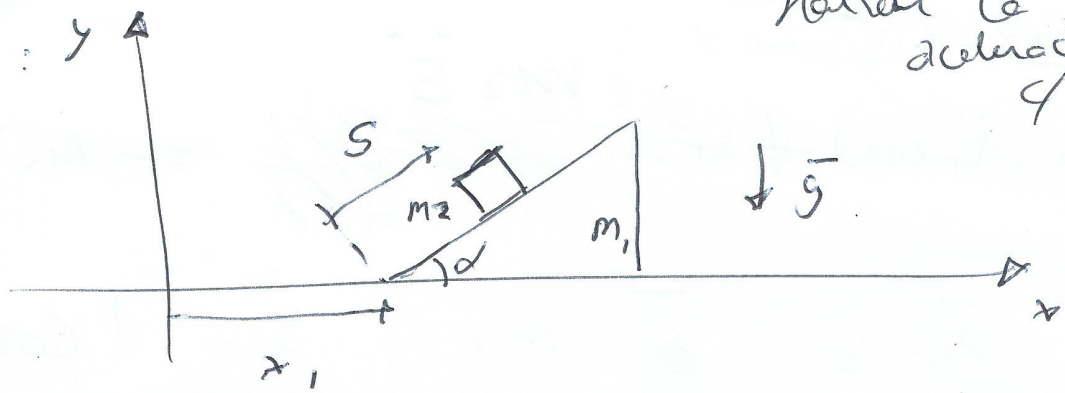
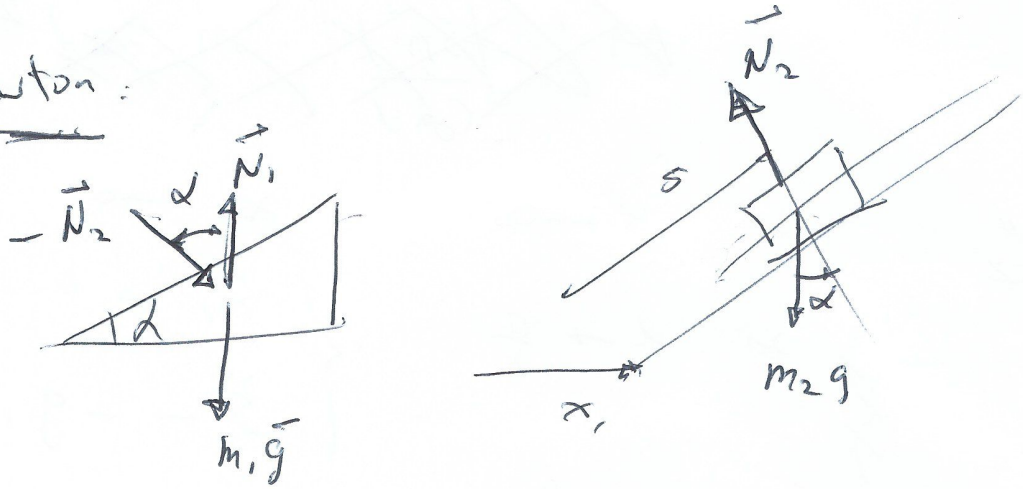
PROB: Hallar la aceleración de c/ masa.



Por Newton:



$$\begin{cases} m_1\ddot{x}_1 = N_2 \sin\alpha & (1) \\ \underbrace{m_1\ddot{y}_1}_{0} = N_1 - m_1 g - N_2\cos\alpha & (2) \end{cases}$$

$x_2 = x_1 + s\cos\alpha$

$y_2 = \cancel{x_1} + s\sin\alpha$

$\ddot{x}_2 = \ddot{x}_1 + \ddot{s}\cos\alpha$

$\ddot{y}_2 = \cancel{\ddot{x}_1} + \ddot{s}\sin\alpha$

$m_2\ddot{x}_2 = -N_2\sin\alpha$

$m_2\ddot{y}_2 = N_2\cos\alpha - m_2 g$

$$\begin{cases} m_2(\ddot{x}_1 + \ddot{s}\cos\alpha) = -N_2\sin\alpha & (3) \\ m_2\ddot{s}\sin\alpha = N_2\cos\alpha - m_2 g. & (4) \end{cases}$$

(1) + (3)

$m_1\ddot{x}_1 + m_2\ddot{x}_2 = 0$ $\quad P^{CM} = cte.$ [No hay F. exteriores horizontales]

$M\ddot{x}_1 + m_2\ddot{s}\cos\alpha = 0$

$$\ddot{s} = -\frac{M\ddot{x}_1}{m_2\cos\alpha} \quad (5)$$

③ $\cos\alpha$ + ④ $\sin\alpha$:

$$m_2 \ddot{x}_1 \cos\alpha + m_2 \ddot{s} = -m_2 g \sin\alpha \quad ⑥$$

de ⑤ en ⑥ se obtiene: (ver desarrollo en D'alembert)

$$\boxed{\ddot{x}_1 = \frac{g \sin\alpha \cos\alpha}{\sin^2\alpha + \frac{m_1}{m_2}}}$$

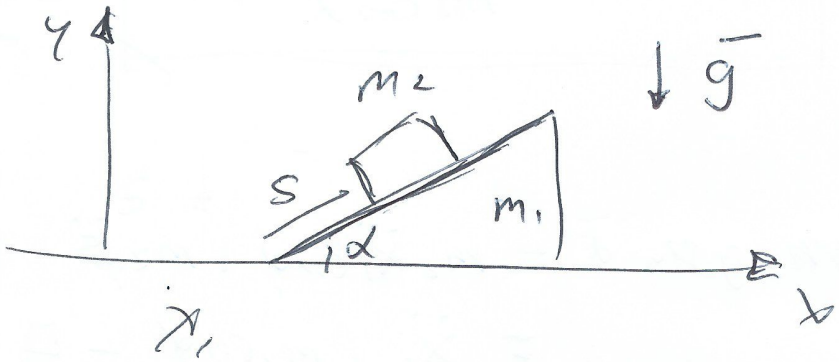
<u>Chequeo</u>: · Si $\alpha \to 0$ $\ddot{x}_1 \to 0$ ok!

· Si $\alpha \to \frac{\pi}{2}$ $\begin{cases} \ddot{x}_1 \to 0 \\ \ddot{s} \to -g. \end{cases}$ ok!

· Si $m_1 \to \infty$ (m, queda fijo) $\begin{cases} \ddot{x}_1 \to 0 \\ \ddot{s} \to -g \sin\alpha \end{cases}$

Por PTV D'alembert:

δs y δx_1 independientes



$$\left(\sum \vec{F}_i - \dot{\vec{p}}_i\right) \cdot \delta \vec{r}_i = 0$$

$$\begin{cases} \vec{F}_1 = -m_1 g \hat{y} \qquad \vec{F}_2 = -m_2 g \hat{y} \\ \delta \vec{r}_1 = \delta x_1 \hat{x} \\ \delta \vec{r}_2 = (\delta x_1 + \delta s \cos\alpha)\hat{x} + \delta s \sin\alpha \hat{y} \\ \dot{\vec{p}}_1 = m_1 \ddot{x}_1 \hat{x} \; ; \; \dot{\vec{p}}_2 = (m_2 \ddot{x}_1 + m_2 \ddot{s} \cos\alpha)\hat{x} \\ \qquad + m_2 \ddot{s} \sin\alpha \, \hat{y} \end{cases}$$

$$\vec{F}_1 \cdot \delta \vec{r}_1 = 0 \qquad \vec{F}_2 \cdot \delta \vec{r}_2 = -m_2 g \sin\alpha \, \delta s$$

$$-m_2 g \sin\alpha \, \delta s - m_1 \ddot{x}_1 \delta x_1$$

$$-(m_2 \ddot{x}_1 + m_2 \ddot{s} \cos\alpha)(\delta x_1 + \delta s \cos\alpha)$$

$$-m_2 \ddot{s} \sin^2\alpha \, \delta s = 0$$

factorizamos δx_1 y δs ; e igualamos a cero:

(1) $-m_2 g \sin\alpha - (m_2 \ddot{x}_1 + m_2 \ddot{s} \cos\alpha)\cos\alpha - m_2 \ddot{s} \sin^2\alpha = 0$ $\quad \delta s$

(2) $-m_1 \ddot{x}_1 - (m_2 \ddot{x}_1 + m_2 \ddot{s} \cos\alpha = 0$ $\quad \delta x_1$

de (2) :

$$\boxed{\ddot{s} = -\frac{M\,\ddot{x}_A}{m_2 \cos\alpha}} \quad (3)$$

OK!

de (1) :

$$-m_2 g \sin\alpha = m_2 \ddot{x}_1 \cos\alpha + m_2 \ddot{s} \quad (4)$$

(3) en (4) :

$$= \ddot{x}_1 \left(m_2 \cos\alpha - \frac{M}{\cos\alpha} \right)$$

$$= \frac{\ddot{x}}{\cos\alpha} \left(m_2 \cos^2\alpha - m_1 - m_2 \right)$$

$$= -\frac{\ddot{x}\, m_2}{\cos\alpha} \left(\sin^2\alpha + \frac{m_1}{m_2} \right)$$

$$\therefore \quad \boxed{\ddot{x}_1 = -\frac{g \sin\alpha \cos\alpha}{\sin^2\alpha + \frac{m_1}{m_2}}}$$

Notar que (4) es la ec. (6) de Newton.