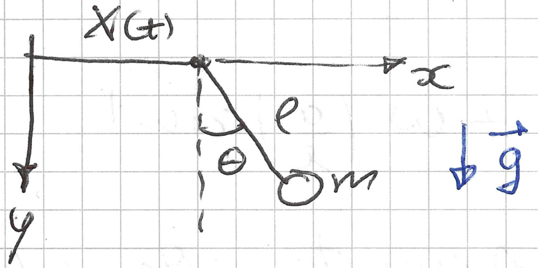
Guia 1 Mecánica

P13

$$X(t) = a\cos\omega t$$



$$\begin{cases} x = X(t) + l\sin\theta \\ y = l\cos\theta \end{cases}$$

$$\begin{cases} \dot{x} = -\omega a\sin\omega t + l\dot{\theta}\cos\theta \\ \dot{y} = -l\dot{\theta}\sin\theta \end{cases}$$

$$v^2 = (-\omega a\sin\omega t + l\dot{\theta}\cos\theta)^2 + (-l\dot{\theta}\sin\theta)^2$$

$$\Rightarrow v^2 = \omega^2 a^2\sin^2\omega t - 2\omega a l\sin\omega t\cos\theta\,\dot{\theta} + l^2\dot{\theta}^2$$

Sin tomar en cuenta la parte que depende sólo del tiempo (derivada de alguna $F(t)$)

$$\mathcal{L} = \underbrace{\frac{m}{2}\left(-2\omega a l\sin\omega t\cos\theta\,\dot{\theta} + l^2\dot{\theta}^2\right)}_{T} + \underbrace{mgl\cos\theta}_{-V}$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\theta}} = ml^2\dot{\theta} - m\omega a l\sin\omega t\cos\theta \Rightarrow \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\theta}}\right) = ml^2\ddot{\theta} - m\omega^2 a l\cos\omega t\cos\theta \cancel{+ m\omega a l\sin\omega t\,\dot{\theta}\sin\theta}$$

$$\frac{\partial\mathcal{L}}{\partial\theta} = \cancel{+ m\omega a l\sin\omega t\,\dot{\theta}\sin\theta} - mgl\sin\theta$$

Ec. de Lagrange:

$$ml^2\ddot{\theta} = -mgl\sin\theta + m\omega^2 a l\cos\omega t\cos\theta$$

$$\ddot{\theta} = -\omega_0^2\sin\theta + \omega^2\left(\frac{a}{l}\right)\cos\omega t\cos\theta$$

Linealizado: θ chico (cerca equilibrio) $\begin{cases} \sin\theta \simeq \theta \\ \cos\theta \simeq 1 \end{cases}$

$$\ddot{\theta} = -\omega_0^2\theta + \omega^2\left(\frac{a}{l}\right)\cos\omega t$$

oscilador forzado.

$$\theta(t) = \theta_h(t) + \theta_p(t)$$

donde: $\ddot{\theta}_h = -\omega_0^2 \theta_h$

$$\theta_h = \theta_0 \cos(\omega_0 t + \varphi_0)$$

$$\ddot{\theta}_p = -\omega_0^2 \theta_p + \omega^2 \left(\frac{a}{\ell}\right) \cos \omega t$$

Proponemos: $\theta_p = A \cos \omega t$; $\dot{\theta}_p = -\omega A \sin \omega t$

$\ddot{\theta}_p = -A\omega^2 \cos \omega t$

(tirando $\cos \omega t$)

$$\Rightarrow \quad -A\omega^2 = -\omega_0^2 A + \omega^2 \left(\frac{a}{\ell}\right)$$

$$\therefore \quad A = \frac{\omega^2}{\omega_0^2 - \omega^2} \left(\frac{a}{\ell}\right)$$

Si $\omega \approx \omega_0$ A es grande (resonancia)

La frecuencia del forzante (ω) es parecida a la la frecuencia natural ω_0 del sistema.