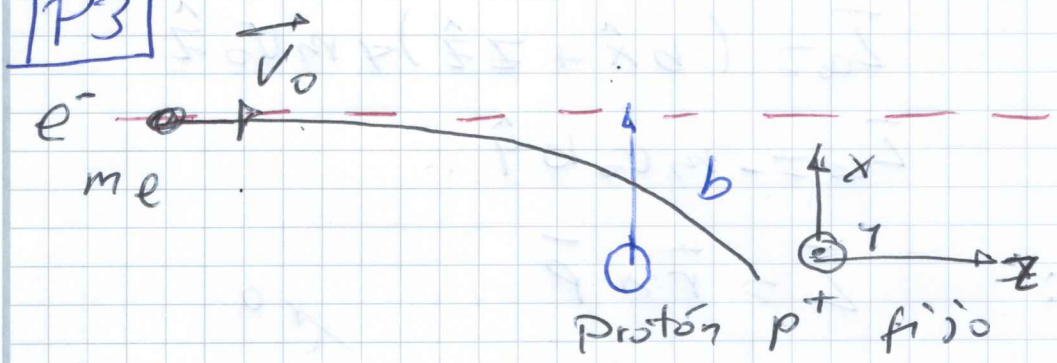
P3



$(M_p \gg m_e)$

$U(r) = \frac{e^2}{r}$ (atractivo)

Sistema gaussiano unidades.

a) 1 part en potencial central $\Rightarrow T+V=E$ es constante.

$\bar{F} = -\nabla U$ $\quad \bar{F} = F(r)\hat{r}$ radial

- Se conserva $E = T+V$

$\Rightarrow \bar{\tau} = \bar{r} \times \bar{F} = 0$ (torque c/r al centro de Fuerzas)

$\Rightarrow \bar{L} = \bar{r} \times \bar{p} = cte$

pue $\frac{d\bar{L}}{dt} = \bar{\tau}$ (c/r a punto fijo o en Mov. uniforme)

- Se conserva $\vec{L}$ (momento angular)

b) Comparamos en $r \to \infty$ con $r = r_c$ (máximo acercamiento)

$E_\infty = \frac{mv_0^2}{2} = \frac{e^2}{r_c} + \frac{mv_c^2}{2}$ (1)

Pero en r_c, $\vec{v}_c \perp \vec{r}_c$ (No hay componente radial)

$|\vec{L}_\infty| = mv_0 b$, $|\vec{L}_c| = mv_c r_c$

$\Rightarrow mv_0 b = mv_c r_c$ (2)

(2) en (1)

$\frac{mv_0^2}{2} = \frac{e^2}{r_c} + \frac{m}{2}\left(\frac{v_0 b}{r_c}\right)^2$

$\frac{mv_0^2}{2} = \frac{e^2}{r_c} + \frac{mv_0^2 b^2}{2r_c^2}$

$\frac{mv_0^2}{2} r_c^2 = e^2 r_c + \frac{mv_0^2 b^2}{2}$

$\frac{mv_0^2}{2} r_c^2 - e^2 r_c - \frac{mv_0^2 b^2}{2} = 0$

$r_c = \frac{e^2 \pm \sqrt{e^4 + m^2 v_0^4 b^2}}{mv_0^2}$

$r_c = \frac{e^2}{mv_0^2}\left(1 \pm \sqrt{1 + \frac{m^2 v_0^4 b^2}{e^4}}\right)$

$r_c = \frac{e^2}{mv_0^2}\left(1 + \sqrt{1 + \frac{m^2 v_0^4 b^2}{e^4}}\right)$ (+ porque $r_c > 0$)

Calculo de $\bar{L}$:

- En $r \to \infty$: $\quad \bar{L}_\infty = \bar{r} \times \bar{p}$

$$\bar{L}_\infty = (b\hat{x} + z\hat{z}) \times m v_0 \hat{z}$$

$$\bar{L}_\infty = -m v_0 b \hat{\varphi}$$

- En $r = r_c$: $\quad \bar{L} = \bar{r}_c \times \bar{p}$

$$\bar{L} = (r\hat{r}) \times m(\underbrace{\dot{r}\hat{r} + r\dot{\varphi}\hat{\varphi}}_{\vec{v}_c}) \quad (\dot{r} \to 0)$$

$$\bar{L}_c = -m r_c^2 \dot{\varphi} \hat{\varphi}$$

$$v_0 b = r_c^2 \dot{\varphi} = r_c v_c \quad (2)$$

$$v_c^2 = \dot{r}_c^2 + r_c^2 \dot{\varphi}^2 \quad (\dot{r}_c \to 0)$$

de (2) en (1)

$$\frac{m v_0^2}{2} = \frac{m v_0^2}{2}\left(\frac{b}{r_c}\right)^2 + \frac{e^2}{r_c}$$

$$\left(\frac{b}{r_c}\right)^2 + \left(\frac{b}{r_c}\right)\frac{2e^2}{b m v_0^2} - 1 = 0$$

- Si $v_0 \to \infty \Rightarrow b/r_c \to 1 \therefore r_c \to b$ ok!

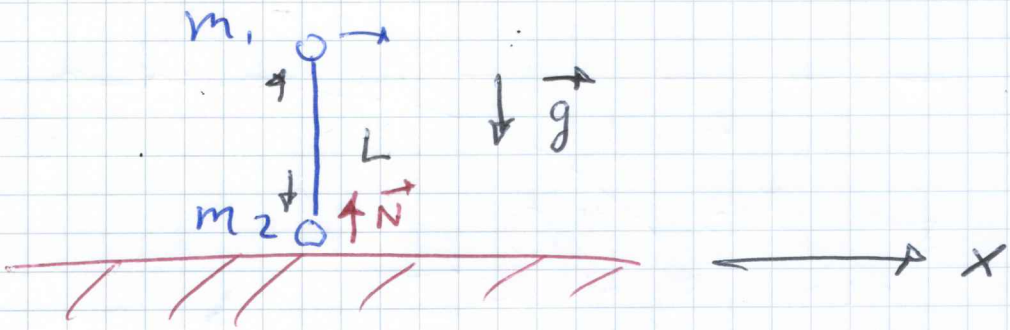
- Si $v_0 \to 0 \Rightarrow \frac{b}{r_c}$.

$$\frac{b}{r_c} = \frac{-\frac{2e^2}{b m v_0^2} \pm \sqrt{\left(\frac{2e^2}{b m v_0^2}\right)^2 + 4}}{2}$$

nos quedamos con el +.

Comentario: F de roce: vínculo es en el fondo F de interacc.

P7 2 partículas.



Hay fuerzas { externas: peso y normal del plano.
internas: de vínculo entre m_1 y m_2.

i) Sobre c/ masa ∃ fuerza ⇒ $\vec{P}_i \neq cte$.

Pero $\left(\sum \vec{F}_{i\,ext}\right) \cdot \hat{x} = 0$

$$\left(m_1\vec{g} + m_2\vec{g} + \vec{N} + \underbrace{\vec{F}_{r1} + \vec{F}_{r2}}_{\text{par acción - reacción}} \right) \cdot \hat{x} = 0$$

$$\Rightarrow \quad \frac{dP_x}{dt} = F_x = 0$$

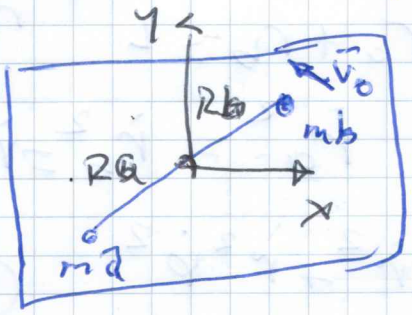
$$\therefore \quad P_x = P_{x1} + P_{x2} = cte = 0 \quad \text{(inicial)}$$

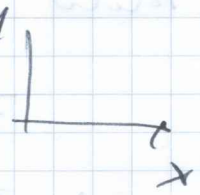
$$\frac{d}{dt}\left(m_1 x_1 + m_2 x_2 \right) = 0$$

$$\Rightarrow \quad m_1 x_1 + m_2 x_2 = x_{CM}\, M = 0 \quad \text{(inicial)}$$

$$\begin{cases} m_1 x_1 = -m_2 x_2 & (1) \\ x_1 - x_2 = L & (2) \end{cases}$$

P8 — 2 partículas.





(a) • Conservación: $P_{z_1} = P_{z_2} = 0$ (la normal cancela el peso)

• Energía: • La normal no realiza W.; $V_g = 0$.

• Ed W total de la tensión es nulo:

$$\bar{T}_a \delta \bar{r}_a + \bar{T}_b \delta \bar{r}_b = -T_a \delta r_a - T_b \delta r_b = -T \delta (r_a + r_b) = 0$$

$\delta r_a < 0$, $\delta r_b > 0$

$r_a + r_b = L$

• Momento angular, c/r al centro de cada masa.

pues $\bar{r}_a \times \bar{F}_a = \bar{r}_b \times \bar{F}_b = 0 \Rightarrow \bar{\tau}_a = \bar{\tau}_b$.

$\bar{r}_a \parallel \bar{F}_a$, $\bar{r}_b \parallel \bar{F}_b$

(b) Energía:

$$\frac{m_b v_b^2}{2} = \frac{m_a v_a^2}{2} + \frac{m_b v_b^2}{2} \quad (1)$$

$$0 = m_a R_a^2 \dot{\varphi}_a \Rightarrow \dot{\varphi}_a = 0 \quad (2)$$

L_a:

L_b:

$$m_b R_b^2 \dot{\varphi}_{b_0} = m_b m_b r_b^2 \dot{\varphi}_b \quad (3)$$

Usamos que: $v_a^2 = \dot{r}_a^2 + r_a^2\dot{\varphi}_a^2 = \dot{r}_a^2$

$v_b^2 = \dot{r}_b^2 + r_b^2\dot{\varphi}_b^2$ (4)

$\begin{cases} \dot{\varphi}_b = \dot{\varphi} \\ r_b = r \end{cases}$

Además por cinch $r_a + r_b = L$

$\Rightarrow \dot{r}_a^2 = \dot{r}_b^2 = \dot{r}^2$ (5)

$\Rightarrow v_a^2 = \dot{r}^2 \qquad v_b^2 = \dot{r}^2 + r^2\dot{\varphi}^2$ (6)

de (3): $\dot{\varphi} = \left(\frac{R_b}{r}\right)^2 \dot{\varphi}_{(0)} = \frac{R_b}{r^2} v_0$ (7)

en (1): $\frac{m_b v_0^2}{2} = \frac{m_a \dot{r}^2}{2} + \frac{m_b}{2}\left(\dot{r}^2 + \frac{R_b^2}{r^2} v_0^2\right)$

$\left(m_a + m_b\right)\frac{\dot{r}^2}{2} = \frac{m_b}{2} \frac{R_b^2}{r^2} v_0^2 \left(1 - \frac{R_b^2}{r^2}\right)$

$$\dot{r} = v_0 \sqrt{\frac{1}{1 + \frac{m_a}{m_b}}} \sqrt{1 - \frac{R_b^2}{r^2}}$$

Si $r = R_b$ $\Rightarrow \dot{r} = 0$ o vc

(c) Movimiento de m_b: (lineal)

$\hat{r}_b$: $-T_b = m_b \ddot{r}_b$

$$T_b = m_b v_0 \sqrt{\frac{1}{1 + \frac{m_a}{m_b}}} \frac{\frac{R_b^2}{r^3}}{\sqrt{1 - \frac{R_b^2}{r^2}}} \dot{r}$$