P7)

$$x = \frac{1}{\sqrt{m\omega}}\left(\sqrt{2p_1}\sin q_1 + P_2\right) \quad (1) \qquad P_x = \frac{\sqrt{m\omega}}{2}\left(\sqrt{2p_1}\cos q_1 - q_2\right) \quad (2)$$

$$y = \frac{1}{\sqrt{m\omega}}\left(\sqrt{2p_1}\cos q_1 + q_2\right) \quad (3) \qquad P_y = \frac{\sqrt{m\omega}}{2}\left(-\sqrt{2p_1}\sin q_1 + P_2\right) \quad (4)$$

$q_i \to \{x, y\}$ $\quad p_i \to \{P_x, P_y\}$ (originales)

$Q_i \to \{q_1, q_2\}$ $\quad P_2 \to \{P_1, P_2\}$ (nuevos)

Probemos:
$$\begin{cases} p = \dfrac{\partial F_1}{\partial q} \\[2ex] P = -\dfrac{\partial F_1}{\partial Q} \end{cases}$$
(así se integra el $\sin q_1, \cos q_1$)

de (3):
$$P_1 = \frac{\left(\sqrt{m\omega}\, y - q_2\right)^2}{2\cos^2 q_1} \quad (5)$$

de (1) y (5):
$$P_2 = \sqrt{m\omega}\, x - \frac{\left(\sqrt{m\omega}\, y - q_2\right)\sin q_1}{\cos q_1} \quad (6)$$

de (2) y (5):
$$P_x = \frac{\sqrt{m\omega}}{2}\left(\sqrt{m\omega}\, y - 2 q_2\right) \quad (7)$$

de (4) y (6):
$$P_y = \frac{\sqrt{m\omega}}{2}\left[-2\left(\sqrt{m\omega}\, y - q_2\right)\operatorname{tg} q_1 + \sqrt{m\omega}\, x\right] \quad (8)$$

de (5)
$$P_1 = \frac{\left(\sqrt{m\omega}\, y - q_2\right)^2}{2\cos^2 q_1} = \frac{\partial F_1}{\partial q_1}$$

integrando:
$$F_1 = \frac{\left(\sqrt{m\omega}\, y - q_2\right)^2}{}\operatorname{tg} q_1 + g_1(q_2, y)$$

en (6)

$$P_2 = \sqrt{m\omega}\, x - (\sqrt{m\omega}\, y - q_2)\, tg\, q = \frac{\partial F_1}{\partial q_2}$$

$$= -(\sqrt{m\omega}\, y - q_2)\, tg\, q_1 + \frac{\partial g_1}{\partial q_2}(q_2, x, y)$$

$$\Longrightarrow \quad g_1 = \sqrt{m\omega}\, x q_2 + g_2(x,y) \quad (10)$$

de (9) y (10) en (7)

$$P_x = \frac{m\omega y}{2} - \sqrt{m\omega}\, q_2 = -\frac{\partial F_1}{\partial x}$$

$$= -\sqrt{m\omega}\, q_2 + \frac{\partial g_2}{\partial x}$$

$$\Longrightarrow \quad g_2 = \frac{m\omega x y}{2} + g_3(y) \quad (11)$$

de (9), (10) y (11) en (8)

$$P_y = \frac{\sqrt{m\omega}}{2}\left[-2(\sqrt{m\omega}\, y - q_2)\, tg(q_1) + \sqrt{m\omega}\, x\right] = -\frac{\partial F_1}{\partial y}$$

$$= -\sqrt{m\omega}(\sqrt{m\omega}\, y - q_2)\, tg\, q_1 + \frac{m\omega x}{2}$$

$$+ \frac{d}{dy} g_3(y)$$

$$\Longrightarrow \quad g_3 = 0$$ (tomamos la constante nula)

$$F_1 = -\frac{(\sqrt{m\omega}\, y - q_2)^2}{2}\, tg\, q_1 + \sqrt{m\omega}\, x q_2 + \frac{m\omega x y}{2} \quad (12)$$

P5(b) $\vec{A} = \frac{1}{2}\vec{B} \times \vec{r} = \frac{1}{2} B \hat{z} \times (x\hat{x} + y\hat{y} + z\hat{z})$

$$\therefore \vec{A} = \frac{1}{2} B (-y\hat{x} + x\hat{y})$$

$$H = \frac{(\vec{p} - \vec{A})^2}{2m} = \frac{(p_x + \frac{m\omega}{2} y)^2}{2m} + \frac{(p_y - \frac{m\omega}{2} x)^2}{2m} + \frac{p_z^2}{2m}$$

← Desacoplado no lo consideramos

donde se usó: $\omega = \frac{B}{m}$

de (2) y (3): $p_x + \frac{m\omega}{2} y = \sqrt{m\omega}\sqrt{2P_1}\cos q_1$

de (4) y (1): $p_y - \frac{m\omega}{2} x = -\sqrt{m\omega}\sqrt{2P_1}\sin q_1$

$$K = H(q(Q,P), p(Q,P), t) + \underbrace{\frac{\partial F_1}{\partial t}}_{0}$$

$$K = \omega P_1$$

Ec. de Hamilton con el nuevo Kamiltoniano.

$$\begin{cases} \dot{q}_1 = \frac{\partial K}{\partial P_1} = \omega \Rightarrow q_1 = \omega t + q_{10} & \dot{q}_2 = \frac{\partial K}{\partial P_2} = 0 \Rightarrow q_2 = q_{20} \\ \dot{P}_1 = -\frac{\partial K}{\partial q_1} = 0 \Rightarrow P_1 = P_{10} & \dot{P}_2 = -\frac{\partial K}{\partial q_2} = 0 \Rightarrow P_2 = P_{20} \end{cases}$$

$$\begin{cases} x = \frac{1}{\sqrt{m\omega}}\left(\sqrt{2P_{10}}\sin(\omega t + q_{10}) + P_{20}\right) \\ y = \frac{1}{\sqrt{m\omega}}\left(\sqrt{2P_{10}}\cos(\omega t + q_{10}) + q_{20}\right) \end{cases}$$