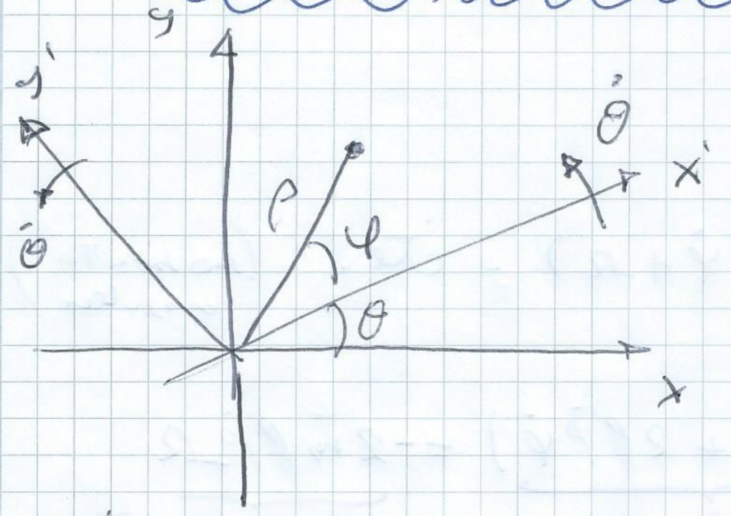
Guía 1

P3) Problema simplificado:



$\dot{\theta} = \Omega$

$\theta = \Omega t$

Lagrangiano en sistema Inercial: $\mathcal{L} = \frac{m v^2}{2}$

Pero expresado en coordenadas del sistema rotante (ρ, φ).

$x = \rho \cos(\varphi + \theta)$

$y = \rho \sin(\varphi + \theta)$

$\dot{x} = \dot{\rho} \cos(\varphi + \theta) - \rho(\dot{\varphi} + \Omega) \times \sin(\varphi + \theta)$

$\dot{y} = \dot{\rho} \sin(\varphi + \theta) + \rho(\dot{\varphi} + \Omega) \times \cos(\varphi + \theta)$

$v^2 = \dot{x}^2 + \dot{y}^2 = \dot{\rho}^2 + \rho^2 (\dot{\varphi} + \Omega)^2$

$\mathcal{L} = \frac{m}{2} \left[\dot{\rho}^2 + \rho^2 (\dot{\varphi} + \Omega)^2 \right]$

$\mathcal{L} : \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\rho}}\right) = m\ddot{\rho} \qquad \frac{\partial \mathcal{L}}{\partial \rho} = m\rho(\dot{\varphi} + \Omega)^2$

$m\ddot{\rho} = m\rho\dot{\varphi}^2 + 2m\rho\dot{\varphi}\Omega + m\rho\Omega^2$

$\underbrace{m(\ddot{\rho} - \rho\dot{\varphi}^2)}_{a\rho} = \underbrace{2m\rho\Omega\dot{\varphi}}_{\text{Coriolis}} + \underbrace{m\Omega^2\rho}_{\text{Centrífuga}}$

$\underset{\sim}{\varphi}$: $\quad \dfrac{\partial \mathcal{L}}{\partial \dot{\varphi}} = m\rho^2(\dot{\varphi}+\Omega)$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\varphi}}\right) = 2m\rho\dot{\rho}(\dot{\varphi}+\Omega) + m\rho^2\ddot{\varphi}$$

$$\frac{\partial \mathcal{L}}{\partial \varphi} = 0$$

$\therefore \quad m\rho^2(\dot{\varphi}+\Omega) = \text{cte.}$ (momento angular)

ó (cancelando un $\rho \neq 0$)

$$m\underbrace{(\rho\ddot{\varphi} + 2\dot{\rho}\dot{\varphi})}_{a_\varphi} = \underbrace{-2m\dot{\rho}\Omega}_{\text{Coriolis}}$$

F. Coriolis

$$m\bar{a}_c = -2m\bar{\Omega}\times\bar{v}$$

$$\bar{\Omega} = \Omega\hat{z} \quad \bar{v} = (\dot{\rho}\hat{\rho} + \rho\dot{\varphi}\hat{\varphi})$$

$$-m2\bar{\Omega}\times\bar{v} = -2m\Omega\dot{\rho}\hat{\varphi} + 2m\Omega\rho\dot{\varphi}\hat{\rho}$$

F. Centrífuga

$$\bar{a}_E = -m\bar{\Omega}\times\bar{\Omega}\times\bar{r}'$$

$$= -m\Omega\hat{z}\times(\Omega\hat{z})\times(\rho\hat{\rho})$$

$$= -\Omega\hat{z}\times\Omega\rho\hat{\varphi}$$

$$= m\Omega^2\rho\hat{\rho}$$