18 - Demuestre las siguientes propiedades de los corchetes de Poisson, siendo f, g, h funciones arbitrarias de p_i, q_i; $F(f)$ es una función de f. Sea c una constante.

a) $$[f,c] = \frac{\partial f}{\partial q}\cdot\frac{\partial c}{\partial p} - \frac{\partial f}{\partial p}\cdot\frac{\partial c}{\partial q} = 0 - 0 = 0$$

$$[f,f] = \frac{\partial f}{\partial q}\frac{\partial f}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial f}{\partial q} = \frac{\partial f}{\partial q}\cdot\frac{\partial f}{\partial p}(1-1) = 0$$

$$[f,g] + [g,f] = \frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}\cdot\frac{\partial g}{\partial q} + \frac{\partial g}{\partial q}\cdot\frac{\partial f}{\partial p} - \frac{\partial g}{\partial p}\cdot\frac{\partial f}{\partial q} = 0$$

$$[f+g,h] = [f,h] + [g,h]$$

$$[f+g,h] = \frac{\partial(f+g)}{\partial q}\cdot\frac{\partial h}{\partial p} - \frac{\partial(f+g)}{\partial p}\cdot\frac{\partial h}{\partial q} = \frac{\partial f}{\partial q}\cdot\frac{\partial h}{\partial p} - \frac{\partial f}{\partial p}\cdot\frac{\partial h}{\partial q} + \frac{\partial g}{\partial q}\frac{\partial h}{\partial p} - \frac{\partial g}{\partial p}\cdot\frac{\partial h}{\partial q} = [f,h] + [g,h]$$

$$[fg,h] = f\,[g,h] + [f,h]\,g$$

$$[fg,h] = \frac{\partial(fg)}{\partial q}\cdot\frac{\partial h}{\partial p} - \frac{\partial(fg)}{\partial p}\cdot\frac{\partial h}{\partial q} = f\left\{\frac{\partial g}{\partial q}\cdot\frac{\partial h}{\partial p} - \frac{\partial g}{\partial p}\cdot\frac{\partial h}{\partial q}\right\} + \left\{\frac{\partial f}{\partial q}\cdot\frac{\partial h}{\partial p} - \frac{\partial f}{\partial p}\cdot\frac{\partial h}{\partial q}\right\}g = f\,[g,h] + [f,h]\,g$$

$$\frac{\partial}{\partial t}[f,g] = \left[\frac{\partial f}{\partial t}, g\right] + \left[f, \frac{\partial g}{\partial t}\right]$$

$$\frac{\partial}{\partial t}[f,g] = \frac{\partial}{\partial t}\left\{\frac{\partial f}{\partial q}\cdot\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}\cdot\frac{\partial g}{\partial q}\right\} = \frac{\partial}{\partial q}\left(\frac{\partial f}{\partial t}\right)\cdot\frac{\partial g}{\partial p} - \frac{\partial}{\partial p}\left(\frac{\partial f}{\partial t}\right)\frac{\partial g}{\partial q} + \frac{\partial f}{\partial q}\cdot\frac{\partial}{\partial p}\left(\frac{\partial g}{\partial t}\right) - \frac{\partial f}{\partial p}\cdot\frac{\partial}{\partial q}\left(\frac{\partial g}{\partial t}\right) = \left[\frac{\partial f}{\partial t}, g\right] + \left[f, \frac{\partial g}{\partial t}\right]$$

$$[f,[g,h]] + [g,[h,f]] + [h,[f,g]] = 0$$

$$[f,[g,h]] = \left[f, \frac{\partial g}{\partial q}\cdot\frac{\partial h}{\partial p} - \frac{\partial g}{\partial p}\cdot\frac{\partial h}{\partial q}\right] = \frac{\partial f}{\partial q}\cdot\frac{\partial}{\partial p}\left(\frac{\partial g}{\partial q}\cdot\frac{\partial h}{\partial p} - \frac{\partial g}{\partial p}\cdot\frac{\partial h}{\partial q}\right) - \frac{\partial f}{\partial p}\cdot\frac{\partial}{\partial q}\left(\frac{\partial g}{\partial q}\cdot\frac{\partial h}{\partial p} - \frac{\partial g}{\partial p}\cdot\frac{\partial h}{\partial q}\right) =$$

$$= \frac{\partial f}{\partial q}\cdot\frac{\partial h}{\partial p}\cdot\frac{\partial^2 g}{\partial q\partial p} + \frac{\partial f}{\partial q}\cdot\frac{\partial g}{\partial q}\cdot\frac{\partial^2 h}{\partial p^2} - \frac{\partial f}{\partial q}\cdot\frac{\partial h}{\partial q}\frac{\partial^2 g}{\partial p^2} - \frac{\partial f}{\partial q}\cdot\frac{\partial g}{\partial p}\cdot\frac{\partial^2 h}{\partial q\partial p} - \frac{\partial f}{\partial p}\cdot\frac{\partial g}{\partial q}\cdot\frac{\partial^2 h}{\partial q\partial p} - \frac{\partial f}{\partial p}\cdot\frac{\partial h}{\partial p}\cdot\frac{\partial^2 g}{\partial q^2} +$$

$$+ \frac{\partial f}{\partial p}\frac{\partial h}{\partial q}\cdot\frac{\partial^2 g}{\partial q\partial p} + \frac{\partial f}{\partial p}\cdot\frac{\partial g}{\partial p}\cdot\frac{\partial^2 h}{\partial q^2}$$