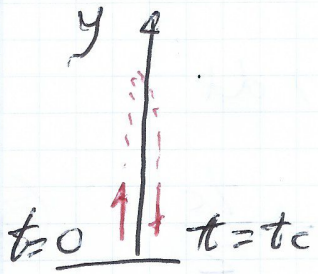
Guía 2: Mecánica Clásica

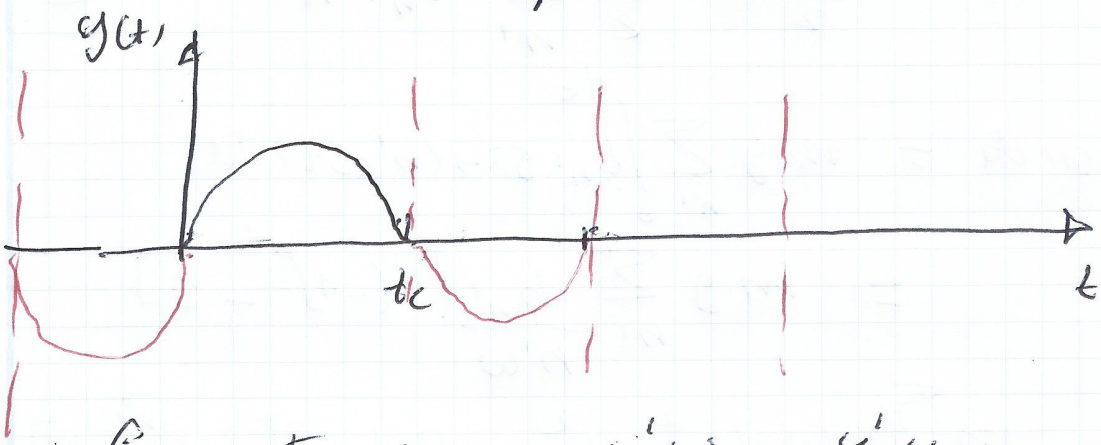
Principios Variacionales

P8



$$\begin{cases} y(0) = y(t_c) = 0 \\ t_c = \dfrac{2v_0}{g} \end{cases} \quad (1)$$

No usamos la serie de Fourier, sólo la prolongación impar.



En este caso $y'(0) = -y'(t_c)$.
Lo cual es físicamente correcto.

$$y(t) = \sum_{n'} b_{n'} \sin(n'\omega t) \quad (2)$$

$$\text{con} \quad \omega = \frac{\pi}{t_c} \Rightarrow \begin{cases} y(0) = 0 \\ y(t_c) = 0 \end{cases} \quad (3)$$

Veremos que n' debe ser impar como se puede verificar gráficamente.

El Lagrangiano es: $\mathcal{L} = \dfrac{m\dot{y}^2}{2} - mgy$ (4)

$$S = \int_0^{t_c} \mathcal{L}\, dt = \int_0^{t_c} T\, dt - \int_0^{t_c} V\, dt \quad (5)$$

húsares

de (2): $\dot{y}(t) = \sum_{n'} b_{n'} n' \omega \cos(n'\omega t)$ (6)

$$\int_0^{t_c} \frac{m}{2} \dot{y}^2(t)\,dt = \sum_{n'n''} \frac{m}{2} \omega^2 \int_0^{t_c} b_{n'} b_{n''} \cos(n'\omega t) \cos(n''\omega t)\,dt.$$

(7)

Usando: $\int_0^{t_c} \cos(n'\omega t)\cos(n''\omega t)\,dt = \delta_{n'n''} \frac{t_c}{2}$ (8)

$$\therefore \int_0^{t_c} T\,dt' = \frac{m}{2}\omega^2 \frac{t_c}{2} \sum_{n'} b_{n'}^2 n'^2 \quad (9)$$

$$\int_0^{t_c} mg\,y(t)\,dt' = mg \sum_{n'} \int_0^{t_c} b_{n'} \sin(n'\omega t)\,dt$$

$$= mg \sum_{n'} \frac{b_{n'}}{n'\omega} \left[(-1)^{n'} - 1\right]$$

$$\therefore S = m \left(\frac{\pi\omega}{4} b_{n'}^2 n'^2 - \frac{g\, b_{n'}}{n'\omega} \left[(-1)^{n'} - 1\right] \right)$$

$$\frac{\partial \Sigma}{\partial n} = m \left(\frac{\pi\omega}{2} b_{n'} n' - \frac{g}{n'\omega} \left[(-1)^{n'} - 1\right] \right) = 0$$

- Si n es par $\Rightarrow b_n = 0$.
- Si n es impar $\Rightarrow b_{2l+1} = \frac{4g}{\pi\omega^2 (2l+1)^3}$

$$y(t) = \sum_{l=0}^{N} \frac{8V_0}{\omega^2 \pi^2} \frac{1}{(2l+1)^3} \sin[(2l+1)\omega]t.$$

Check:

$$\dot{y}(0) = \frac{8V_0}{\pi^2} \sum_{l=0} \frac{1}{(2l+1)^2} \quad (\omega) \Rightarrow \boxed{\dot{y}(0) \to V_0}$$

$\pi^2/8$