Cuadrivectores

$$x^\mu = (\underbrace{ct}_{x^0}, x^1, x^2, x^3) \quad \text{contravariante}$$

Componente de un cuadrivector.

$\exists$ producto escalar:

$$x_\mu = (ct, -x^1, -x^2, -x^3) \quad \text{covariante.}$$

Producto escalar:

$$x^\mu x_\mu = \text{escalar. de Lorentz}$$

$$x^\mu y_\mu = \text{escalar} \quad \left(\begin{array}{l}\text{de lo mismo} \\ \text{en } S \text{ y } S' \\ \text{usando las} \\ \text{trafo. de Lorentz}\end{array}\right)$$

y^μ = transforma como x^μ $\Rightarrow$ y^μ es cuadrivector.

Ejemplo:

$$p^\mu = \left(\frac{E}{c}, \vec{p}\right)$$

$$A^\mu = \left(\frac{\phi}{c}, \vec{A}\right)$$

Onda plana en:

$$\omega t - \vec{k}\vec{r} = k^\mu x_\mu$$

$$k^\mu = (k, \vec{k}) \qquad k^\mu k_\mu = 0$$

$$= \left(\frac{\omega}{c}, \vec{k}\right)$$

número de onda

$$u^\mu = \frac{dx^\mu}{d\tau}$$

cuadrivelocidad

Relatividad

P2) Lorentz:

$$\begin{cases} x' = \gamma(x - vt) & (2) \\ t' = \gamma\left(t - \frac{vx}{c^2}\right) & (1) \end{cases}$$

$$y' = y \quad (3)$$

$$z' = z \quad (4)$$

esto vale si $v = v_{\parallel} = \vec{v}\cdot\hat{x}$

Sea $\vec{r} = \underbrace{\vec{r}_{\parallel}}_{\hat{v}(\vec{r}\cdot\hat{v})} + \underbrace{\vec{r}_{\perp}}_{\vec{r}-(\vec{r}\cdot\hat{v})\hat{v}}$ $\qquad \hat{v} = \frac{\vec{v}}{v}$

de (3) y (4) $\qquad \vec{r}_{\perp}' = \vec{r}_{\perp} \quad (5)$

de (1) $\qquad \boxed{t' = \gamma\left(t - \frac{\vec{r}\cdot\vec{v}}{c^2}\right)} \quad (6)$

de (2): $\qquad \vec{r}_{\parallel}' = \gamma\left(\vec{r}_{\parallel} - \vec{v}t\right) \quad (7)$

de (5) y (7): $\qquad \vec{r}' = \vec{r} - (\vec{r}\cdot\hat{v})\hat{v} + \gamma(\vec{r}\cdot\hat{v})\hat{v} - \gamma t\vec{v}$

$$\boxed{\vec{r}' = \vec{r} + (\gamma - 1)\underbrace{(\vec{r}\cdot\hat{v})\hat{v}}_{\vec{r}_{\parallel}} - \gamma t\vec{v}} \quad (8)$$

En la onda plana:

$$kct - \vec{k}\cdot\vec{r} = k^{\nu}x_{\nu}$$

$$k^{\nu} = (k, \vec{k}) = \left(\frac{\omega}{c}, \vec{k}\right) \quad (9)$$

uso (6): $\qquad \frac{\omega'}{c} = \gamma\left(\frac{\omega}{c} - \frac{\vec{k}\cdot\vec{v}}{c^2}\right)$

$$\therefore \quad \omega' = \gamma\left(\omega - \frac{\vec{k}\cdot\vec{v}}{c}\right)$$

Sea $\vec{k} = k\hat{k}$

$$\boxed{\omega' = \gamma\left(1 - \frac{\vec{v}}{c}\cdot\hat{k}\right)\omega}$$

La dirección de propagación es:

$$\hat{n} = \frac{\vec{k}}{k} \qquad k = \frac{\omega}{c}$$

$$\hat{k}' = \frac{\vec{k}'}{k'} = \frac{c}{\omega'}\vec{k}'$$

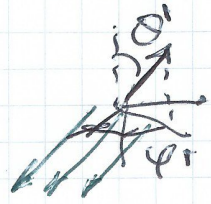
$$\therefore \quad \boxed{\hat{k}' = \frac{\hat{k} + (\gamma - 1)(\hat{k}\cdot\hat{v})\hat{v} - \gamma\vec{v}/c}{\gamma(1 - \vec{v}\cdot\hat{k}/c)}}$$

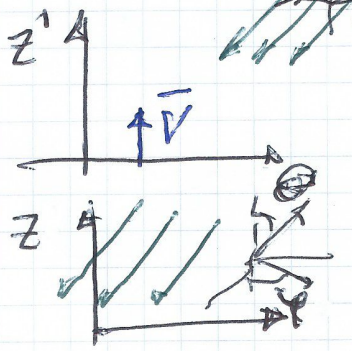
(se dividió por ω')

Caso $\vec{V} = V\hat{z}$ $\hat{K} = -(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

Efecto Doppler:

$$\hat{K}\cdot\vec{V} = -V\cos\theta$$





$$\nu' = \gamma\left(1 + \frac{V}{c}\cos\theta\right)\nu \quad (1)$$

S' se acerca a la fuente, la frecuencia aumenta si $\theta < \frac{\pi}{2}$, además $\gamma = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} > 1$

Aberración de la luz:

Comp-z:

$$\beta = \frac{V}{c}$$

$$\cos\theta' = \frac{\cos\theta + \beta}{1 + \beta\cos\theta} \quad (2)$$

Dividiendo Comp x entre y:

$$\varphi = \varphi' \quad (3)$$

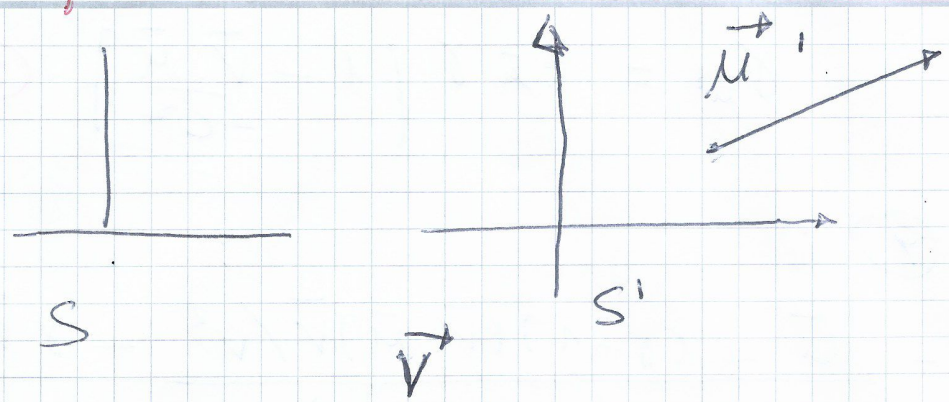
de (2):

$$\operatorname{ctg}^2\left(\frac{\theta'}{2}\right) = \frac{1+\cos\theta'}{1-\cos\theta'} = \frac{(1+\beta)\cancel{(1+\cos\theta)}}{(1-\beta)\cancel{(1-\cos\theta)}}$$

$$\begin{cases} 1+\cos\theta = 2\cos^2\frac{\theta}{2} \\ 1-\cos\theta = 2\sin^2\frac{\theta}{2} \end{cases}$$

$$\operatorname{tg}\left(\frac{\theta'}{2}\right) = \sqrt{\frac{1-\beta}{1+\beta}}\,\operatorname{tg}\left(\frac{\theta}{2}\right)$$

(Cálculo alternativo) Transformación de velocidades



queremos hallar $\vec{u}'$ visto desde S: $\vec{u}$

$$U'^{\mu} = \frac{d}{d\tau}(c\,t, \vec{r}) \quad (1)$$

pero: $d\tau = dt \cdot \gamma_{\vec{u}}^{-1}$ $\qquad \gamma_{\vec{u}} = \dfrac{1}{\sqrt{1 - \frac{|\vec{u}|^2}{c^2}}}$

$$U^{\mu} = \gamma_{\vec{u}}(c, \vec{u})$$

$$\gamma_{\vec{u}}^2 c^2 - \gamma_{\vec{u}}^2 u^2$$

$$= \gamma_u^2 \cdot c^2 \left(1 - \frac{u^2}{c^2}\right)$$

$$U^{\mu} U_{\mu} = c^2 \quad \text{(invariante)} \quad (2)$$

$$\gamma_{\vec{u}'} c = \gamma(\gamma_{\vec{u}})\left(c - \frac{\vec{u} \cdot \vec{V}}{c}\right) \quad (3)$$

$$\gamma_{\vec{u}'} \vec{u}' = \gamma_{\vec{u}}\left[\vec{u} + (\gamma - 1)(\vec{V} \cdot \vec{u})\frac{\vec{V}}{c^2} - \gamma \vec{V}\right] \quad (4)$$

de (3):

$$\gamma_{\bar{u}'} = \gamma_{\bar{u}}\,\gamma\left(1 - \frac{\bar{v}\cdot\bar{u}}{c^2}\right) \quad (5)$$

(5) en (4):

$$\bar{u}' = \frac{\bar{u} + (\gamma-1)(\bar{v}\cdot\bar{u})\bar{v}/v^2 - \gamma\bar{v}}{\gamma\left(1 - \frac{\bar{v}\cdot\bar{u}}{c^2}\right)} \quad (6)$$

con 5.13) de Mizoti

Si $\bar{u} = \lambda\bar{v}$

$$\bar{v}\cdot\bar{u} = \lambda v^2$$

$$\bar{u}' = \frac{\bar{u} - \bar{v}}{1 - \frac{\bar{v}\cdot\bar{u}}{c^2}}$$

$\vec{v} = v\hat{z}$

$$u_z' = \frac{\gamma(u_z - v)}{\gamma\left(1 - \frac{v u_z}{c^2}\right)}$$

ok!

Aberración de la luz:

$$\bar{u} = c\hat{n} \qquad \bar{u}' = c\hat{n}'$$

$$\hat{n}' = \frac{\hat{n} + (\gamma-1)(\bar{v}\cdot\hat{n})\vec{v}/v^2 - \gamma\vec{v}/c}{\gamma\left(1 - \frac{\bar{v}\cdot\hat{n}}{c}\right)}$$