Mecánica, Cuerpo Rígido, Ec. Euler

$\boxed{P18}$

- $\bar{\Omega} = \dot{\phi}\hat{z} + \dot{\theta}\hat{n} + \dot{\psi}\hat{3}$ (1)

$$\begin{cases} \hat{z} = \cos\theta\,\hat{3} + \sin\theta\sin\psi\,\hat{1} + \sin\theta\cos\psi\,\hat{2} & (2) \\ \hat{n} = \cos\psi\,\hat{1} - \sin\psi\,\hat{2} & (3) \end{cases}$$

$$\begin{cases} \Omega_1 = \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi & (4) \\ \Omega_2 = \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi & (5) \\ \Omega_3 = \dot{\phi}\cos\theta + \dot{\psi} & (6) \end{cases}$$

- Torque: $\bar{\tau} = \bar{d} \times (-mg\,\hat{z}) = mgd\sin\theta\,\hat{n}$ (7)

$$\begin{cases} \tau_1 = mgd\sin\theta\cos\psi & (8) \\ \tau_2 = -mgd\sin\theta\sin\psi & (9) \\ \tau_3 = 0 & (10) \end{cases}$$

- Ecuaciones de Euler, $I_1 = I_2 = I,\ I_3$
(Caso simétrico)

$$\left.\frac{d\bar{L}}{dt}\right|_m + \bar{\Omega}\times\underbrace{\bar{L}}_{\mathbb{I}\Omega} = \bar{\tau} \begin{cases} I\dot{\Omega}_1 + (I_3 - I)\Omega_2\Omega_3 = \tau_1 & (11) \\ I\dot{\Omega}_2 - (I_3 - I)\Omega_1\Omega_3 = \tau_2 & (12) \\ I_3\dot{\Omega}_3 = \tau_3 & (13) \end{cases}$$

$i(11) + (12)$

necesitamos:
se usa:
$e^{i\psi} = \cos\psi + i\sin\psi$
(Euler)

$$\begin{cases} i\Omega_1 + \Omega_2 = \dot{\phi}\sin\theta\, e^{i\psi} + i\dot{\theta}\, e^{i\psi} \\ i\Omega_2 - \Omega_1 = i\dot{\phi}\sin\theta\, e^{i\psi} - \dot{\theta}\, e^{i\psi} \\ i\tau_1 + \tau_2 = i\,mgd\sin\theta\, e^{i\psi} \end{cases}$$

$\mu = i(\dot{\theta} - i\dot{\phi}\sin\theta)$

$$\begin{cases} I\left(\frac{d\mu}{dt} + i\dot{\psi}\mu\right) + (I_3 - I)\Omega_0\, i\mu = i\,mgd\sin\theta \\ \dot{\Omega}_3 = 0 \Rightarrow \Omega_3 = \Omega_0 \ (\text{cte}) \end{cases}$$

Linealizamos:
$\lambda = \theta - i\phi\sin\theta_0$

$$\begin{cases} \mu \approx i(\dot{\theta} - i\dot{\phi}\sin\theta_0) = i\dot{\lambda} \\ mgd\sin\theta \approx mgd\sin\theta_0 \quad m \text{ chico} \end{cases}$$

$$I\ddot{\lambda} + iI_3\Omega_0\dot{\lambda} = mgd\sin\theta_0$$

Solución: $\lambda = \lambda_h + \lambda_p$ $\left(\begin{matrix}\text{homogénea} \\ + \\ \text{particular}\end{matrix}\right)$

- λ_p : $\ddot{\lambda} = 0$

$$\lambda_p = -i\underbrace{\frac{mgd}{I_3\Omega_0}}_{\omega_p}\sin\theta_0 t + \underbrace{C_1}_{c - if\sin\theta_0}$$

ω_p : velocidad angular de precesión de trompo pesado

- λ_h : $\lambda_h = \lambda_0 e^{-i\omega_L t}$

$$-I\omega_L^2 + I_3\Omega_0\omega_L = 0$$

$$\therefore \quad \omega_L = \frac{I_3}{I}\Omega_0$$

precesión de trompo libre

Usamos : $\lambda_0 = a e^{-i\varphi_0}\sin\theta_0$

$$\therefore \quad \lambda = \underbrace{a\sin\theta_0\cos(\omega_L t + \varphi_0) + c}_{\theta(t)} - i\sin\theta_0\underbrace{(\omega_p t + a\sin(\omega_L t + \varphi_0) + f)}_{\phi(t)}$$

Hallemos : $\{a, \varphi_0, c, f\}$ con c.i $\begin{cases}\theta(0) = \theta_0, & \dot{\theta}(0) = \dot{\theta}_0 \\ \phi(0) = 0, & \dot{\phi}(0) = \dot{\phi}_0\end{cases}$

$$\left.\begin{aligned}\dot{\theta}(0) &= \omega_L a\sin\theta_0\sin\varphi_0 = \dot{\theta}_0 \\ \dot{\phi}(0) &= \omega_p + a\,\omega_L\cos\varphi_0 = \dot{\phi}_0\end{aligned}\right\} \quad \begin{aligned}\tan\varphi_0 &= \frac{\dot{\theta}_0\sin\theta_0}{\dot{\phi}_0 - \omega_p} \\ a^2 &= \left(\frac{\dot{\phi}_0 - \omega_p}{\omega_L}\right)^2 + \left(\frac{\dot{\theta}_0}{\omega_L\sin\theta_0}\right)^2\end{aligned}$$

$$\left.\begin{aligned}\theta(0) &= a\sin\theta_0\cos\varphi_0 + c = \theta_0 \\ \phi(0) &= a\sin\varphi_0 + f = 0\end{aligned}\right\} \quad \begin{aligned}c &= \theta_0 - a\sin\theta_0\cos\varphi_0 \\ f &= -a\sin\varphi_0\end{aligned}$$