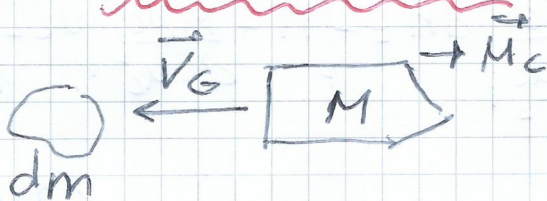


Relatividad.

P7) Propulsión del cohete.

No Relativista



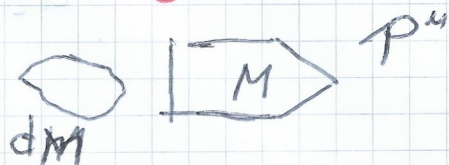
$$d\vec{P}_{tot} = 0$$

impulso: $\vec{u}_G dm + d(M\vec{u}_c) = 0$ (1)

masa: $d(m + M) = 0$ (2)

Composición velocidades: $\vec{u}_G = \vec{u}_c + \vec{v}_G$ (3)

Relativista



$$dP_{tot}^u = 0$$

$$\frac{dM \vec{u}_G}{\sqrt{1 - \frac{u_G^2}{c^2}}} + d\left(\frac{M \vec{u}_c}{\sqrt{1 - \frac{u_c^2}{c^2}}}\right) = 0$$

$$\frac{dM c}{\sqrt{1 - \frac{u_G^2}{c^2}}} + d\left(\frac{M c}{\sqrt{1 - \frac{u_c^2}{c^2}}}\right) = 0$$

$$\vec{u}_G = \frac{\vec{u}_c + \vec{v}_G}{1 + \frac{\vec{u}_c \cdot \vec{v}_G}{c^2}}$$

(2) y (3) en (1):

$$d(M\vec{u}_c) = (\vec{u}_c + \vec{v}_G) dM$$

$$\text{(4)} \quad d\left(\frac{M \vec{u}_c}{\sqrt{1 - \frac{u_c^2}{c^2}}}\right) = \frac{\vec{u}_c + \vec{v}_G d}{1 + \frac{\vec{u}_c \cdot \vec{v}_G}{c^2}} \left(\frac{M}{\sqrt{1 - \frac{u_c^2}{c^2}}}\right)$$

Cálculo auxiliar: $d\left(\frac{1}{\sqrt{1 - \frac{u_c^2}{c^2}}}\right) = \frac{\vec{u}_c \cdot d\vec{u}_c}{c^2 (1 - \frac{u_c^2}{c^2})^{3/2}}$

Dirección X:

$$M du_c + u_c dM = (u_c + v_G) dM$$

$$\text{(5)} \quad M d\vec{u}_c + u_c dM + \frac{M u_c^2 du_c}{c^2 - u_c^2} = \left(\frac{u_c + v_G}{1 - \frac{u_c v_G}{c^2}}\right) \left(\frac{dM + M u_c du_c}{c^2 - u_c^2}\right)$$

factorizando dM y dM_c :

$$V_G dM = -M du_c$$

$$\left(M_c - \frac{M_c - V_G}{1 - \frac{V_G u_c}{c^2}} \right) dM = - \left[M + \frac{M M_c}{c^2 - u_c^2} \left(M_c - \frac{M_c - V_G}{1 - \frac{V_G u_c}{c^2}} \right) \right] \times$$

$\times du_c$

Simplificando finalmente: (Mesa de álgebra)

$$\frac{dM}{M} = - \frac{du_c}{V_G} \quad (7)$$

$$\frac{dM}{M} = - \frac{dM_c}{V_G \left(1 - \frac{u_c^2}{c^2} \right)}$$

Integrando:

$$\ln \left(\frac{M_f}{M_0} \right) = - \frac{(u_{cf} - u_{c0})}{V_G} \quad (8) \quad \ln \left(\frac{M_f}{M_0} \right) = - \frac{c \tanh^{-1} \left(\frac{u_c}{c} \right) \Big|_{u_{c0}}^{u_{cf}}}{V_G}$$

Si $u_{c0} = 0$

$$M_{cf} = V_G \ln \left(\frac{M_0}{M_f} \right) \quad (9) \quad M_{cf} = c \tanh \left[\frac{V_G \ln \left(\frac{M_0}{M_f} \right)}{c} \right]$$

Cálculo no relativista

M_{cf} puede tomar cualquier valor.

Cálculo relativista

$M_{cf} \leq c$,
pues $\tanh(x) \leq 1$.