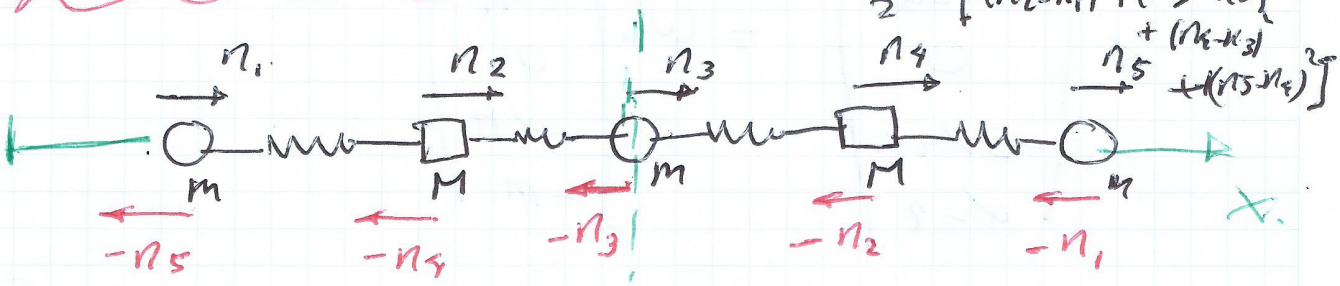
Guía 4: Mecánica Clásica

Ejemplo Simetrías

$$\mathcal{L} = \frac{1}{2} m(\dot{x}_1^2 + \dot{x}_3^2 + \dot{x}_5^2) + \frac{M}{2}(\dot{x}_2^2 + \dot{x}_4^2) - \frac{1}{2} k\left[(x_2 - x_1)^2 + (x_3 - x_2)^2 + (x_4 - x_3)^2 + (x_5 - x_4)^2\right]$$



$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \qquad \Phi \vec{x} = \begin{pmatrix} -x_5 \\ -x_4 \\ -x_3 \\ -x_2 \\ -x_1 \end{pmatrix}$$

Modos Simétricos:

$$\vec{x} = \Phi \vec{x}$$

$$x_5 = -x_1$$

$$x_4 = -x_2$$

$$x_3 = -x_3 \Rightarrow x_3 = 0$$

$$\therefore \quad \vec{q}_s = \begin{pmatrix} 1 \\ \alpha \\ 0 \\ -\alpha \\ -1 \end{pmatrix}$$

Ojo: $\mathbb{T} = \begin{pmatrix} m & & & & \\ & M & & & \\ & & m & & \\ & & & M & \\ & & & & m \end{pmatrix} \quad \mathbb{V} = k \begin{pmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{pmatrix}$

$$V\vec{a}_s = \omega_s^2 \Pi \vec{a}$$

$$K\begin{pmatrix} 1-\alpha \\ -1+2\alpha \\ 0 \\ -2\alpha+1 \\ \alpha-1 \end{pmatrix} = \omega_s^2 \begin{pmatrix} m \\ \alpha M \\ 0 \\ -\alpha M \\ -m \end{pmatrix}$$

Sólo hay 2 ecuaciones independientes, con 2 incógnitas α y ω_s^2

$$K(1-\alpha) = \omega_s^2 m \qquad (1)$$

$$K(-1+2\alpha) = \alpha\omega_s^2 M. \qquad (2)$$

$\frac{(1)}{(2)}$: $\quad \frac{1-\alpha}{-1+2\alpha} = \frac{1}{\alpha}$ $\qquad$ [Como $m = M$]

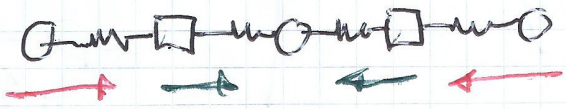
$$\alpha - \alpha^2 = -1 + 2\alpha.$$

$$\therefore \quad \alpha^2 + \alpha - 1 = 0$$

$$\alpha_\pm = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

de (1) : $\quad \omega_{s\pm}^2 = \frac{K}{m}\left(\frac{3 \mp \sqrt{5}}{2}\right)$

Modos antisimétricos $\qquad \vec{n} = -D\vec{n}$



$n_5 = n_1$

$n_4 = n_2$

$n_3 = n_3$

Cont. ejemplo simetrías:

Modo antisimétrico:

$$\vec{a}_a = \begin{pmatrix} 1 \\ \alpha \\ \beta \\ \alpha \\ 1 \end{pmatrix}$$

- Modo de frecuencia cero: traslación

$$\vec{a}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \qquad V\vec{a}_3 = 0 \Rightarrow \omega_3 = 0$$

es un modo antisimétrico. $(\alpha = \beta = 1)$

- Los otros dos modos antisimétricos son $\perp$ según Π a $\vec{a}_3$: (no desplazan el CM)

$$\vec{a}_a^{\,t}\, \Pi\, \vec{a}_3 = 0$$

$$m + \alpha M + \beta m + \alpha M + m = 0$$

$$\Rightarrow \boxed{\beta = -\frac{2\alpha M + 2m}{m}}$$

$$V\vec{a}_a = k \begin{pmatrix} 1-\alpha \\ -1+2\alpha-\beta \\ -2\alpha+2\beta \\ -1+2\alpha-\beta \\ 1-\alpha \end{pmatrix} = \omega^2 \begin{pmatrix} m \\ M\alpha \\ \beta m \\ m\alpha \\ m \end{pmatrix}$$

[Caso $m = M$]

$$k(1-\alpha) = \omega^2 m \qquad (3)$$

$$k(-1+2\alpha-\beta) = \omega^2 m \alpha \qquad (4)$$

$\frac{(3)}{(4)}$:

$$\frac{1-\alpha}{-1+2\alpha-\beta} = \frac{1}{\alpha}$$

$$\alpha - \alpha^2 = -1 + 2\alpha + 2\alpha + 2$$

$$\alpha^2 + 3\alpha + 1 = 0$$

$$\alpha_{\pm} = \frac{-3 \pm \sqrt{5}}{2}$$

Ambos modos tienen movimientos en contrafase.