2) $\vec{E} = -\frac{E_0 t}{a}\hat{e}_x \longrightarrow \varphi = \frac{E_0 t x}{a}$ $\quad (\vec{E} = -\vec{\nabla}\varphi)$

(A lo sumo $\vec{\nabla}\times\vec{B} \sim \frac{\partial \vec{E}}{\partial t} = cte \Rightarrow \frac{\partial \vec{A}}{\partial t} = 0$)

¡Esta justif. No hace falta!

$V = q\varphi = \frac{qE_0 t x}{a} \longrightarrow \boxed{\mathcal{L} = \frac{1}{2}m\dot{x}^2 - \frac{qE_0 t x}{a}}$

Traslaciones: $x \mapsto x' = x + \epsilon L \Rightarrow \dot{x} \mapsto \dot{x}' = \dot{x}$ $\quad (t' \mapsto t)$

$\Rightarrow \mathcal{L}' = \frac{1}{2}m\dot{x}'^2 - \frac{qE_0 t' x'}{a} = \frac{1}{2}m\dot{x}^2 - \frac{qE_0 t x}{a} + \epsilon\left(-\frac{qE_0 t L}{a}\right)$

Llamo $f_{tras} = -\frac{qE_0 t^2 L}{2a} \Rightarrow \mathcal{L}' = \mathcal{L} + \frac{d f_{tras}}{dt}$ con $f_{tras}(t)$

$\Rightarrow$ ¡Simetría! $\Rightarrow$ Teo Noether: $C_1 = p_x g_x - f_{tras}$

$\Rightarrow C_1 = m\dot{x}L + \frac{qE_0 t^2 L}{2a} \Rightarrow \frac{C_1}{L} = \tilde{C}_1 = m\dot{x} + \frac{qE_0 t^2}{2a}$

$\Rightarrow \boxed{\dot{x} = \left(\tilde{C}_1 - \frac{qE_0 t^2}{2a}\right)\frac{1}{m}}$

Boost Galileo: $x \mapsto x' = x + \epsilon v t \Rightarrow \dot{x} \mapsto \dot{x}' = \dot{x} + \epsilon v$ $\quad (t' \mapsto t)$

$\Rightarrow \mathcal{L}' = \frac{1}{2}m\dot{x}'^2 - \frac{qE_0 t' x'}{a} = \frac{1}{2}m(\dot{x} + \epsilon v)^2 - \frac{qE_0 t(x + \epsilon v t)}{a}$

$= \frac{1}{2}m\dot{x}^2 - \frac{qE_0 t x}{a} +$ [tachado] $+ \epsilon\left(m\dot{x}v - \frac{qE_0 t^2 v}{a}\right) + \frac{\epsilon^2 m v^2}{2}$

$\mathcal{O}(\epsilon^2)$

$\Rightarrow f_{Galileo} = m x v - \frac{qE_0 t^3 v}{3a} \Rightarrow \mathcal{L}' = \mathcal{L} + \frac{d f_{Galileo}}{dt}$

$\Rightarrow$ Teo Noether: $\frac{C_2}{v} = m\dot{x}t - m x + \frac{qE_0 t^3}{3a} = \tilde{C}_2$

$$\Rightarrow x = \dot{x} t + \frac{qE_0 t^3}{3m\tau} - \tilde{C}_2 = \frac{\tilde{C}_1}{m} t - \frac{qE_0 t^3}{2\tau m} + \frac{qE_0 t^3}{3\tau m} - \tilde{C}_2$$

$$\boxed{x = \frac{\tilde{C}_1 t}{m} - \frac{qE_0 t^3}{6\tau m} - \tilde{C}_2}$$

$\rightarrow$ Con $x(0) = 0$, $x(\tau) = 0$

$\rightsquigarrow \tilde{C}_1, \tilde{C}_2$

$x(0)=0 \rightarrow \tilde{C}_2 = 0, \quad x(\tau)=0 \Rightarrow 0 = \frac{\tilde{C}_1 \tau}{m} - \frac{qE_0 \tau^3}{6\tau m} \rightarrow \tilde{C}_1 = \frac{qE_0\tau}{6}$

$\rightarrow x(t) = \frac{qE_0 \tau}{6m} t - \frac{qE_0 t^3}{6m\tau} = \boxed{\frac{qE_0}{6m} t\left(\tau - t^2/\tau\right)}$

b) $\begin{cases} x(t) = at + bt^3 \\ \dot{x}(t) = a + 3bt^2 \end{cases}$ $\quad x(0)=0$ ✓ $\quad x(\tau)=0 \rightarrow 0 = a\tau + b\tau^3 \rightarrow \boxed{a = -b\tau^2}$

$\Rightarrow \begin{cases} x(t) = b(t^3 - t\tau^2) \\ \dot{x}(t) = b(3t^2 - \tau^2) \end{cases}$ Ahora $\mathcal{L} = \frac{1}{2} m \dot{x}^2 - \frac{qE_0 t x}{\tau}$

$\Rightarrow \mathcal{L} = \frac{1}{2} m b^2 (3t^2 - \tau^2)^2 - \frac{qE_0 b}{\tau}(t^4 - t^2\tau^2)$

$\rightarrow \mathcal{L} = \frac{1}{2} m b^2 (9t^4 + \tau^4 - 6t^2\tau^2) - \frac{qE_0 b}{\tau}(t^4 - t^2\tau^2) =$

$= \left(\frac{9}{2} m b^2 - \frac{qE_0 b}{\tau}\right) t^4 + \left(-3mb^2\tau^2 + qE_0 b\tau\right) t^2 + \frac{1}{2} m b^2 \tau^4$

$\Rightarrow S = \left(\frac{9}{2} m b^2 - \frac{qE_0 b}{\tau}\right)\frac{\tau^5}{5} + \left(qE_0 b\tau - 3mb^2\tau^2\right)\frac{\tau^3}{3} + \frac{1}{2} m b^2 \tau^5 \quad \left(S = \int_0^\tau \mathcal{L}\, dt\right)$

$\rightarrow \frac{\partial S}{\partial b} = \left(9mb - \frac{qE_0}{\tau}\right)\frac{\tau^5}{5} + \left(qE_0\tau - 6mb\tau^2\right)\frac{\tau^3}{3} + mb\tau^5 = 0$

$\rightarrow b\left(\frac{9m\tau^2}{5} - 2m\tau^2 + m\tau^2\right) = \frac{qE_0\tau}{5} - \frac{qE_0\tau}{3}$

$\rightarrow b\left(+\frac{4m\tau^2}{5}\right) = -\frac{2qE_0\tau}{15} \Rightarrow \boxed{b = \frac{-qE_0}{6m\tau}}$

$\Rightarrow x(t) = \frac{qE_0}{6m\tau} t(\tau^2 - t^2) = \boxed{\frac{qE_0}{6m} t\left(\tau - t^2/\tau\right)}$ ~ Igual que el pto a!