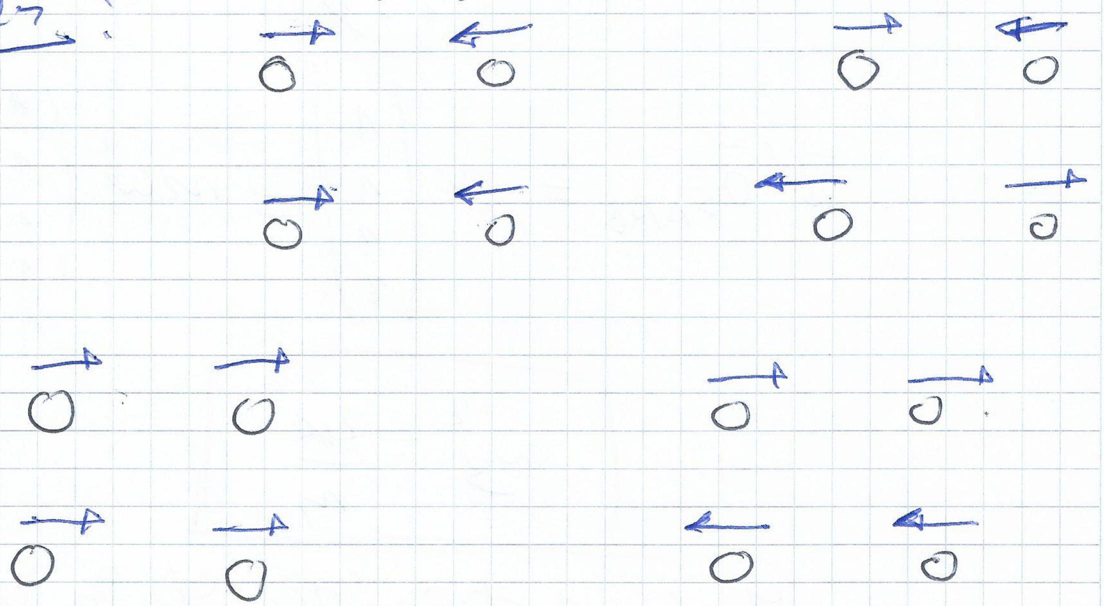
$$V \vec{a}_{A_A A_B} = 4k \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} = m\omega^2 \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$$

$$\boxed{\omega_4^2 = \frac{4k}{m}}$$

Hay 2 modos degenerados (igual frecuencia)

Resumen:

$S_A S_B$ $\qquad$ $A_A S_B$



$S_A A_B$ $\qquad$ $A_A A_B$