Mecánica Clásica

Ej-15 Guia 0
Coordenadas

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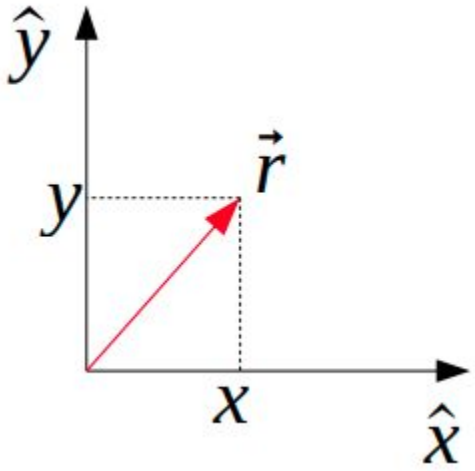
Coordenadas Cartesianas: (x,y)

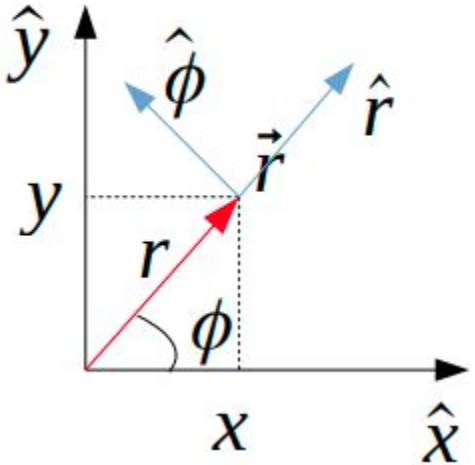


$$\vec{r} = x\,\hat{x} + y\,\hat{y}$$

$$\dot{\vec{r}} = \dot{x}\,\hat{x} + \dot{y}\,\hat{y} \qquad \dot{x} = \frac{dx}{dt} \qquad \dot{\hat{x}} = \frac{d\,\hat{x}}{dt} = 0$$

$$\ddot{\vec{r}} = \ddot{x}\,\hat{x} + \ddot{y}\,\hat{y}$$

Coordenadas Polares: (x, y) $\rightarrow$(r, ϕ)



$$x = r\cos(\phi)$$
$$y = r\,sen(\phi)$$

$r \ni [0, r]; \phi \ni [0, 2\pi]$

$$\vec{r} = r\,\hat{r}$$
$$\dot{\vec{r}} = \dot{r}\,\hat{r} + r\,\dot{\hat{r}} \longrightarrow \hat{r}(t), \hat{\phi}(t)$$

$$\hat{r} = \cos(\phi)\,\hat{x} + sen(\phi)\,\hat{y}$$

$$\hat{\phi} = -sen(\phi)\,\hat{x} + \cos(\phi)\,\hat{y}$$

$$\rightarrow \dot{\hat{r}} = \dot{\phi}\,\hat{\phi}$$

$$\dot{\vec{r}} = \dot{r}\,\hat{r} + r\,\dot{\hat{r}} = \dot{r}\,\hat{r} + r\,\dot{\phi}\,\hat{\phi}$$

Coordenadas Polares: (x, y) $\rightarrow$ (r, ϕ)

Velocidad $\rightarrow$ $\dot{\vec{r}} = \dot{r}\,\hat{r} + r\,\dot{\phi}\,\hat{\phi}$

Aceleración $\rightarrow$

$$\ddot{\vec{r}} = \ddot{r}\,\hat{r} + \dot{r}\,\dot{\hat{r}} + \dot{r}\,\dot{\phi}\,\hat{\phi} + r\,\ddot{\phi}\,\hat{\phi} + r\,\dot{\phi}\,\dot{\hat{\phi}} \qquad \dot{\hat{r}} = \dot{\phi}\,\hat{\phi}$$

$$\ddot{\vec{r}} = \ddot{r}\,\hat{r} + \dot{r}\,\dot{\phi}\,\hat{\phi} + \dot{r}\,\dot{\phi}\,\hat{\phi} + r\,\ddot{\phi}\,\hat{\phi} - r\,\dot{\phi}^2\,\dot{\hat{r}} \qquad \dot{\hat{\phi}} = -\dot{\phi}\,\hat{r}$$

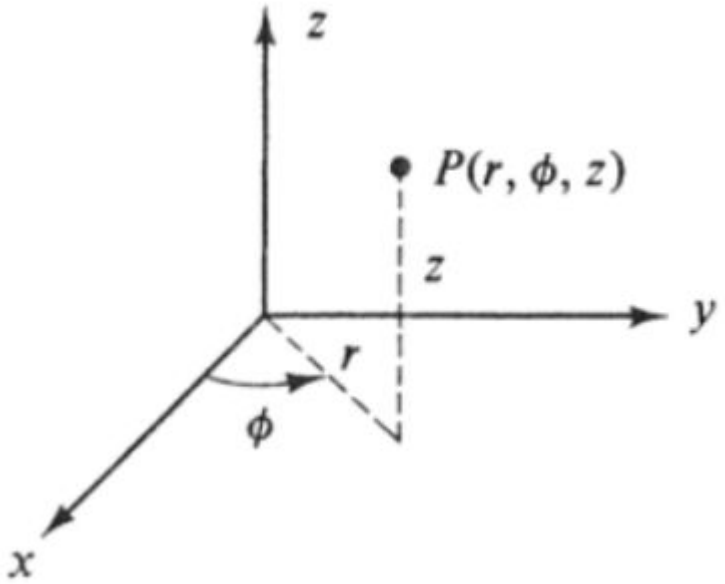
$$\ddot{\vec{r}} = \left(\ddot{r} - r\,\dot{\phi}^2\right)\hat{r} + \left(2\dot{r}\,\dot{\phi} + r\,\ddot{\phi}\right)\hat{\phi}$$

Coordenadas Cilíndricas: $(x, y, z) \rightarrow (r, \phi, z)$

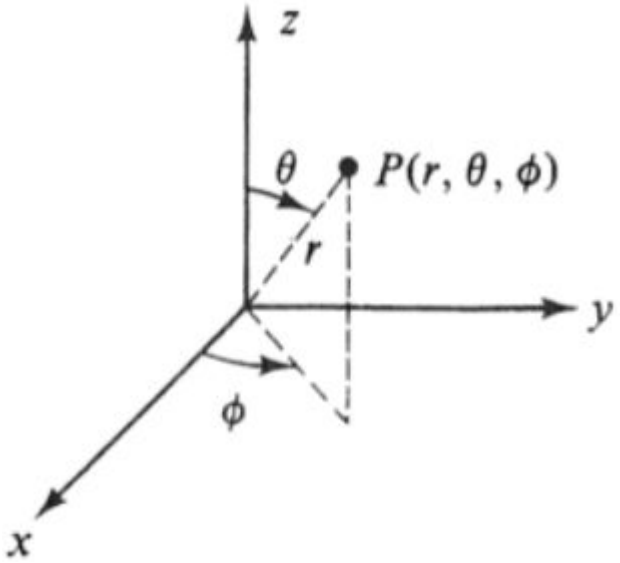
$$\vec{r} = r\,\hat{r} + z\,\hat{z}$$

$$\dot{\vec{r}} = \dot{r}\,\hat{r} + r\,\dot{\phi}\,\hat{\phi} + \dot{z}\,\hat{z}$$

$$\ddot{\vec{r}} = \left(\ddot{r} - r\,\dot{\phi}^2\right)\hat{r} + \left(2\dot{r}\,\dot{\phi} + r\,\ddot{\phi}\right)\hat{\phi} + \ddot{z}\,\hat{z}$$



Coordenadas Esféricas: (x, y, z)→(r, θ, ϕ)



$$\hat{r} = \cos(\phi)\, sen(\theta)\, \hat{x} + sen(\phi)\, sen(\theta)\, \hat{y} + \cos(\theta)\, \hat{z}$$

$$\hat{\phi} = -sen(\phi)\, \hat{x} + \cos(\phi)\, \hat{y}$$

$$\hat{\theta} = \cos(\phi)\cos(\theta)\, \hat{x} + sen(\phi)\cos(\theta)\, \hat{y} - sen(\theta)\, \hat{z}$$

$$r \ni [0, r]; \phi \ni [0, 2\pi]; \theta \ni [0, \pi]$$

Coordenadas Esféricas: (x, y, z)→(r, θ, ϕ)

$$\vec{r} = r\,\hat{r}$$

$$\dot{\vec{r}} = \dot{r}\,\hat{r} + r\,\dot{\hat{r}}$$

$$\dot{\hat{r}} = \frac{d\hat{r}}{dt} = \frac{d\phi}{dt}\frac{d\hat{r}}{d\phi} + \frac{d\theta}{dt}\frac{d\hat{r}}{d\theta} = \dot{\theta}\frac{d\hat{r}}{d\theta} + \dot{\phi}\frac{d\hat{r}}{d\phi}$$

$$\hat{r} = \cos(\phi)\,sen(\theta)\,\hat{x} + sen(\phi)\,sen(\theta)\,\hat{y} + \cos(\theta)\,\hat{z}$$

$$\hat{\phi} = -sen(\phi)\,\hat{x} + \cos(\phi)\,\hat{y}$$

$$\hat{\theta} = \cos(\phi)\cos(\theta)\,\hat{x} + sen(\phi)\cos(\theta)\,\hat{y} - sen(\theta)\,\hat{z}$$

$$\frac{d\hat{r}}{d\theta} = \hat{\theta} \qquad \frac{d\hat{r}}{d\phi} = sen(\theta)\,\hat{\phi}$$

Velocidad

↓

$$\dot{\vec{r}} = \dot{r}\,\hat{r} + r\,\dot{\theta}\hat{\theta} + r\,\dot{\phi}\,sen(\theta)\,\hat{\phi}$$

Coordenadas Esféricas: (x, y, z)→(r, θ, ϕ)

$$\frac{d\hat{r}}{d\phi} = sen(\theta)\hat{\phi} \qquad \frac{d\hat{r}}{d\theta} = \hat{\theta}$$

$$\frac{d\hat{\theta}}{d\phi} = \cos(\theta)\hat{\phi} \qquad \frac{d\hat{\theta}}{d\theta} = -\hat{r}$$

$$\frac{d\hat{\phi}}{d\phi} = -(sen(\theta)\hat{r} + \cos(\theta)\hat{\phi}) \qquad \frac{d\hat{\phi}}{d\theta} = 0$$

$$\ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2 sen(\theta)^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2 \cos(\theta) sen(\theta))\hat{\theta} + (r\ddot{\phi} sen(\theta) + 2\dot{r}\dot{\phi} sen(\theta) + 2r\dot{\theta}\dot{\phi}\cos(\theta))\hat{\phi}$$

Aceleración