Guía 3: Centrales

P4) Usaremos: $V(r) = -\frac{k}{r} + \frac{c}{2r^2}$ (1)

Partimos de las magnitudes conservadas:

$$\begin{cases} mr^2\dot{\theta} = l & (2) \\ \frac{m\dot{r}^2}{2} + \frac{l^2}{2mr^2} + V(r) = E & (3) \end{cases}$$

$$u(\theta) = \frac{1}{r(\theta)}$$

$$\frac{du}{d\theta} = \underbrace{\frac{1}{r^2(\theta)}}_{u^2} \cdot \frac{dr}{dt} \cdot \underbrace{\frac{dt}{d\theta}}_{\frac{1}{\dot{\theta}} = \frac{m}{l u^2}}$$

$$\Rightarrow \dot{r} = \frac{l}{m}\frac{du}{d\theta} \quad (4)$$

(4) en (3):

$$\frac{l^2}{2m}\left(\frac{du}{d\theta}\right)^2 + \frac{l^2}{2m}u^2 - ku + \frac{c}{2}u^2 = E$$

$\times \frac{2m}{l^2} \rightarrow$

$$\left(\frac{du}{d\theta}\right)^2 + \left(\sqrt{1+\frac{mc}{l^2}}\,u - \frac{mk}{l^2\sqrt{1+\frac{mc}{l^2}}}\right)^2 = \frac{2mE}{l^2} + \frac{m^2k^2}{(l^2+mc)l^2}$$

o mejor:

$$\left(\frac{du}{d\theta}\right)^2 + \left(\sqrt{1+\frac{mc}{l^2}}\right)^2\left(u - \frac{mk}{l^2+mc}\right)^2 =$$

$$= \left(\sqrt{\frac{2Em}{l^2} + \frac{m^2k^2}{(l^2+mc)l^2}}\right)^2$$

Proponemos:

$$u = u_0 \cos[\alpha(\theta-\theta_0)] + \frac{mk}{l^2+mc}$$

$$\frac{du}{d\theta} = -\alpha u_0 \sin[\alpha(\theta-\theta_0)]$$

$$\alpha^2 u_0^2 \sin^2[\alpha(\theta-\theta_0)] + \left(\sqrt{1+\frac{mc}{l^2}}\right)^2 u_0^2 \cos^2[\alpha(\theta-\theta_0)]$$

$$= \left(\sqrt{\frac{2Em}{l^2} + \frac{m^2k^2}{(l^2+mc)l^2}}\right)^2$$

identificamos:

$$\alpha = \sqrt{1+\frac{mc}{l^2}} \qquad u_0 = \sqrt{\frac{2Em}{l^2} + \frac{m^2k^2}{(l^2+mc)l^2}}\ \frac{1}{\alpha}$$

Caso $1+\frac{mc}{l^2} > 0$

$$\frac{1}{r} = \frac{1}{\alpha}\sqrt{\frac{2Em}{l^2} + \frac{m^2k^2}{(l^2+mc)l^2}}\cos[\alpha(\theta-\theta_0)] + \frac{mk}{l^2+mc}$$

Si $1+\frac{mc}{l^2} < 0$ $\qquad \alpha \to i|\alpha|$

$$\frac{1}{r} = \sqrt{\frac{2Em}{\alpha^2 l^2} + \frac{m^2k^2}{(l^2+mc)^2}}\cosh[|\alpha|(\theta-\theta_0)] + \frac{mk}{l^2+mc}$$