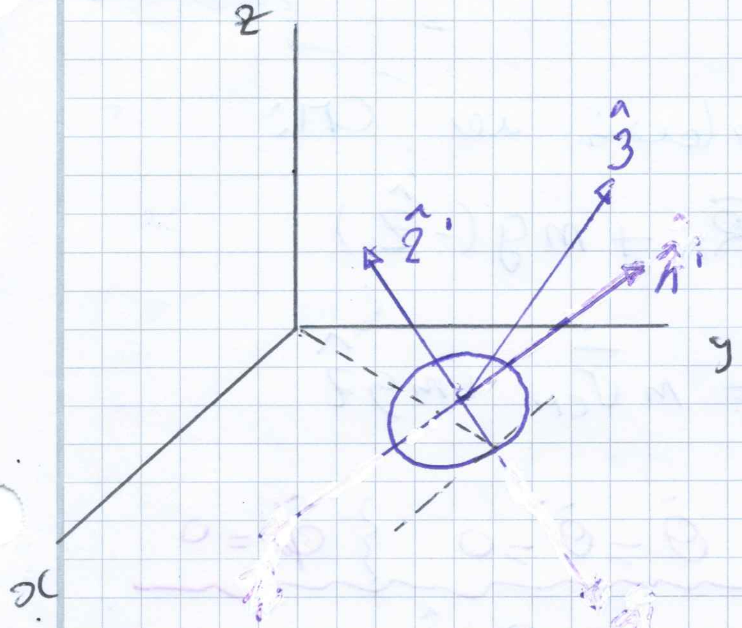
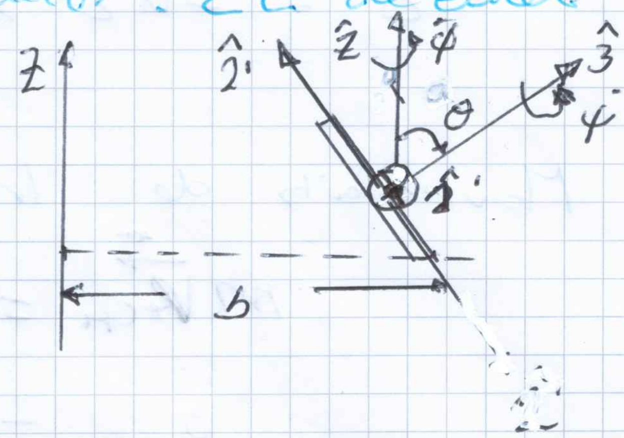
Moneda que rueda sin deslizar en círculos: Ec de Euler





$$\vec\Omega = \dot\phi\,\hat z + \dot\psi\,\hat 3 + \dot\theta\,\hat 1'$$

$$\hat z = \hat 3\cos\theta + \hat 2'\sin\theta$$

$$\vec\Omega = \dot\theta\,\hat 1' + \dot\phi\sin\theta\,\hat 2' + (\dot\psi + \dot\phi\cos\theta)\hat 3, \quad \vec\omega = \dot\theta\,\hat 1' + \dot\phi\sin\theta\,\hat 2' + \dot\phi\cos\theta\,\hat 3$$

Condición de rodadura:

$$\vec v_{CM} = \underbrace{\vec v_Q}_{0} + \vec\Omega \times a\,\hat 2'$$

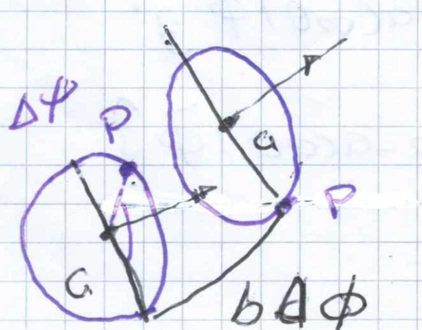
$$\left\{ \begin{aligned} \vec r_{CM} &= (b - a\cos\theta)\,\hat e_r + a\sin\theta\,\hat e_z \\ \dot{\vec r}_{CM} &= (b - a\cos\theta)\,\hat 1' + a\underbrace{(\cos\theta\,\hat e_r + \sin\theta\,\hat e_z)}_{3}\dot\theta \end{aligned} \right.$$

$$\vec v_{CM} = a\dot\theta\,\hat 3 - a(\dot\psi + \dot\phi\cos\theta)\,\hat 1'$$

$$= a\dot\theta\,\hat 3 + \dot\phi(b - a\cos\theta)\,\hat 1'$$

$$\therefore \quad -a\dot\psi = b\dot\phi$$

$$-a\,d\psi = b\,d\phi$$



Torque c/r al CM en ejes $\hat{1}', \hat{2}', \hat{3}$:

$$\bar{\tau} = -a\hat{2}' \times \vec{R}$$

$\vec{R}$ = fuerza de reacción

Movimiento de Traslación del CM:

$$m\dot{\vec{V}}_{CM} = \vec{R} + mg(-\hat{z})$$

$$\therefore \quad \bar{R} = m\dot{\vec{V}}_{CM} + mg\hat{z}$$

Caso estacionario: $\dot{\theta} = \ddot{\theta} = 0$ } $\ddot{\phi} = 0$

$$\bar{V}_{CM} = \dot{\phi}(b - a\cos\theta)\hat{1}'$$

$$\frac{d\bar{V}_{CM}}{dt} = \dot{\phi}(b - a\cos\theta)\frac{d\hat{1}}{dt} + \ddot{\phi}(b - a\cos\theta)\hat{1}'$$

$$\frac{d\hat{1}'}{dt} = \vec{\omega} \times \hat{1}' = -\dot{\phi}\sin\theta\,\hat{3} + \dot{\phi}\cos\theta\,\hat{2}'$$

$$\therefore \quad \frac{d\bar{V}_{CM}}{dt} = \dot{\phi}^2(b - a\cos\theta)(-\sin\theta\,\hat{3} + \cos\theta\,\hat{2}')$$

(aceleración centrípeta)

$$\bar{\tau} = ma\dot{\phi}^2(b - a\cos\theta)\sin\theta\,\hat{1}' - amg\cos\theta\,\hat{1}'$$
$$+\ddot{\phi}(b - a\cos\theta)a\,\hat{3}$$

$$\Omega = \dot{\phi}\sin\theta\,\hat{2}' - \frac{1}{a}(b - a\cos\theta)\dot{\phi}\,\hat{3}$$

$$\bar{L} = I\dot{\phi}\sin\theta\,\hat{2}' - \frac{I_3}{a}(b - a\cos\theta)\dot{\phi}\,\hat{3}$$

Ecuaciones de Euler:

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

$$\underbrace{\left.\frac{d\vec{L}}{dt}\right|_{rot}}_{=0 \text{ (estacionario)}} + \vec{\omega}\times\vec{L} = \vec{\tau}$$

con $\vec{\omega} = \dot{\phi}\sin\theta\,\hat{2}' + \dot{\phi}\cos\theta\,\hat{3}$

$$-\frac{I_3}{a}\sin\theta\,(b-a\cos\theta)\dot{\phi}^2\,\hat{1}' - I\dot{\phi}^2\sin\theta\cos\theta\,\hat{1}'$$
$$= ma\dot{\phi}^2(b-a\cos\theta)\sin\theta\,\hat{1}'$$
$$- mag\cos\theta\,\hat{1}'$$
$$+ a\ddot{\phi}(b-a\cos\theta)\,\hat{3}$$

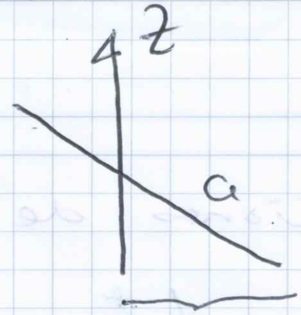
Comp 3 $\boxed{\ddot{\phi}=0}$

Com $\hat{1}'$: $$\dot{\phi}^2 = \frac{g\cot\theta}{\left[\left(\frac{I_3}{ma^2}+1\right)(b-a\cos\theta) + \frac{I}{ma}\cos\theta\right]}$$

Caso Disco uniforme: $I_3 = \frac{ma^2}{2}$, $I = \frac{ma^2}{4}$

$$\dot{\phi}^2 = \frac{4g\cot\theta}{[6b-5a\cos\theta]}$$

Caso especial



$b = a\cos\theta$

CM fijo.

$$\dot\phi^2 = \frac{4g}{a\sin\theta}$$

en este caso: $\vec\Omega = \dot\phi \sin\theta\, \hat{2}'$ a lo largo del eje instantaneo de rotación.

pues $\Omega_3 = 0 = \dot\psi + \dot\phi\cos\theta$.

Si $\theta \to 0$ $\dot\phi \to \infty$ (singularidad)

$$\vec\Omega \to 0$$

y visto desde arriba

$$\Omega_z = \dot\phi + \dot\psi\cos\theta$$

pero $\dot\psi = -\frac{b}{a}\dot\phi = -\frac{a\cos\theta}{a}\dot\phi$

$$\therefore \quad \Omega_z = \dot\phi\,(1-\cos\theta)$$

Si $\theta \to 0$ $\Omega_z \to 0$ deja de rotar la cara vista desde arriba.