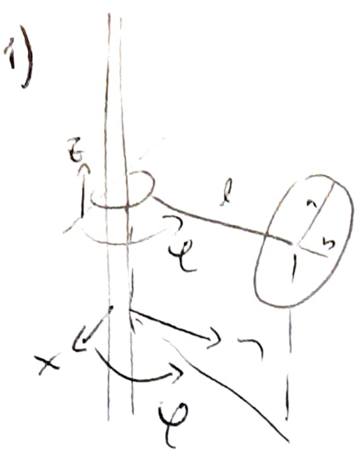
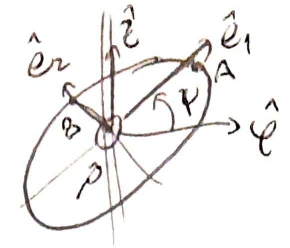
1)





$\hat{e}_1 = \cos\psi\,\hat{\varphi} + \sin\psi\,\hat{z}$

$\hat{e}_2 = -\sin\psi\,\hat{\varphi} + \cos\psi\,\hat{z}$

$\hat{e}_3 = \hat{\rho}$

$\cancel{\varphi = \varphi_E + \frac{\pi}{2}}$

Con Euler:

$\theta_E = \frac{\pi}{2}$

$\varphi_E = \varphi - \frac{\pi}{2}$

$\psi_E = \psi$

$\bar{r}_{cm} = l\hat{\rho} + z\hat{z} \qquad \bar{v}_{cm} = l\dot{\varphi}\hat{\varphi} + \dot{z}\hat{z}$

$\bar{r}_A - \bar{r}_{cm} = a\hat{e}_1 = a(\cos\psi\,\hat{\varphi} + \sin\psi\,\hat{z})$

$\bar{v}_A - \bar{v}_{cm} = a\dot{\psi}\hat{e}_2 - a\dot{\varphi}\cos\psi\,\hat{e}_3 = \bar{\Omega}\times(a\hat{e}_1)$

$\rightarrow \Omega_2 = \dot{\varphi}\cos\psi \qquad \Omega_3 = \dot{\psi}$

$\bar{r}_B - \bar{r}_{cm} = b\hat{e}_2 = b(-\sin\psi\,\hat{\varphi} + \cos\psi\,\hat{z})$

$\bar{v}_B - \bar{v}_{cm} = -b\dot{\psi}\hat{e}_1 + b\dot{\varphi}\sin\psi\,\hat{e}_3 = \bar{\Omega}\times(b\hat{e}_2)$

$\bar{\Omega} = \dot{\varphi}\sin\psi\,\hat{e}_1 + \dot{\varphi}\cos\psi\,\hat{e}_2 + \dot{\psi}\hat{e}_3 \qquad \Omega_1 = \dot{\varphi}\sin\psi$

$= \dot{\varphi}\hat{z} + \dot{\psi}\hat{e}_3 \rightarrow$ mismo resultado con Euler

$L = \frac{1}{2}M(l^2\dot{\varphi}^2 + \dot{z}^2) + \frac{1}{2}(I_1\sin^2\psi + \underbrace{I_2\cos^2\psi}_{})\dot{\varphi}^2 + \frac{1}{2}I_3\dot{\psi}^2 + Mgz$

$I_1 = \frac{1}{4}Mb^2 < I_2 = \frac{1}{4}Ma^2$

$I_1 + (I_2 - I_1)\cos^2\psi$

$L = \frac{1}{2}M\underbrace{(l^2 + \frac{1}{4}b^2\sin^2\psi + \frac{1}{4}a^2\cos^2\psi)}_{l^2 + \frac{1}{4}b^2 + \frac{1}{4}(a^2-b^2)\cos^2\psi}\dot{\varphi}^2 + \underbrace{\frac{1}{2}I_3}_{\frac{1}{8}M(a^2+b^2)}\dot{\psi}^2 + \frac{1}{2}M\dot{z}^2 + Mgz$

z) $\ddot{z} + g = 0$

$L = \frac{1}{8}M(a^2-b^2)\cos^2\psi\,\dot{\varphi}^2 + \dots$

φ) $P_\varphi = M(l^2 + \frac{1}{4}b^2 + \frac{1}{4}(a^2-b^2)\cos^2\psi)\dot{\varphi} = cte$

ψ) $P_\psi = \frac{1}{4}M(a^2+b^2)\dot{\psi} \qquad \frac{\partial L}{\partial \psi} = -\frac{1}{4}M(a^2-b^2)\cos\psi\sin\psi\,\dot{\varphi}^2$

$\rightarrow (a^2+b^2)\ddot{\psi} = -(a^2-b^2)\sin\psi\cos\psi\,\dot{\varphi}^2$

c) Si $\dot{\psi} = \omega$

$$L = \frac{1}{2} M\left(\ell^2 + \frac{1}{4} b^2 + \frac{1}{4}(a^2-b^2)\cos^2\psi\right)\omega^2 + \frac{1}{8} M(a^2+b^2)\dot{\psi}^2 + \frac{1}{2} M\dot{z}^2 + Mgz$$

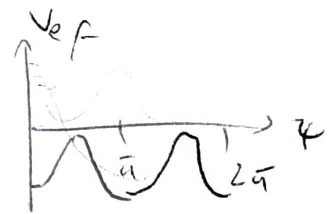
$$p_\psi = \frac{1}{4} M(a^2+b^2)\dot{\psi}$$

$$\frac{\partial L}{\partial \psi} = -\frac{1}{4} M(a^2-b^2)\omega^2 \operatorname{sen}\psi\cos\psi$$

$\rightarrow \frac{1}{2}\operatorname{sen} 2\psi \quad \rightarrow V_{ef} = -\frac{1}{4}\cos 2\psi$

$$(a^2+b^2)\ddot{\psi} + (a^2-b^2)\omega^2 \operatorname{sen}\psi\cos\psi = 0$$

$$\frac{1}{2}(a^2+b^2)\dot{\psi}^2 \underbrace{- \frac{1}{2}(a^2-b^2)\omega^2\cos^2\psi}_{V_{ef}} = cte$$



2) $Q = \beta q^2 \qquad P = \frac{\alpha p}{q}$

a) $\{Q, P\} = 2\beta q \frac{\alpha}{q} = 2\alpha\beta = 1 \rightarrow P = \frac{p}{2\beta q}$, $\cancel{\frac{p}{2\beta}}$

$p = 2\beta q P$

Con una alcantza

F_1: no se puede

F_2: $p = 2\beta q P = \frac{\partial F_2}{\partial q} \rightarrow \boxed{F_2 = \beta q^2 P} + f(P)$

$Q = \beta q^2 = \frac{\partial F_2}{\partial P} = \beta q^2 + \cancel{f'(P)}$

F_3: $q = \sqrt{\frac{Q}{\beta}} = -\frac{\partial F_3}{\partial p} \rightarrow \boxed{F_3 = -p\sqrt{\frac{Q}{\beta}}} + \cancel{f(Q)}$

$P = \frac{p}{2\beta}\sqrt{\frac{\beta}{Q}} = \frac{p}{2\sqrt{\beta Q}} = -\frac{\partial F_3}{\partial Q}$

F_4: $q = \frac{p}{2\beta P} = -\frac{\partial F_4}{\partial p} \rightarrow \boxed{F_4 = -\frac{p^2}{4\beta P}} + \cancel{f(P)}$

$Q = \beta\left(\frac{p}{2\beta P}\right)^2 = \frac{p^2}{4\beta P^2} = \frac{\partial F_4}{\partial P}$

b) $q = \sqrt{\frac{Q}{\beta}} \qquad p = 2\beta q P = 2\beta\sqrt{\frac{Q}{\beta}} P = 2\sqrt{\beta Q}\, P$

$H = \frac{1}{2m}(4\beta Q P^2) + \frac{m\omega^2}{2}\frac{Q}{\beta} = Q\left(\frac{m\omega^2}{2\beta} + \frac{2\beta}{m}P^2\right)$

$\dot{Q} = \frac{\partial H}{\partial P} = \frac{4\beta}{m} Q P \qquad \dot{P} = -\frac{\partial H}{\partial Q} = -\left(\frac{m\omega^2}{2\beta} + \frac{2\beta}{m}P^2\right)$

$\hookrightarrow P = \frac{m}{4\beta}\frac{\dot{Q}}{Q}$

c) $L = \frac{m}{4\beta}\frac{\dot{Q}^2}{Q} - \frac{m\omega^2}{2\beta}Q - \frac{2\beta}{m}\frac{m^2}{16\beta^2}\frac{\dot{Q}^2}{Q}$

$= \frac{m}{8\beta}\frac{\dot{Q}^2}{Q} - \frac{m\omega^2}{2\beta}Q$

$\rightarrow \ddot{Q} - \frac{\dot{Q}^2}{2Q} + 2\omega^2 Q = 0$

$\frac{\partial L}{\partial \dot{Q}} = \frac{m}{4\beta}\frac{\dot{Q}}{Q} \qquad \frac{\partial L}{\partial Q} = -\frac{m}{8\beta}\frac{\dot{Q}^2}{Q^2} - \frac{m\omega^2}{2\beta}$

(no es necesario)

$\frac{m}{4\beta}\frac{\ddot{Q}}{Q} - \frac{m}{4\beta}\frac{\dot{Q}^2}{Q^2} + \frac{m}{8\beta}\frac{\dot{Q}^2}{Q^2} + \frac{m\omega^2}{2\beta} = 0$

$-\frac{m}{8\beta}$

3) a) $H = \frac{1}{2m}\left(p_r^2 + \frac{p_\varphi^2}{r^2} + p_z^2\right) + \frac{1}{2} k r^2 + \frac{1}{2} k_z z^2$

$$S = S_r(r) + p_\varphi \varphi + S_z(z) - Et$$

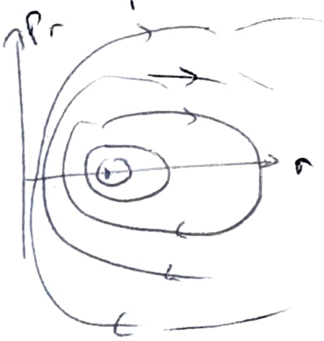
b) $\frac{1}{2m}\left(S_r'^2 + \frac{p_\varphi^2}{r^2} + S_z'^2\right) + \frac{1}{2} k r^2 + \frac{1}{2} k_z z^2 = E$

$$\underbrace{S_r'^2 + \frac{p_\varphi^2}{r^2} + kmr^2}_{=\kappa} + \underbrace{S_z'^2 + k_z m z^2 - 2mE}_{=-\kappa} = 0$$

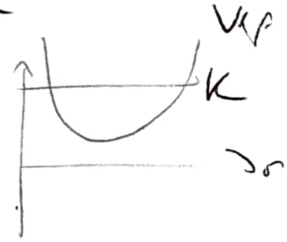
$$S_r = \int dr \sqrt{\kappa - \frac{p_\varphi^2}{r^2} - kmr^2}$$

$$S_z = \int dz \sqrt{2mE - \kappa - k_z m z^2}$$

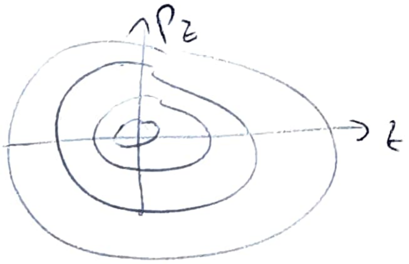
$$p_r = \sqrt{\kappa - \frac{p_\varphi^2}{r^2} - kmr^2}$$



$$p_r^2 + \underbrace{\frac{p_\varphi^2}{r^2} + kmr^2}_{V_{ef}(r)} = \kappa$$



$$p_z = \sqrt{2mE - \kappa - k_z m z^2}$$



$$p_z^2 + k_z m z^2 = 2mE - \kappa$$

$\hookrightarrow \kappa < 2mE$

con 2π:

c) $J_\varphi = \frac{1}{2\pi}\oint p_\varphi \, d\varphi = p_\varphi$

$$J_r = \frac{1}{2\pi}\oint dr \sqrt{K - \frac{J_\varphi^2}{r^2} - kmr^2} = \frac{1}{\pi}\int_{r_1}^{r_2} dr \sqrt{K - \frac{J_\varphi^2}{r^2} - kmr^2}$$

$$= \frac{1}{\pi}\cdot\frac{\pi}{2}\left(\frac{K}{2\sqrt{km}} - J_\varphi\right) \qquad b = \frac{K}{2} \quad a = km \quad c = J_\varphi^2$$

$$\frac{K}{2\sqrt{km}} = 2J_r + J_\varphi$$

$$J_z = \frac{1}{2\pi}\cdot 2\int_{-z_0}^{z_0} dz \sqrt{2mE - K - k_z m z^2} \qquad x = \sqrt{\frac{k_z m}{2mE-K}}\, z$$

$$= \frac{1}{\pi}\sqrt{2mE-K}\int_{-z_0}^{z_0} dz \sqrt{1 - \frac{k_z m z^2}{2mE-K}} =$$

$$= \frac{2mE-K}{\pi\sqrt{k_z m}}\int_{-1}^{1} dx \sqrt{1-x^2} = \frac{2mE-K}{2\sqrt{k_z m}}$$

$$2mE = 2\sqrt{k_z m}\, J_z + K$$

$$E = \sqrt{\frac{k_z}{m}}\frac{J_z}{\cancel{\pi}} + \frac{K}{2m} = \sqrt{\frac{k_z}{m}}\frac{J_z}{\cancel{\pi}} + \frac{1}{m}\sqrt{km}\left(2J_r + J_\varphi\right)$$

$$= \omega_z J_z + 2\omega_r J_r + \omega_r J_\varphi$$