Mecánica Clásica

Ej-5 Guia 3

Leandro E. Fernández

Potencial Efectivo

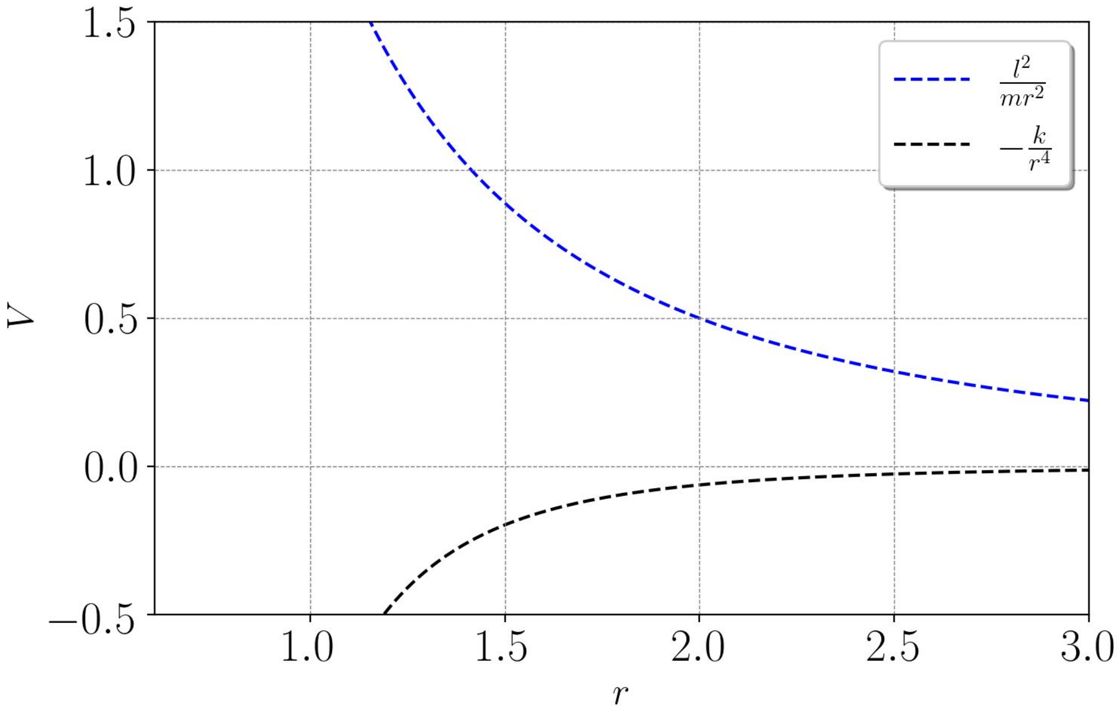
$$E = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\varphi}^2) - \frac{k}{r^4} \longrightarrow \text{Tenemos 2 coordenadas generalizadas}$$

$$\dot{L} = \bar{r} \times \bar{F} = 0 \longrightarrow L = cte \longrightarrow L_z = l = mr^2\dot{\varphi}$$

$$E = T + V_{ef} = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{mr^2} - \frac{k}{r^4}$$

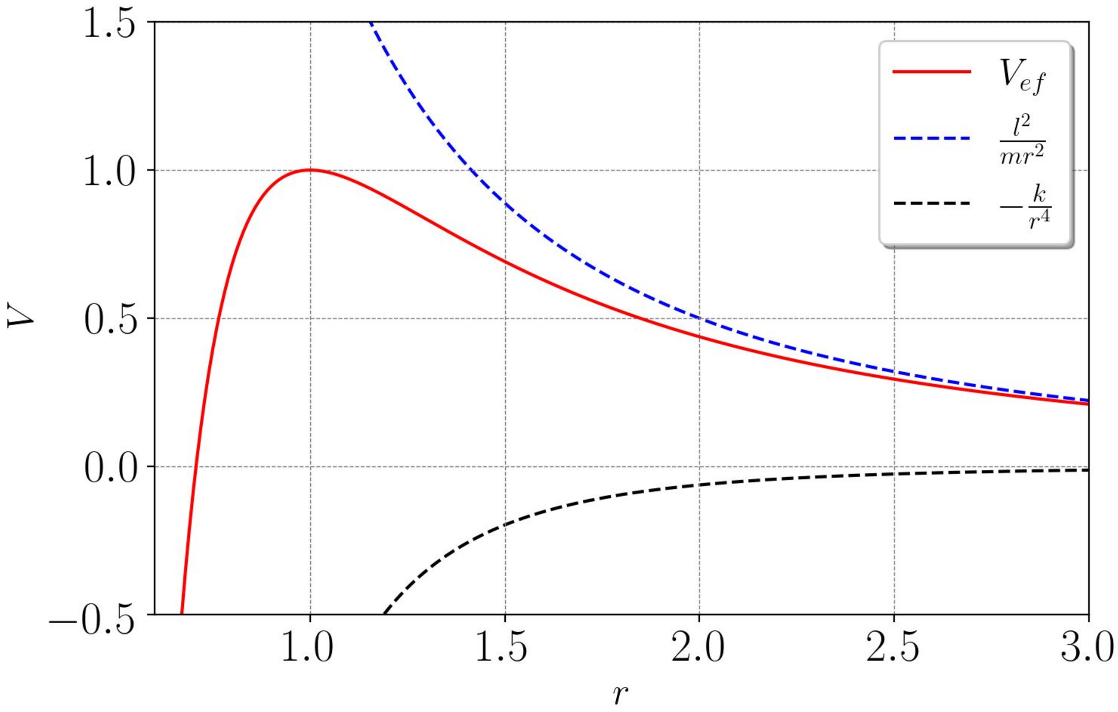
Potencial Efectivo

$$V_{ef} = \frac{l^2}{mr^2} - \frac{k}{r^4}$$



Potencial Efectivo

$$V_{ef} = \frac{l^2}{mr^2} - \frac{k}{r^4}$$



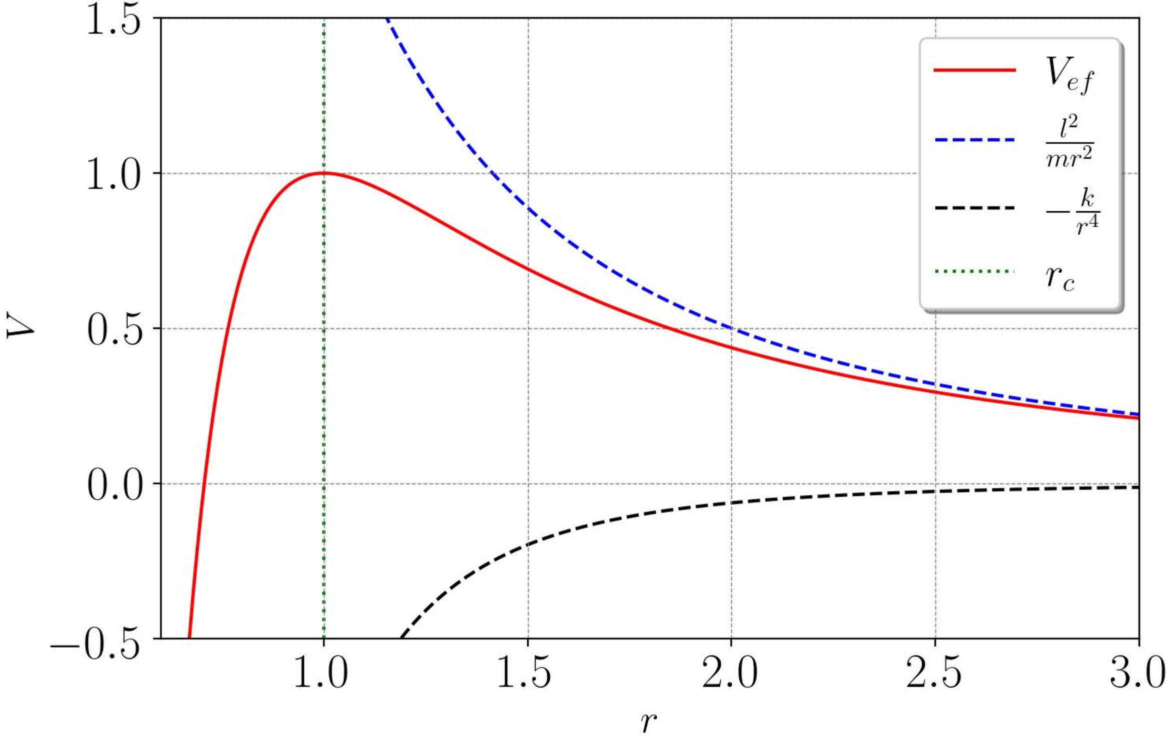
Potencial Efectivo

$$\frac{\partial V_{ef}}{\partial r} = \frac{4k}{r^5} - \frac{l^2}{mr^3} \longrightarrow r_c = \sqrt{\frac{4km}{l^2}}$$

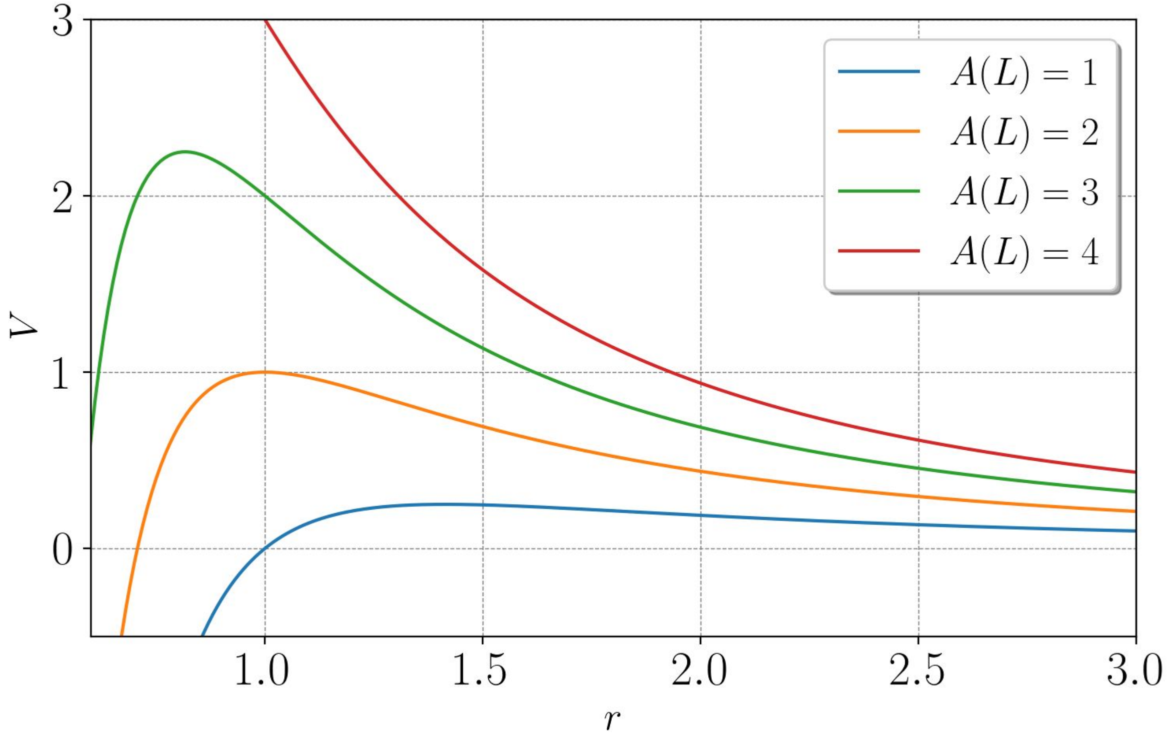
$$E_c = V_{ef}(r_c) = \frac{l^4}{16km^2}$$

$$\dot{\varphi} = \frac{l}{mr_c^2} \longrightarrow \varphi = \frac{l}{mr_c^2}t \longrightarrow 2\pi = \frac{l}{mr_c^2}\tau \longrightarrow \tau_c = \frac{2\pi mr_c^2}{l}$$

Trayectorias



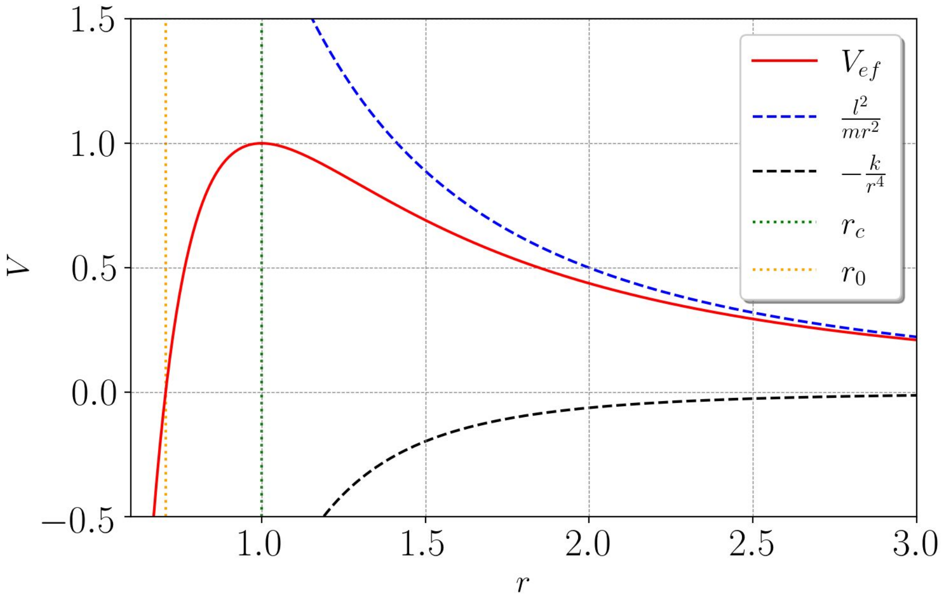
Trayectorias



Anular el Potencial efectivo

$$V_{ef}(r_0) = 0 = \frac{l^2}{mr_o^2} - \frac{k}{r_0^4}$$

$$r_0 = \sqrt{\frac{2mk}{l^2}} = \frac{r_c}{\sqrt{2}}$$



Binet

$$m\ddot{r} = -\frac{\partial V_{ef}}{\partial r} = \frac{l^2}{mr^3} + f(r) \longrightarrow u \equiv r^{-1}$$

$$\frac{\partial}{\partial t} = \dot{\varphi}\frac{\partial}{\partial \varphi} \longrightarrow \dot{r} = \frac{\partial}{\partial t}\left(\frac{1}{u}\right) = -\frac{1}{u^2}\frac{\partial u}{\partial t} = -\frac{l}{m}\frac{\partial u}{\partial \varphi}$$

$$\ddot{r} = \frac{l}{mr^2}\frac{\partial}{\partial \varphi}\dot{r} = -\frac{l^2u^2}{m^2}\frac{\partial^2 u}{\partial \varphi^2}$$

$$\frac{\partial^2 u}{\partial \varphi^2} + u + \frac{m}{l^2u^2} f\left(\frac{1}{u}\right) = 0$$

Binet

$$u(\varphi) = \frac{1}{Acos(\varphi)}$$

Proponemos esta solución a la ecuación de Binet

$$\frac{\partial u(\varphi)}{\partial \varphi} = \frac{sen(\varphi)}{Acos^2(\varphi)}$$

$$\frac{\partial^2 u(\varphi)}{\partial \varphi^2} = \frac{2}{Acos^3(\varphi)} + \frac{1}{Acos(\varphi)}$$

$$A = \sqrt{\frac{2mk}{l^2}} = r_0$$

Se cae al origen