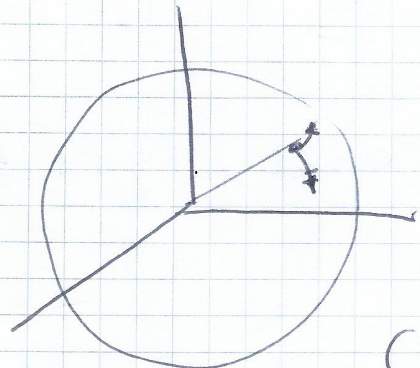
Geodésicas en la Esfera



$$d\vec{r} = R\sin\theta\, d\phi\, \hat{e}_\phi + R\, d\theta\, \hat{e}_\theta$$

$$dS = \sqrt{R^2 \sin^2\theta\, d\phi^2 + R^2 d^2\theta}$$

$$\begin{cases} dS_1 = R\sqrt{\sin^2\theta + \left(\dfrac{d\theta}{d\phi}\right)^2}\, d\phi \\[2ex] dS_2 = R\sqrt{\sin^2\theta \left(\dfrac{d\phi}{d\theta}\right)^2 + 1}\, d\theta \end{cases}$$

i) $$L = R\int_{\theta(\phi_1)}^{\theta(\phi_2)} \sqrt{\sin^2\theta + \theta'^2}\, d\phi$$

$F = R\sqrt{\sin^2\theta + \theta'^2}$ no depende de ϕ (Rol del tiempo)

$\therefore\ H = \theta' \dfrac{\partial F}{\partial \theta'} - F = cte.$ (Símil del Hamiltoniano)

$$\frac{\theta'\,\theta'}{\sqrt{\sin^2\theta + \theta'^2}} - \sqrt{\sin^2\theta + \theta'^2} = C$$

$$\frac{-\sin^2\theta}{\sqrt{\sin^2\theta + \theta'^2}} = C$$

$$\sin^4\theta = C^2(\sin^2\theta + \theta'^2)$$

$$C^2\theta'^2 = \sin^2\theta(\sin^2\theta - C^2)$$

$$\int \frac{c\, d\theta}{\sin\theta \sqrt{\sin^2\theta - c^2}} = \int d\phi$$

$$u = \cot\theta$$

$$du = -\csc^2\theta\, d\theta$$

$$\phi = -\int \frac{c\, du}{\sqrt{1 - c^2\csc^2\theta}}$$

$$\phi = -\int \frac{c\, du}{\sqrt{1 - c^2(1+u^2)}} = -\int \frac{du}{\sqrt{a^2 - u^2}}$$

con $a = \dfrac{\sqrt{1-c^2}}{c}$

$$\phi = \cos^{-1}(u/a) + \phi_0 .$$

o,

$$\cot\theta = a\cos(\phi - \phi_0)$$ (gran círculo circunmeridiano)

$$\cos\theta = a\sin\theta\cos\phi\cos\phi_0 + a\sin\theta\sin\phi\sin\phi_0 .$$

<u>por R:</u>

$$Z = AX + BY$$

está contenida en el plano que contiene el origen (y corta a la esfera en 2 partes iguales) y contiene los puntos inicial y final.