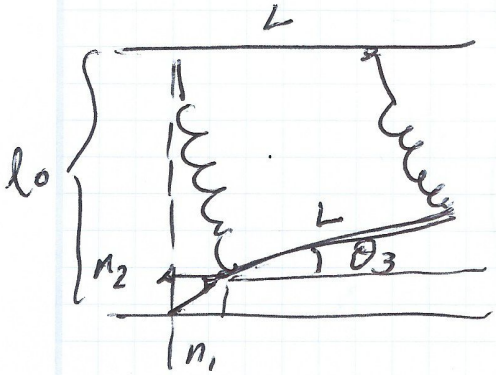
Pequeñas Oscilaciones



$n_3 = L\theta_3$

$2k(l_0 - a) = mg$

$$\boxed{l_0 = a + \frac{mg}{2k}}$$

$J_{cm} = \frac{mL^2}{12}$

Tomamos $a = 0$

$n_3 = L\theta_3$

3 GL

Equilibrio: $n_1 = n_2 = \theta_3 = 0$

$x_1 = n_1 \qquad y_1 = n_2$

$x_2 = n_1 + L\cos\theta_3$

$y_2 = n_2 + L\sin\theta_3$

$$T = \frac{m}{2}(\dot{x}_{cm}^2 + \dot{y}_{cm}^2) + \frac{I}{2}\dot{\theta}_3^2$$

$$V = \frac{k}{2}[(l_0 - n_2)^2 + n_1^2] + \frac{k}{2}[(l_0 - n_2 - L\sin\theta_3)^2 + (L\cos\theta_3 + n_1 - L)^2] - mg\left[l_0 - n_2 - \frac{L}{2}\sin\theta_3\right]$$

$x_{cm} = n_1 + \frac{L}{2}\cos\theta_3 \qquad \dot{x}_{cm} = \dot{n}_1 - \frac{L}{2}\sin\theta_3\,\dot{\theta}_3$

$y_{cm} = n_2 + \frac{L}{2}\sin\theta_3 \qquad \dot{y}_{cm} = \dot{n}_2 + \frac{L}{2}\cos\theta_3\,\dot{\theta}_3$

$$T \simeq \frac{m}{2}\left[\dot{n}_1^2 + \left(\dot{n}_2 + \frac{\dot{n}_3}{2}\right)^2\right] + \frac{m}{24}\dot{n}_3^2$$

$$V \simeq \frac{k}{2}(n_2^2 + n_1^2) + \frac{k}{2}(n_2 + n_3)^2 + \frac{k}{2}n_1^2$$

$$\Pi = m \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/2 \\ 0 & 1/2 & 1/3 \end{pmatrix}$$

$$V = k \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

i) n_1 : desacoplado. $\vec{a}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\rightarrow n_2 = 0$
$\rightarrow \theta_3 = 0.$

es autovector con frecuencia:

$$\omega_1 = \sqrt{\frac{2k}{m}} = \sqrt{\frac{g}{l_0}}$$

Movimiento horizontal.

ii) Probamos $\bar{a}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ Movimiento vertical.

$$V\bar{a}_2 = \omega^2 \Pi \bar{a}_2 \Rightarrow V\bar{a}_2 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \qquad \Pi\bar{a}_2 = \begin{pmatrix} 0 \\ 1 \\ 1/2 \end{pmatrix}$$

$$\Rightarrow \omega_2^2 = 2\frac{k}{m} \Rightarrow \omega_2 = \sqrt{\frac{2k}{m}}$$

resortes en paralelo suman sus k's.

iii) Probamos $\bar{a}_3 = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$ deja fijo el CM (sólo rota)

$$V\bar{a}_3 = k\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad \Pi\bar{a}_3 = m\begin{pmatrix} 0 \\ 0 \\ -1/6 \end{pmatrix} \Rightarrow \omega_3 = \sqrt{\frac{6k}{m}}$$

Equilibrio:

$$\frac{\partial V}{\partial n_1} = k n_1 + k\,(L\cos\theta_3 + n_1 - L) = 0 \qquad (1)$$

$$\frac{\partial V}{\partial n_2} = -k\,(l_0 - n_2) - k\,(l_0 - n_2 - L\sin\theta_3) + mg = 0 \qquad (2)$$

$$\frac{\partial V}{\partial \theta_3} = -kL\,(l_0 - n_2 - L\sin\theta_3)\cos\theta_3 - kL\sin\theta_3\,(L\cos\theta_3 + n_1 - L) + \frac{mgL}{2}\cos\theta_3 = 0 \qquad (3)$$

Si $\theta_3 = 0$, de (1) $n_1 = 0$

de (2) si $n_2 = 0 \Rightarrow 2k l_0 = mg$.
(longitud de equilibrio)

en (3) si $l_0 = \frac{mg}{2k}$ se satisface.

$$\frac{\partial^2 V}{\partial n_1^2} = 2k, \quad \frac{\partial^2 V}{\partial n_2^2} = 2k, \quad \frac{\partial^2 V}{\partial \theta_3^2} =$$

$$\frac{\partial^2 V}{\partial \theta_3^2} = kL^2\cos\theta_3\Big|_{\theta_3=0} + \frac{mgL}{2}\sin\theta_3\Big|_{\theta_3=0} = kL^2$$

$$\frac{\partial^2 V}{\partial n_2 \partial n_1} = 0 = \frac{\partial^2 V}{\partial \theta_3 \partial n_1} \qquad \frac{\partial^2 V}{\partial \theta_3 \partial n_1} = -kL\sin\theta_3\Big|_{\theta_3=0} = 0$$

$$\frac{\partial^2 V}{\partial \theta_3 \partial n_2} = kL\cos\theta_3\Big|_{\theta_3=0} = kL = \frac{\partial^2 V}{\partial n_2 \partial \theta_3}$$