$$\vec V_p = \vec V_o' + \vec\omega' \times (\vec r_p \underbrace{- \vec r_o + \vec r_o}_{} - \vec r_o') = \underbrace{\vec V_o' + \vec\omega' \times (\vec r_o - \vec r_o')}_{\vec V_o} + \vec\omega' \times (\vec r_p - \vec r_o)$$

$$\vec V_p = \vec V_o + \vec\omega' \times (\vec r_p - \vec r_o) \Rightarrow \boxed{\vec\omega = \vec\omega'}$$

b) Si $\vec V_o \perp \vec\Omega$, $\vec V_o$ es de la forma $\vec V_o = \vec\Omega \times \vec b$

en otro sistema:

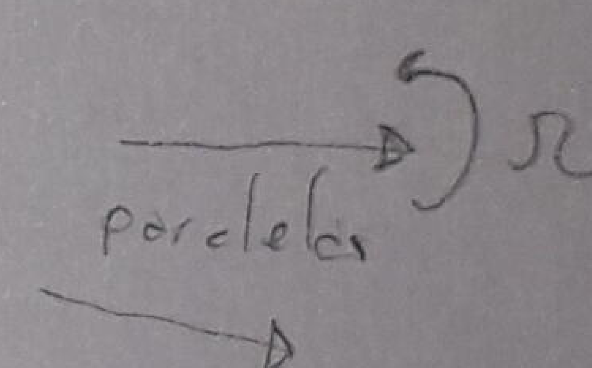
$$\vec V_o' = \vec V_o + \vec\Omega \times (\vec r_o' - \vec r_o) = \vec\Omega \times \vec b + \vec\Omega \times (\vec r_o' - \vec r_o)$$

$$\vec V_o' = \vec\Omega \times (\vec b + \vec r_o - \vec r_o') \; ; \quad \text{si } \vec b + \vec r_o \neq \vec r_o' \Rightarrow \boxed{\vec V_o' \perp \vec\Omega}$$

c) Sale del punto anterior si $\vec b + \vec r_o = \vec r_o' \Rightarrow \boxed{\vec V_o' = 0}$

d) Supongamos que tenemos $\vec r_s = \vec r_o' + \lambda \vec\Omega$ con $\vec r_o' = \vec b + \vec r_o$

$$\Rightarrow \vec V_{s'} = \vec V_o' + \vec\Omega \times (\vec r_s - \vec r_o') = \vec\Omega \times \lambda \vec\Omega = 0$$



e)

$$\vec V_p = \vec V_o + \vec\Omega \times \vec r_{op}$$

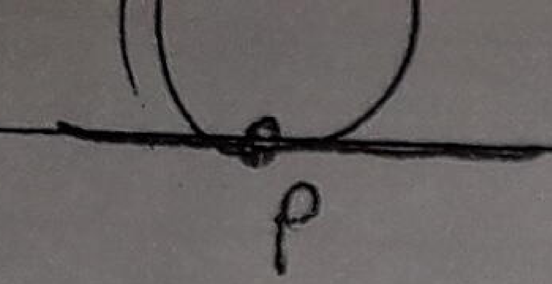
$$\vec V_p - \vec V_o = \vec\Omega \times \vec r_{op}$$

$A \times (B \times C) = (A \cdot C) B - (A \cdot B) C$

$$(\vec V_p - \vec V_o) \times \vec\Omega = (\vec\Omega \times \vec r_{op}) \times \vec\Omega = -\vec\Omega \times (\vec\Omega \times \vec r_{op}) = +\left[(\vec\Omega \cdot \vec\Omega) \vec r_{op} - (\vec r_{op} \cdot \vec\Omega) \vec\Omega \right]$$

$$\vec V_p \times \vec\Omega - \vec V_o \times \vec\Omega = +\left[\Omega^2 \vec r_{op} - (\vec r_{op} \cdot \vec\Omega) \vec\Omega \right] \Rightarrow \Omega^2 \vec r_{op} = -\vec V_o \times \vec\Omega + \vec V_p \times \vec\Omega + (\vec r_{op} \cdot \vec\Omega) \vec\Omega$$

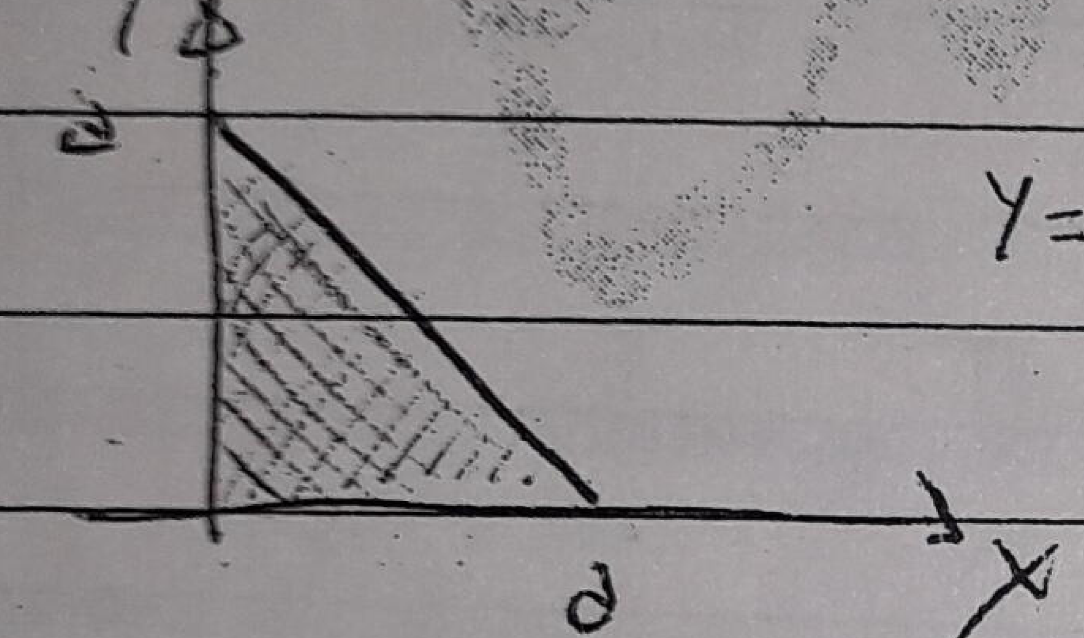
$$\boxed{\vec r_{op} = \frac{1}{\Omega^2} \left[\vec V_o \times \vec\Omega + \vec V_p \times \vec\Omega + (\vec r_{op} \cdot \vec\Omega) \vec\Omega \right]}$$



$V_P = 0$

2 Hallar el centro de masa y calcular el tensor de inercia respecto del mismo, para el cuerpo plano de la figura (su densidad es ρ). Hallar los ejes principales de inercia y expresar el tensor de inercia en dichos ejes.

$\int_0^a \int_{a-x}^0 \rho\, dx\, dy = -\rho(a-x)dx = -\rho\left(ax - \frac{x^2}{2}\right)\Big|_0^a$



$Y = a - X$

$$M = \int \rho\, dx\, dy = \int_0^a \int_0^{a-x} \rho\, dx\, dy = \rho \int_0^a (a-x)\, dx = \rho\left(aX - \frac{X^2}{2}\right)\Bigg|_0^a$$

$$M = \frac{\rho a^2}{2}$$

¡Ojo se reemplaza después de integrar!

Calculo del c.m.

$$X_{cm} = \frac{1}{M}\int_0^a \int_0^{a-X} \rho\, x\, dx\, dy = \frac{\rho}{M}\int_0^a x(a-x)\, dx = \frac{2}{a^2}\left[\frac{a x^2}{2} - \frac{x^3}{3}\right]\Bigg|_0^a = 2a\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{2a}{6} = \frac{a}{3}$$

$$Y_{cm} = \frac{a}{3}$$

$$\Rightarrow \vec{r}_{cm} = \frac{a}{3}(1,1)$$

$$Z_{cm} = 0 \longrightarrow \vec{r}_{cm} = \frac{a}{3}(1,1,0)$$

TRABAJOS EN EL ACTO Y POR ENCARGO *

$$I_{je} = \sum_i \left[m_i r_i^2 \delta_{je} - r_j r_e \right] \qquad dm = \rho\, dx\, dy\, dz$$

$$I_{je} = \int dm \left(r_i^2 \delta_{je} - r_j r_e \right)$$

Cálculo del tensor de inercia respecto del c.m

$$I_{xx} = \int \rho \left\{ \left(x - \frac{a}{3}\right)^2 + \left(y - \frac{a}{3}\right)^2 - \left(x - \frac{a}{3}\right)^2 \right\} dx\,dy = \rho \int_0^a \int_0^{a-x} \left(y - \frac{a}{3}\right)^2 dx\,dy = \rho \int_0^a \frac{1}{3}\left(y - \frac{a}{3}\right)^3 \Big|_0^{a-x} dy =$$

$$= \frac{1}{3}\rho \int_0^a \left\{ \left(a - x - \frac{a}{3}\right)^3 - \left(-\frac{a}{3}\right)^3 \right\} dx = \frac{1}{3}\rho \int_0^a \left\{ \left(\frac{2}{3}a - x\right)^3 + \frac{a^3}{27} \right\} dx$$

$$I_{xx} = \frac{1}{3}\rho \left\{ -\frac{1}{4}\left(\frac{2}{3}a - x\right)^4 + \frac{a^3 x}{27} \right\} \Big|_0^a = \frac{1}{3}\rho \left\{ -\frac{1}{4}\left(-\frac{1}{3}a\right)^4 + \frac{1}{4}\left(\frac{2}{3}a\right)^4 + \frac{a^4}{27} \right\} = \frac{1}{3}\rho \left\{ -\frac{1}{4}\frac{a^4}{81} + \frac{1}{4}\frac{a^4 16}{81} + \frac{a^4}{27} \right\}$$

$$I_{xx} = \frac{1}{3}\rho \left\{ \frac{15}{4 \cdot 81} + \frac{12}{4 \cdot 81} \right\} a^4 = \frac{1}{36}\rho a^4 = \frac{1}{18} M a^2$$

$$I_{xy} = -\rho \int_0^a \int_0^y \left(x - \frac{a}{3}\right)\left(y - \frac{a}{3}\right) dx\,dy = -\frac{\rho}{2} \int_0^a \left(x - \frac{a}{3}\right)\left(y - \frac{a}{3}\right)^2 \Bigg|_0^{a-x} dx$$

$$I_{xy} = -\frac{1}{2}\rho \int_0^a \left\{ \left(x - \frac{a}{3}\right)\left(\frac{2a}{3} - x\right)^2 - \left(x - \frac{a}{3}\right)\left(-\frac{a}{3}\right)^2 \right\} dx$$

$$I_{xy} = -\frac{1}{2}\rho \int_0^a \left\{ \left(x - \frac{a}{3}\right)\left(\frac{4}{9}a^2 - \frac{4}{3}ax + x^2\right) - \frac{a^2}{9}\left(x - \frac{a}{3}\right) \right\} dx$$

$$I_{xy} = -\frac{1}{2}\rho \int_0^a \left\{ \frac{4}{9}a^2x - \frac{4}{3}ax^2 + x^3 - \frac{4}{27}a^3 + \frac{4}{9}a^2x - \frac{a}{3}x^2 - \frac{a^2}{9}x + \frac{a^3}{27} \right\} dx$$

$$I_{xy} = -\frac{1}{2}\rho \int_0^a \left\{ \frac{7}{9}a^2x - \frac{5}{3}ax^2 + x^3 - \frac{1}{9}a^3 \right\} dx$$

$$I_{xy} = -\frac{1}{2}\rho \left\{ \frac{7}{18}a^4 - \frac{5}{9}a^4 + \frac{a^4}{4} - \frac{1}{9}a^4 \right\} = -\frac{1}{2}\rho a^4 \left\{ \frac{14 - 20 + 9 - 4}{36} \right\} = \frac{1}{72}$$

$$I_{xy} = -\rho\left[\frac{1}{12} + \frac{1}{8} - \frac{2}{9} - \frac{1}{18} - \frac{1}{6} + \frac{2}{18}\right]$$

$$\Rightarrow \quad \mathbb{I} = \frac{1}{72}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\rho a^4$$

Busco los momentos principales de inercia

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0 \Rightarrow (2-\lambda)^2 - 1 = 0 \Rightarrow \lambda - 2 = \pm 1 \Rightarrow \lambda_1 = 3 \,;\, \lambda_2 = 1$$

Busco la dirección de los ejes principales de inercia

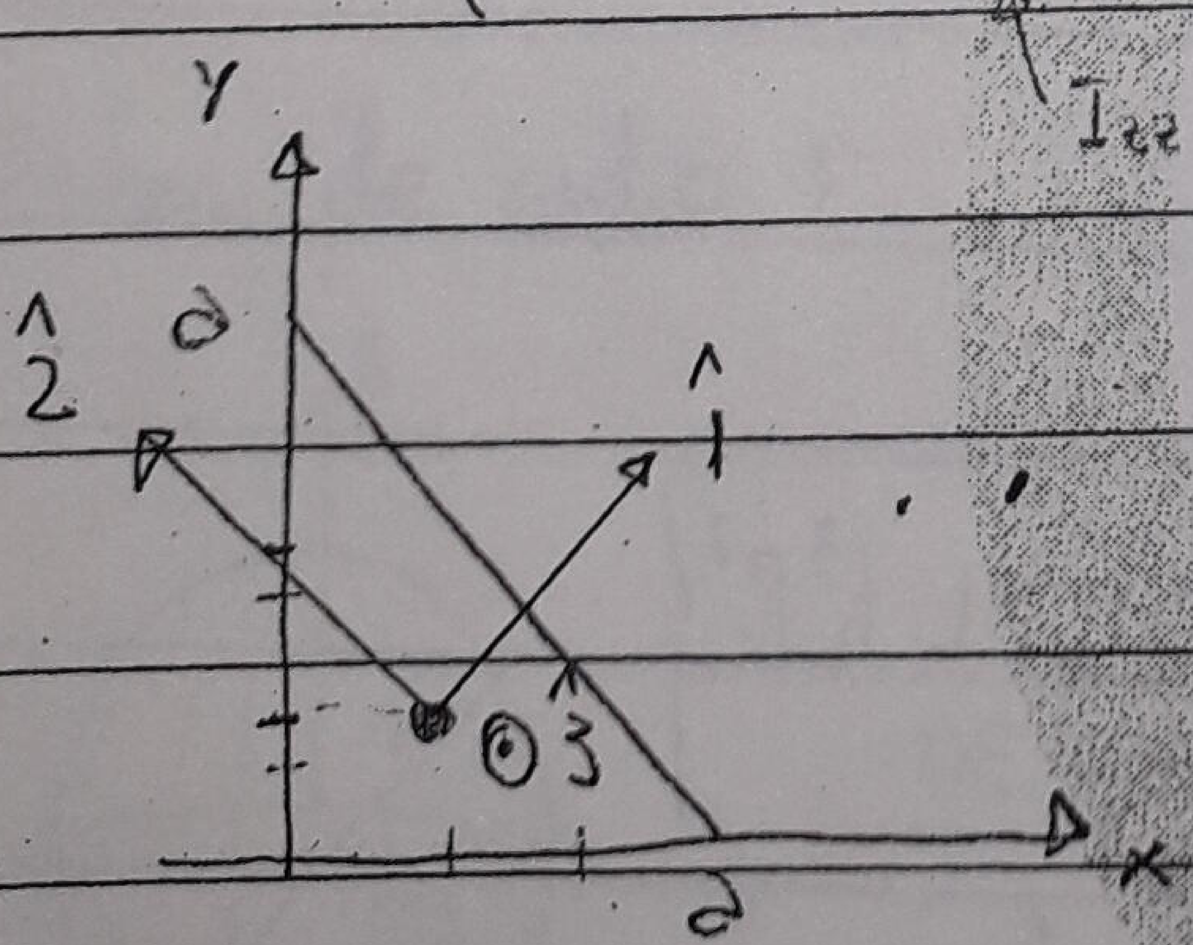
$$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow x_1 = x_2 \Rightarrow \bar{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\frac{1}{\sqrt{2}} \rightarrow \bar{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\frac{1}{\sqrt{2}}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow x_1 = -x_2 \Rightarrow V_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}\frac{1}{\sqrt{2}} \rightarrow \bar{V}_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}\frac{1}{\sqrt{2}}$$

$$\bar{V}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$I = \frac{\rho a^4}{72}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$I_{zz} = I_{xx} + I_{yy}$



Determinar los ejes principales de inercia y calcular el tensor de inercia r...
...tro de masa para los siguientes sistemas:

$$X_A = \frac{l}{2}\cos\theta_A \quad \Rightarrow \quad \dot{X}_A = -\frac{1}{2} l\,\dot{\theta}_A \,\text{sen}\,\theta_A$$

$$Y_A = \frac{l}{2}\,\text{sen}\,\theta_A \quad \Rightarrow \quad \dot{Y}_A = \frac{1}{2} l\,\dot{\theta}_A \cos\theta_A$$

$$X_B = l\cos\theta_A + \frac{1}{2} l\cos\theta_B \quad \Rightarrow \quad \dot{X}_B = -l\dot{\theta}_A\,\text{sen}\,\theta_A - \frac{1}{2} l\,\dot{\theta}_B\,\text{sen}\,\theta_B$$

$$Y_B = \frac{1}{2} l\,\text{sen}\,\theta_B \quad \Rightarrow \quad \dot{Y}_B = \frac{1}{2} l\,\dot{\theta}_B\cos\theta_B$$

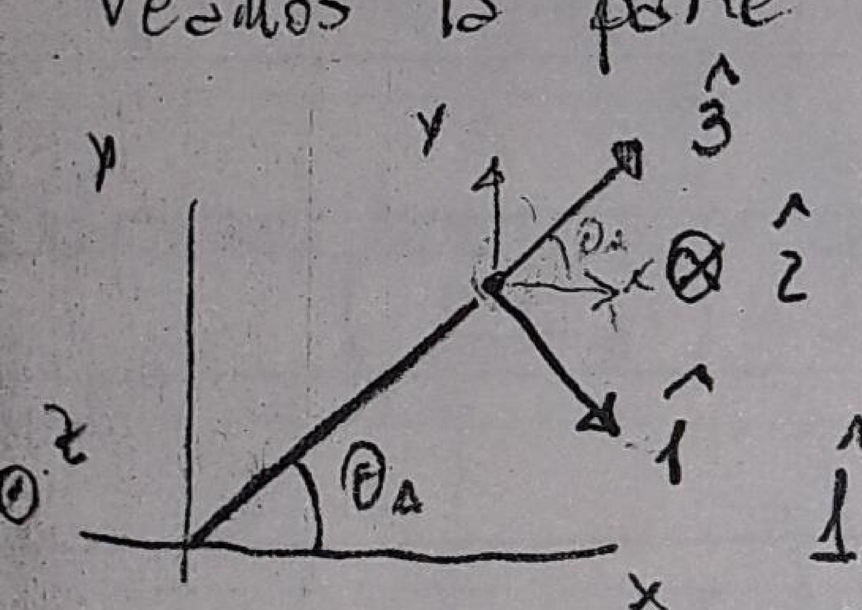
Las velocidades de los c.m. contribuyen a T como:

$$\theta_B = \pi - \theta_A$$

$$T_1 = \frac{1}{2} m\left(\dot{X}_A^2 + \dot{Y}_A^2\right) = \frac{1}{2} m \frac{1}{4} l^2\dot{\theta}_A^2$$

$$T_2 = \frac{1}{2} m\left(\dot{X}_B^2 + \dot{Y}_B^2\right) = \frac{1}{2} m\left[\frac{1}{4} l^2\dot{\theta}_B^2 + l^2\dot{\theta}_A^2\,\text{sen}^2\theta_A + l^2\dot{\theta}_A\dot{\theta}_B\,\text{sen}\,\theta_A\,\text{sen}\,\theta_B\right]$$

Veamos la parte rotacional:

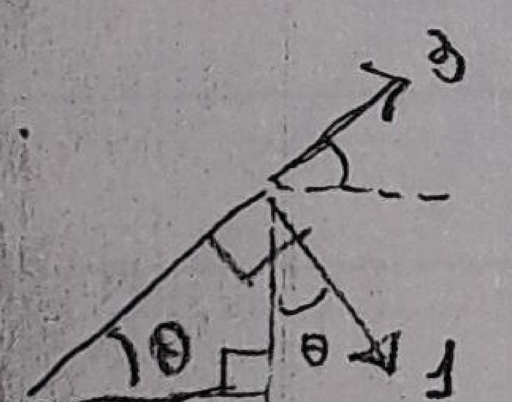


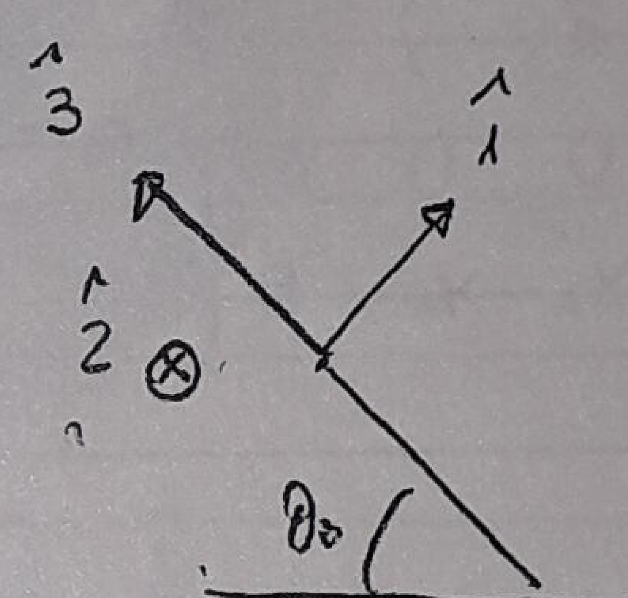
$$\vec{\Omega}_A = -\dot{\theta}_A\,\hat{2}$$

$$\hat{1}_A = \text{sen}\,\theta_A\,\hat{x} - \cos\theta_A\,\hat{y}$$

$$\hat{2}_A = -\hat{z}$$

$$\hat{3}_A = \cos\theta_A\,\hat{x} + \text{sen}\,\theta_A\,\hat{y}$$





$$\vec{\Omega}_B = -\dot{\theta}_B\,\hat{2}$$

$$\hat{1}_B = -\text{sen}\,\theta_B\,\hat{x} + \cos\theta_B\,\hat{y}$$

$$\hat{2}_B = -\hat{z}$$

$$\hat{3}_B = -\cos\theta_A\,\hat{x} + \text{sen}\,\theta_A\,\hat{y}$$

$$I_1 = I_2 = \frac{1}{12} m l^2 \quad ; \quad I_3 = 0$$

$$T_{rot} = \frac{1}{2} I_1\Omega_1^2 + \frac{1}{2} I_2\Omega_2^2 + \frac{1}{2} I_3\Omega_3^2$$

$$T_{rot} = \frac{1}{2}\left(\frac{1}{12} m l^2\right)\dot{\theta}_A^2 + \frac{1}{2}\left(\frac{1}{12} m l^2\right)\dot{\theta}_B^2$$

$$\Rightarrow T = \frac{1}{2} m\left\{\frac{1}{4} l^2\dot{\theta}^2 + \frac{1}{4} l^2\dot{\theta}^2 + l^2\dot{\theta}^2\,\text{sen}^2\theta + l^2\dot{\theta}^2\,\text{sen}^2\theta\right\} + \frac{1}{2}\left(\frac{1}{12} m l^2\right) 2\dot{\theta}^2$$