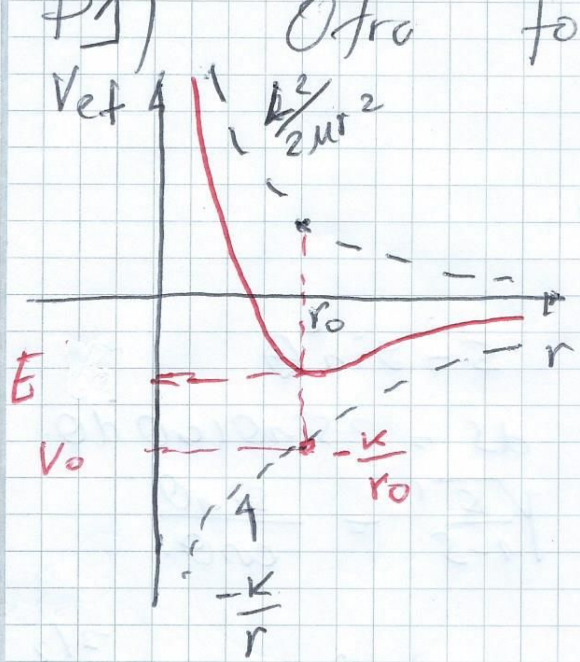
Guia 3: Fuerzas Centrales

P1) Otra forma:



$$\frac{\mu \dot{r}^2}{2} + \overbrace{\frac{L^2}{2\mu r^2} - \frac{K}{r}}^{V_{ef}(r)} = E \quad (1)$$

$$V_{ef}(r) = \frac{L^2}{2\mu r^2} - \frac{K}{r} \quad (2)$$

$$\left.\frac{dV_{ef}}{dr}\right|_{r_0} = 0 \quad \text{órbita circular} \quad (3)$$

$$-\frac{L^2}{\mu r_0^3} + \frac{K}{r_0^2} = 0$$

$$\therefore \quad \frac{K}{r_0} = \frac{L^2}{\mu r_0^2} \quad (4)$$

En órbita circular $\dot{r} = 0$

(4) en (1)

$$E = \frac{L^2}{2\mu r_0^2} - \frac{K}{r_0} = -\frac{K}{2r_0} = \frac{V_0}{2}$$

Si se frena, $L \to 0$ $\quad E' = V_0 = -\frac{K}{r_0}$ (sólo potencial)

de (1) $\quad \dot{r} = \left[\frac{2}{\mu}\left(E - \frac{L^2}{2\mu r^2} + \frac{K}{r}\right)\right]^{1/2}$

con $L = 0$
$E = E'$

$$\int_{r_0}^{0} \frac{dr}{\sqrt{\frac{2}{\mu}}\left[E - \frac{L^2}{2\mu r^2} + \frac{K}{r}\right]^{1/2}} = \int_0^{t_{caída}} dt \quad (5) \; ; \; L = 0, \; E = V_0$$

Necesitamos: $I = \int_0^{r_0} \frac{dr}{\left(\frac{1}{r} - \frac{1}{r_0}\right)^{1/2}}$

Sea $s = \frac{r}{r_0}$

$$I = r_0^{3/2} \int_0^1 \sqrt{\frac{s}{1-s}}\, ds.$$

ahora sed: $s = \sin^2\theta$

$ds = 2 \sin\theta \cos\theta\, d\theta.$

$\sqrt{\frac{s}{1-s}} = \frac{\sin\theta}{\cos\theta}.$

$$I = r_0^{3/2} \cancel{2} \int_0^{\pi/2} \frac{(1 - \cos 2\theta)}{\cancel{2}} d\theta. = \frac{\pi}{2} r_0^{3/2} \quad (6)$$

$\therefore$ $t_{caida} = \frac{\pi}{2\sqrt{2}} \frac{r_0^{3/2}}{\sqrt{K}} \sqrt{\mu}$ (7)

pero el $\tau_{circular}$: $L = \mu r_0^2 \dot{\varphi} = \underbrace{\sqrt{\mu K r_0}}_{\text{de } (4)}$

$$\tau = \frac{2\pi}{\dot{\varphi}} = \frac{2\pi \mu r_0^2}{L} \quad (8)$$

(7)/(8): $\frac{t_{caida}}{\tau} = \frac{1}{4\sqrt{2}}$

$\therefore$ $\boxed{t_{caida} = \frac{\tau}{4\sqrt{2}}}$