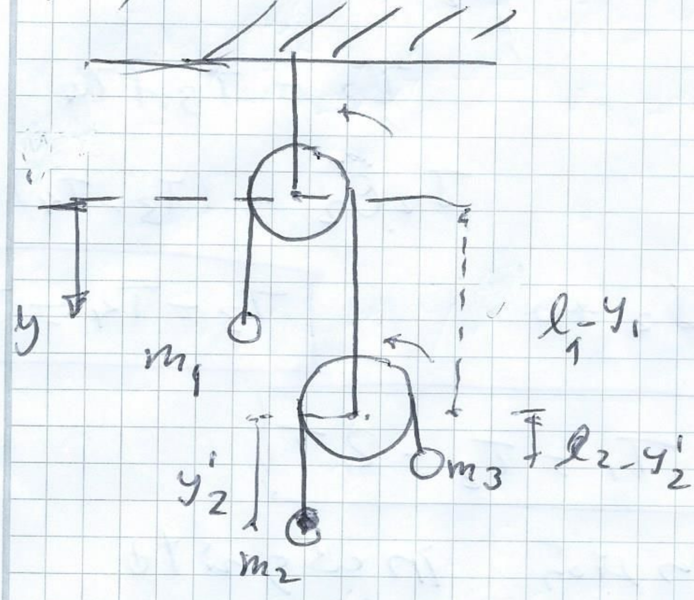
# Guia 1

P4)




$\{y_1, y_2, y_3\}$ no son independientes

tomamos $l_1, l_2 \gg R_1, R_2$ (de todos modos sólo agrega una cte)

$$y_2 = l_1 - y_1 + y_2'$$

$$y_3 = l_1 - y_1 + l_2 - y_2'$$

$$\therefore \quad y_3 = l_1 - y_1 + l_2 - (y_2 - l_1 + y_1)$$

$$\boxed{y_3 = -2y_1 - y_2 + cte.}$$

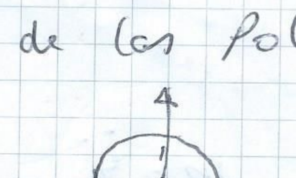
$$\delta y_3 = -2\,\delta y_1 - \delta y_2$$

**Newton:**

$$\ddot{y}_3 = -2\ddot{y}_1 - \ddot{y}_2 \qquad (1)$$

Ecuacion de las poleas:

$M_1$ :




$$\dot{\vec{L}} = \vec{\tau} \qquad (CM)$$

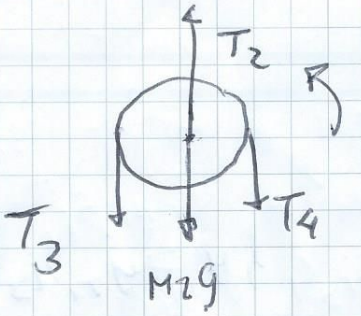
$$I_1 \frac{d\theta_1}{dt} = (T_1 - T_2) . R$$

pero $I_1 \to 0$

La cuerda no desliza

$$\Rightarrow \quad T_1 = T_2$$

$M_2$ :




$$M_2\ddot{y}_2 = -T_2 + (T_3 + T_4)$$

$$M_2 \to 0$$

$$T_2 = T_3 + T_4$$

La cuerda no desliza

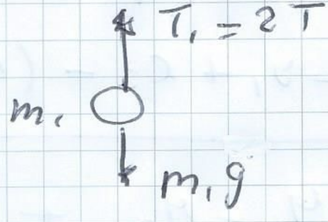
$$I_2\ddot{\theta}_2 = (T_3 - T_4)R$$

$$I_2 \to 0 \qquad \boxed{T_3 = T_4 = T}$$

$$\boxed{T_1 = T_2 = 2T}$$

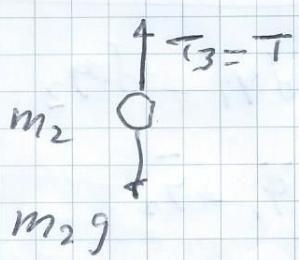
Ya una sola tensión incógnita

$m_1$ :



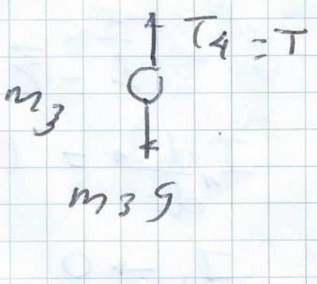

$$m_1\ddot{y}_1 = -2T + m_1 g \quad (2)$$

$m_2$ :




$$m_2\ddot{y}_2 = -T + m_2 g \quad (3)$$

$m_3$ :




$$m_3\ddot{y}_3 = -T + m_3 g \quad (4)$$

4 ecuaciones con 4 incógnitas.

de (1) : $\quad -m_3(2\ddot{y}_1 + \ddot{y}_2) = -T + m_3 g \quad$ (5)

(2) - 2(3): $\quad m_1\ddot{y}_1 - 2m_2\ddot{y}_2 = (m_1 - 2m_2)g \quad$ (6)

(2) - 2(5): $\quad m_1\ddot{y}_1 + 2m_3(2\ddot{y}_1 + \ddot{y}_2) = (m_1 - 2m_3)g$

$$(m_1 + 4m_3)\ddot{y}_1 + 2m_3\ddot{y}_2 = (m_1 - 2m_3)g \quad (7)$$

(6) $m_3$ + (7) $m_2$:

$$(m_1m_3 + m_1m_2 + 4m_2m_3)\ddot{y}_1$$

$$= (m_1m_3 - 4m_2m_3 + m_1m_2)g$$

Checu $\quad$ Si $\quad m_2 = m_3 = m \quad ; \quad m_1 = 2m$

$$\boxed{\ddot{y}_1 = 0}$$

$$\ddot{y}_2 = g\,\frac{(m_1m_2 - 3m_1m_3 + 4m_2m_3)}{(m_1m_3 + m_1m_2 + 4m_2m_3}$$

$$\ddot{y}_3 = g\,\frac{(m_1m_3 - 3m_2m_3 + 4m_2m_3)}{(m_1m_3 + m_1m_2 + 4m_2m_3}$$

$$T = g \cdot \frac{4m_1m_2m_3}{m_1m_2 + m_1m_3 + 4m_2m_3}.$$

Checu $\quad m_2$ es intercambiable con $m_3$

# D'alembert (PTV)

$$\sum (\bar{F}_i - m_i \ddot{\bar{r}}_i)\delta \bar{r}_i = 0$$

$\bar{F}_1 = m_1 g \hat{y}$ $\quad \bar{r}_1 = y_1 \hat{y}$ $\quad \ddot{\bar{r}}_1 = \ddot{y}_1 \hat{y}$

$\bar{F}_2 = m_2 g \hat{y}$ $\quad \bar{r}_2 = y_2 \hat{y}$ $\quad \ddot{\bar{r}}_2 = \ddot{y}_2 \hat{y}$

$\bar{F}_3 = m_3 g \hat{y}$ $\quad \bar{r}_3 = y_3 \hat{y}$ $\quad \ddot{\bar{r}}_2 = \ddot{y}_3 \hat{y}$

$\delta \bar{r}_1 = \delta y_1 \hat{y}$ , $\delta \bar{r}_2 = \delta y_2 \hat{y}$ , $\delta \bar{r}_3 = \delta y_3 \hat{y}$

$$(m_1 g - m_1 \ddot{y}_1)\delta y_1 + (m_2 g - m_2 \ddot{y}_2)\delta y_2$$
$$+ (m_3 g - m_3 \ddot{y}_3)\delta y_3 = 0$$

No calculamos $T$ $\quad \Rightarrow \quad \delta y_3 = -2\delta y_1 - \delta y_2$

$$[m_1 g - m_1 \ddot{y}_1 - 2\{m_3 g - m_3(-2\ddot{y}_1 - \ddot{y}_2)\}]\delta y_1$$
$$+ [m_2 g - m_2 \ddot{y}_2 - \{m_3 g - m_3(-2\ddot{y}_1 - \ddot{y}_2)\}]\delta y_2 = 0$$

$\delta y_1$ y $\delta y_2$ independientes:

anulo cada coeficiente:

$$\begin{cases} (m_1 - 2m_3) g = (m_1 + 4m_3)\ddot{y}_1 + 2m_3 \ddot{y}_2 \\ (m_2 - m_3) g = +2m_3 \ddot{y}_1 + (m_2 + m_3)\ddot{y}_2 \end{cases}$$

$$\ddot{y}_1 = \frac{g(m_1 m_2 + m_1 m_3 - 4m_2 m_3)}{m_1 m_2 + m_1 m_3 + 4m_2 m_3}$$

$$\ddot{y}_2 = \frac{g(m_1 m_2 - 3m_1 m_3 + 4m_2 m_3)}{m_1 m_2 + m_1 m_3 + 4m_1 m_3}$$

Lagrange $\quad \mathcal{L} = T - V$

$$T = \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2 + \frac{m_3}{2} \dot{y}_3^2$$

$$V = -m_1 g y_1 + m_2 g y_2 - m_3 g y_3$$

Vínculo: $\quad \dot{y}_3 = -2\dot{y}_1 - \dot{y}_2$

$$\begin{cases} T = \frac{1}{2}(m_1 + 4m_3)\dot{y}_1^2 + \frac{1}{2}(m_2 + m_3)\dot{y}_2^2 + 2m_3 \dot{y}_1 \dot{y}_2 \\ V = -m_1 g y_1 - m_2 g y_2 + m_3 g (2y_1 + y_2) + cte \end{cases}$$

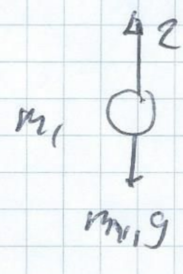
$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}_1}\right) = (m_1 + 4m_3)\ddot{y}_1 + 2m_3 \ddot{y}_2 = \frac{\partial \mathcal{L}}{\partial y_1} = (m_1 - 2m_3) g$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}_2}\right) = (m_2 + m_3)\ddot{y}_2 + 2m_3 \ddot{y}_1 = \frac{\partial \mathcal{L}}{\partial y_2} = (m_2 - m_3) g$$

OK!

Cálculo de T

(alternativo a multiplicadores)




Usamos Newton:

$$m_1 \ddot{y}_1 = -2T + m_1 g$$

$$\boxed{T = \frac{m_1}{2}(g - \ddot{y}_1)}$$