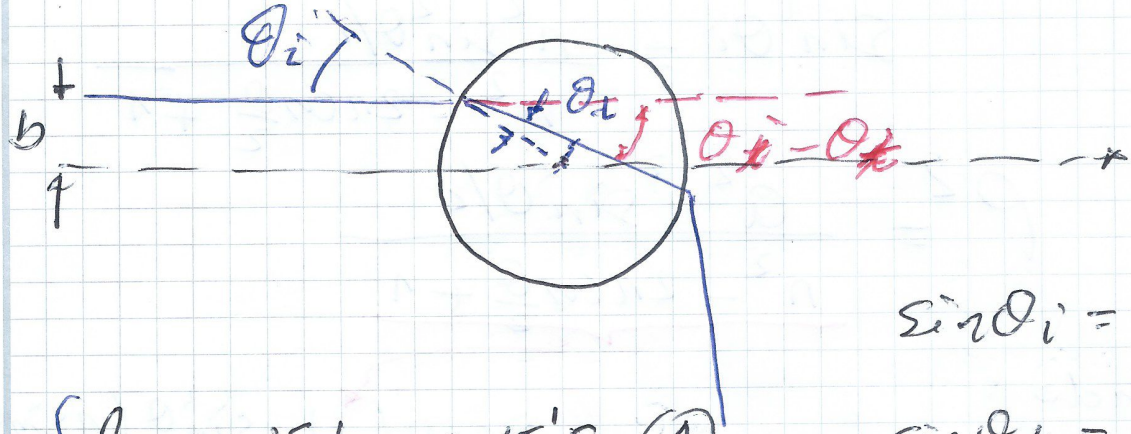
Guía 3: Mecánica

Fuerzas Centrales - Scattering

10 Pozo $V(r) \begin{cases} -V_0 & , r < a \\ 0 & , r \geq a \end{cases}$



$$\left.\begin{aligned} \sin\theta_i &= \frac{b}{a} \\ \sin\theta_t &= \frac{r_0}{a} \end{aligned}\right\} \quad (3)$$

$$\begin{cases} l = m v_0 b = m v' r_0 & (1) \\ E = \dfrac{m v_0^2}{2} = \dfrac{m v'^2}{2} - V_0 & (2) \end{cases}$$

de (3) en (1): $$\frac{\sin\theta_i}{\sin\theta_t} = \frac{v'}{v_0} = n \quad (4)$$

de (2) $$\left(\frac{v'}{v_0}\right)^2 = 1 + \frac{V_0}{E} = n^2 \quad (5)$$

$$\chi = \theta = 2(\theta_i - \theta_t) \Rightarrow \theta_t = \theta_i - \frac{\theta}{2} \quad (6)$$

(6) en (4): $$\frac{\sin\left(\theta_i - \frac{\theta}{2}\right)}{\sin\theta_i} = \cos\frac{\theta}{2} - \cot\theta_i \sin\frac{\theta}{2} = \frac{1}{n} \quad (7)$$

de (7): $$\cot\theta_i = \frac{\sqrt{1-\sin^2\theta_i}}{\sin\theta_i} = \frac{n\cos\theta/2 - 1}{n\sin\theta/2} \quad (8)$$

de (8) despejamos $\sin\theta_i$ (elevamos al cuadrado)

$$1-\sin^2\theta_i = \frac{\left(1-n\cos\frac{\theta}{2}\right)^2\sin^2\theta_i}{\sin^2\frac{\theta}{2}}$$

de (8) $$1+\cot^2\theta_i = \frac{1}{\sin^2\theta_i} = 1+\frac{\left(n\cos\frac{\theta}{2}-1\right)^2}{n^2\sin^2\theta/2}$$

$$\sin^2\theta_i = \frac{n^2\sin^2\theta/2}{n^2-2n\cos\frac{\theta}{2}+1}$$

de (3): $$\rho^2 = \frac{a^2n^2\sin^2\theta/2}{n^2-2n\cos\frac{\theta}{2}+1}$$

derivando:

$$2\rho\frac{d\rho}{d\theta} = a^2n^2\,\frac{\sin\frac{\theta}{2}\cos\frac{\theta}{2}(\quad)-\sin^2\frac{\theta}{2}\left(n\sin\frac{\theta}{2}\right)}{\left(n^2-2n\cos\frac{\theta}{2}+1\right)^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{\cancel{2\pi}}{\cancel{2\pi}\sin\theta}\cdot\rho\frac{d\rho}{d\theta}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{a^2n^2}{4\cos\frac{\theta}{2}}\,\frac{\left(n\cos\frac{\theta}{2}-1\right)\left(n-\cos\frac{\theta}{2}\right)}{\left(n^2+1-2n\cos\frac{\theta}{2}\right)^2}}$$

$$* \quad \cos\frac{\theta}{2}\left(1+n^2-2n\cos\frac{\theta}{2}\right)-\underbrace{n\sin^2\frac{\theta}{2}}_{n\left(1-\cos^2\frac{\theta}{2}\right)} =$$

$$= \cos\frac{\theta}{2}+n^2\cos\frac{\theta}{2}-\cancel{2}n\cos^2\frac{\theta}{2}-n+\cancel{n\cos^2\frac{\theta}{2}}$$