Partícula Libre

Relativista

Queremos usar el principio de mínima acción.

Si las ecuaciones se escriben igual en todos los sistemas

$\Rightarrow$ la acción debe ser un escalar de Lorentz (la misma en todos los sistemas)

$$S = \int \alpha \, d\tau$$

pero

$$\alpha \, d\tau = \alpha \sqrt{1 - \frac{|\vec{u}|^2}{c^2}} \, dt$$

$$\simeq \alpha \left(1 - \frac{|\vec{u}|^2}{2c^2}\right) dt$$

$$\simeq \mathcal{L}_{nR} \, dt.$$

$$\mathcal{L}_{nr} = \alpha - \frac{\alpha |\vec{u}|^2}{2c^2} = cte + \frac{m|\vec{u}|^2}{2}$$

$$\therefore \quad \alpha = -mc^2$$

$$S = -\int mc^2 \, dt \sqrt{1 - \frac{|u|^2}{c^2}}$$

$$\mathcal{L} = -mc^2 \sqrt{1 - \frac{|\bar{u}|^2}{c^2}}$$

$$\vec{p} = \frac{\partial \mathcal{L}}{\partial \vec{u}} = -\left(-\frac{1}{2}\right) \frac{mc^2}{\left(1 - \frac{|\bar{u}|^2}{c^2}\right)^{1/2}} \left(\frac{2\vec{u}}{c^2}\right)$$

$$\vec{p} = \frac{m\vec{u}}{\sqrt{1 - \frac{|\bar{u}|^2}{c^2}}}$$

$$\frac{p^2}{c^2} = \frac{m^2 \frac{|\bar{u}|^2}{c^2}}{1 - \frac{|\bar{u}|^2}{c^2}} \Rightarrow \frac{|\bar{u}|^2}{c^2} = \frac{p^2}{p^2 + m^2c^2}$$

$$E = \bar{p}\cdot\bar{u} - \mathcal{L} = \frac{m|\bar{u}|^2}{\sqrt{1 - \frac{|\bar{u}|^2}{c^2}}} + mc^2\sqrt{1 - \frac{|\bar{u}|^2}{c^2}}$$

$$E = \frac{m\left[|\bar{u}|^2 + c^2\left(1 - \frac{|\bar{u}|^2}{c^2}\right)\right]}{\sqrt{1 - \frac{|\bar{u}|^2}{c^2}}}$$

$$\boxed{E = \frac{mc^2}{\sqrt{1 - \frac{|\bar{u}|^2}{c^2}}}}$$

relación entre E y $\vec{p}$:

$$E = \frac{mc^2}{\sqrt{\frac{m^2c^2}{p^2 + m^2c^2}}} = c\sqrt{p^2 + m^2c^2}$$

$$p^\mu = (E, \vec{p}) \qquad \Rightarrow \boxed{E^2 - \bar{p}^2 = m^2c^2}$$

Prob 3 — Partícula en campo em

No relativista: $\mathcal{L} = \frac{mv^2}{2} - (q\phi - q\vec{v}\cdot\vec{A})$

Relativista: $\mathcal{L} = -mc^2\sqrt{1-\frac{v^2}{c^2}} - q\phi + q\vec{v}\cdot\vec{A}$

$q\phi - q\vec{v}\cdot\vec{A} = u^\nu A_\nu \sqrt{1-\frac{v^2}{c^2}}$ (suma sobre índices repetidos)

con $u^\nu = \frac{1}{\sqrt{1-v^2/c^2}}(c, \vec{v}) = \frac{dx^\nu}{d\tau}$ $\quad \nu = \underbrace{0}_{\text{temporal}}, \underbrace{1,2,3}_{\text{espaciales}}$

cuadrivelocidad

$\underbrace{A^\nu}_{A} = \left(\frac{\phi}{c}, \vec{A}\right) \Rightarrow \underbrace{A_\nu}_{A^*} = \left(\frac{\phi}{c}, -\vec{A}\right)$

(dual)

Contravariante — covariante

$S_{\text{interacción}} = \int \overbrace{u^\nu A_\nu}^{\text{escalar de Lorentz}} \underbrace{d\tau}_{\sqrt{1-\frac{v^2}{c^2}}\cdot dt}$

$S_{\text{interacción}} = \int (q\phi - q\vec{v}\cdot\vec{A})\, dt$

Euler Lagrange:

$\frac{d}{dt}\left(\frac{m\vec{v}}{\sqrt{1-v^2}}\right) = q(\vec{E} + \vec{v}\times\vec{B})$

donde: $\bar{B} = \nabla \times \bar{A}$

$$\bar{E} = -\nabla\phi + \frac{1}{c}\frac{\partial \bar{A}}{\partial t}$$

Conservación de H si $\mathcal{L}$ no dep. de t: $H = \bar{v}\cdot\frac{\partial \mathcal{L}}{\partial \bar{v}} - \mathcal{L} = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} + q\phi$

Prob. 4) Caso ~~$\phi = 0$~~ La parte de $\bar{A}$ se cancela.

$$\bar{E} = 0 \qquad \bar{B} = B_0\hat{z}$$

$$\bar{v}\times\bar{B} = (v_x\hat{x} + v_y\hat{y} + v_z\hat{z})\times B_0\hat{z}$$

$$\bar{v}\times\bar{B} = B_0(-v_x\hat{y} + v_y\hat{x})$$

$$\frac{d}{dt}\left(\frac{m v_z}{\sqrt{1-\frac{v^2}{c^2}}}\right) = 0 \qquad (1)$$

$$\frac{d}{dt}\left(\frac{m v_x}{\sqrt{1-\frac{v^2}{c^2}}}\right) = qB_0 v_y \qquad (2)$$

$$\frac{d}{dt}\left(\frac{m v_y}{\sqrt{1-\frac{v^2}{c^2}}}\right) = -qB_0 v_x \qquad (3)$$

Conservación de la energía $\mathcal{E}$ ($\phi = 0$)

$$\mathcal{E} = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = cte \quad (4) \quad (\text{parte cinética})$$

(4) en (1): $v_z = cte.$

(4) en (2) + i(3): $\frac{d}{dt}(v_x + iv_y) = -i\frac{qB_0c^2}{\mathcal{E}}(v_x + iv_y)$

Sea $\quad Z = V_x + iV_y$

$$\frac{d}{dt} Z = -i\omega Z$$

Con $\boxed{\omega = \frac{qBc^2}{\mathcal{E}}} \longrightarrow \frac{qB}{m}$

$\underline{}$ no relativista

solución : $\quad Z = z_0 e^{-i\omega t}$

$z_0 = A e^{-i\varphi_0} \qquad Z = A e^{-i(\omega t + \varphi_0)}$

$$= V_x + iV_y$$

$$\therefore \quad \begin{cases} V_x = A\cos(\omega t + \varphi_0) = \dot{x} & (5) \\ V_y = -A\sin(\omega t + \varphi_0) = \dot{y} & (6) \end{cases}$$

de (5) $\qquad x(t) = x_0 + \frac{A}{\omega}\sin(\omega t + \varphi_0)$

$$y(t) = y_0 + \frac{A}{\omega}\cos(\omega t + \varphi_0)$$

$$\therefore \quad (x - x_0)^2 + (y - y_0)^2 = \frac{A^2}{\omega^2}$$

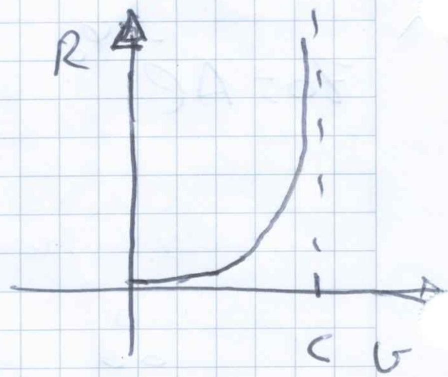
de (5) y (6) $\quad v^2 = A^2 \quad$ en

Como: $\quad \mathcal{E} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$R^2 = \frac{A^2}{\omega^2} = \frac{}{q^2 B^2 c^2}\left(1 - \frac{}{\mathcal{E}^2}\right)$$

$$R^2 = \frac{v^2}{q^2 B^2 \cancel{c^4}} \cdot \frac{m^2 \cancel{c^4}}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$R = \frac{m v}{q B \sqrt{1 - \frac{v^2}{c^2}}}$$



Si $v \to c$ $\quad R \to \infty$

ó $\boxed{R = \frac{v}{c}\frac{\mathcal{E}}{qBc}}$

Caso protones: $\quad mc^2 = 938{,}26 \text{ MeV}$

$$\mathcal{E} = \frac{mc^2}{\sqrt{1-\beta^2}}$$

ojo!

(Máximo $\simeq 6.5$ TeV)

$\mathcal{E} = 7 \text{ TeV} = 7 \cdot 10^6 \text{ MeV}$

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{7 \cdot 10^6}{938{,}26} = 7460$$

$R = 4300$ m

$\beta = 0{,}99999991$

$$R \simeq \frac{\mathcal{E}}{qBc} = \frac{7 \cdot 10^{12} \text{ eV}}{e\, B\, 3 \cdot 10^8 \text{ m/s}}$$

$\boxed{B = 4{,}86 \text{ Tesla}}$