Feynman's wobbling plate

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In the book *Surely You Are Joking, Mr. Feynman!* Richard Feynman tells a story of a Cornell cafeteria plate being tossed into the air. As the plate spun, it wobbled. Feynman noticed a relation between the two motions. He solved the motion of the plate by using the Lagrangian approach. This solution didn't satisfy him. He wanted to understand the motion of the plate by analyzing the motion of its individual particles and the forces acting on them. He was successful, but he didn't tell us how he did it. We provide an elementary explanation for the two-to-one ratio of wobble to spin frequencies, based on an analysis of the motion of the particles and the forces acting on them. We also demonstrate the power of numerical simulation and computer animation to provide insight into a physical phenomenon and guidance on how to do the analysis. © *2007 American Association of Physics Teachers.*

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I. INTRODUCTION

One day Feynman was sitting in the Cornell cafeteria when someone tossed a dinner plate into the air.[1] The plate had a university seal imprinted on the rim. As the plate spun, it wobbled. While Feynman watched the seal on the rim, he noticed that the spin and the wobble were not in synchrony, but were related. He had nothing to do, so he started to analyze the motion of the plate using the Lagrangian approach.[2] The equations were quite complicated, but he discovered that, when the wobble was small, the equations predicted that the plate would wobble twice as fast as it would spin.[3]

He was not satisfied with this explanation of the two-to-one ratio and wanted to understand it by looking at the elementary dynamics. In his account of the story Feynman says that he does not remember how he did it, but that he ultimately worked out the motion of the particles of mass in the plate, showing how their accelerations balanced to make the ratio of wobble to spin frequencies come out two to one.[4]

Feynman reported that the experience of solving the motion of the wobbling plate was important in returning him to physics research after his wartime experience at Los Alamos. He thought he was burned out and would never accomplish anything again, so he decided just to play with physics. He said it was "piddling around" with the wobbling plate that brought him to thinking about electron orbits in relativity, then to the Dirac equation, and finally to the research for which he was awarded the Nobel Prize.

In this paper we present what might be close to Feynman's forgotten elementary explanation of the two-to-one wobble to spin ratio. Unlike Feynman, we will use computer simulation and animation to discover the simple character of trajectories of individual particles of the thrown plate. Then we will show how the two-to-one wobble to spin ratio emerges as a direct geometrical consequence of the motion of the particles that make up the plate. Finally we will use elementary Newtonian mechanics to account for particle motion.

II. TRAJECTORIES OF INDIVIDUAL PARTICLES DISCOVERED

The motion of the plate thrown into the air can be observed by throwing the plate with a mark on its rim. It is difficult to see much in such an experiment because the motion is fast and brief. It is possible to see that wobbling is faster than rotation, but the conclusion that it is approximately twice as fast is difficult to reach. Ordinary video analysis can greatly improve our insight and can lead to the desired conclusion about the frequency ratio. But the video analysis didn't give us any clue on how the individual particles of the plate actually move. Therefore, to discover their motion we used computer modeling and visualization instead.

We wrote the Lagrange equations for the torque-free motion of a thin rigid disc with fixed center of mass to represent the cafeteria plate,[5] disregarding the irrelevant motion of its center of mass. To visualize the motion predicted by these equations, we wrote a Java applet that solves the resulting differential equations for the plate numerically and animates its motion (see Fig. 1).[6]

The applet displays the unit orthogonal axes vectors $\hat{\boldsymbol{x}}_1$, $\hat{\boldsymbol{x}}_2$, and $\hat{\boldsymbol{x}}_3$ fixed in the plate with their origin at the center of mass of the plate and $\hat{\boldsymbol{x}}_3$ perpendicular to the plate (see Fig. 1). Because we want the end points of the axes vectors to be visible in the animation, we arbitrarily chose their length to be a bit longer than the radius of the plate. The plate is stationary in the reference frame defined by these vectors. The motion of the tips of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ trace out the motion of the particles of the plate.

In what follows we summarize our observations of the computer animation. We recommend that readers play with the applet independently and reach their own conclusions. Our online materials are on EPAPS.[6] To obtain more insight into the features of the motion of the plate, we suggest that you follow the "Student exercises for the computer model" link, which opens a set of exercises devised for the applet.

The animation shows that as the plate wobbles, the $\hat{\boldsymbol{x}}_3$ vector traces out a cone as its tip moves around a small circle. We call the half-angle of this cone the angle of wobble

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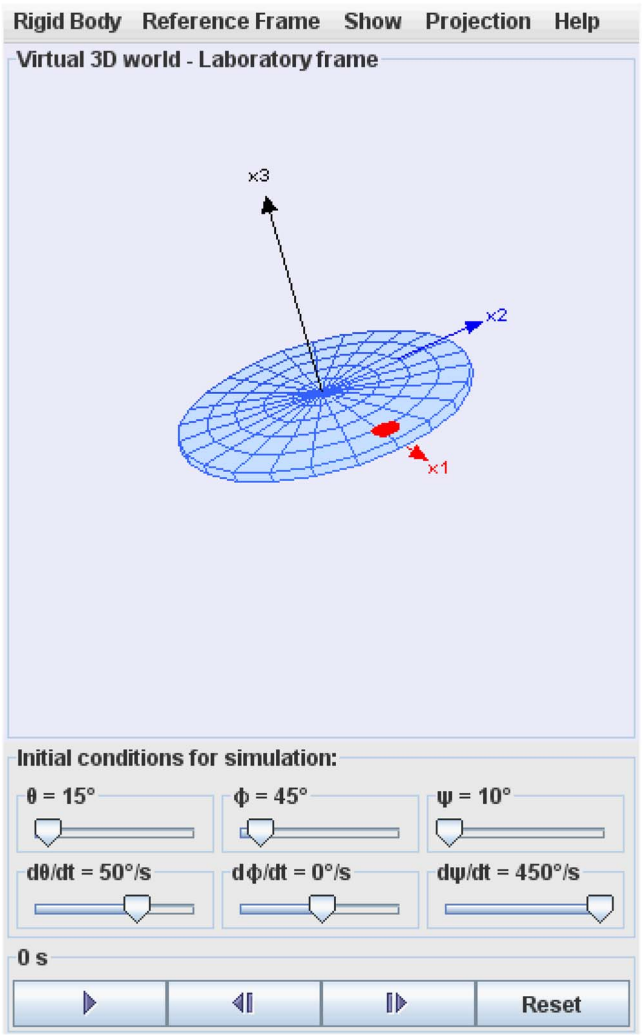


Fig. 1. Screen shot of the Java applet that animates the motion of the plate. The unit vectors $\hat{\boldsymbol{x}}_1$, $\hat{\boldsymbol{x}}_2$, and $\hat{\boldsymbol{x}}_3$ are fixed with respect to the plate. The circle on the rim represents the seal. The user sets the initial conditions of the plate motion using the sliders at the bottom (Ref. 7).

and denote it by θ. What is surprising is the character of the motion of the vectors $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$. When the angle of wobble is small, the tips of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ move along almost circular trajectories with a common origin at the center of mass of the plate, lying in slightly different planes (both at angle θ with the horizontal plane) and intersecting at only two points, as shown in Fig. 2. When the angle of wobble is not small, the tips of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ trace out curves that are not closed. The smaller the angle of wobble, the closer the traces are to circles.

III. GEOMETRICAL EXPLANATION OF THE FREQUENCY RATIO

We provide here an explanation for the two-to-one wobble to spin frequency ratio based on a knowledge of the trajectories of the individual particles of the plate that was provided by our animation. In other words we will show that once the tips of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ move on the circles described in Sec. II, the plate must wobble twice as fast as it rotates. This motion is a consequence of the fact that $\hat{\boldsymbol{x}}_3$ is always perpendicular to $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$.

We first examine the left column of Fig. 2, which displays a series of three sequential snapshots of the plate depicting a quarter-period of its spin. The frequency of motion of the tip of $\hat{\boldsymbol{x}}_3$ on its small circle is what we call the frequency of wobble. The frequency of motion of the tips of the vectors $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ on their large circles is the frequency of spin. What is the ratio of these two frequencies?

Denote by $T_{\rm spin}$ the spin period of the plate. Start at time $t=0$ with the plate in the position shown in Fig. 2(a). The tip of $\hat{\boldsymbol{x}}_1$ is in the front intersection point of the circles, lying in the horizontal plane. The tip of $\hat{\boldsymbol{x}}_2$ is 90° ahead on its circle, tilted slightly downward. The $\hat{\boldsymbol{x}}_3$ vector is always perpendicular both to $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$, and hence its tip has to be situated on the east side of its small circle.

At time $t=T_{\rm spin}/8$ the tip of $\hat{\boldsymbol{x}}_1$ has moved 45° on its circle, as shown in Fig. 2(b), and points slightly above the horizontal plane. The tip of $\hat{\boldsymbol{x}}_2$ has also moved 45° and is pointing slightly below the horizontal plane. Because $\hat{\boldsymbol{x}}_3$ is again perpendicular both to $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$, its tip must be at the north side of its small circle.

At time $t=T_{\rm spin}/4$ the tip of $\hat{\boldsymbol{x}}_1$ has moved an additional 45° on its circle, as shown in Fig. 2(c), and its tip is located at the east side of its circle, in a position highest above the horizontal plane. The $\hat{\boldsymbol{x}}_2$ vector is at the north side of its circle, positioned at the back intersection point of the circles and pointing horizontally. Therefore the tip of $\hat{\boldsymbol{x}}_3$ must be at the western side of its small circle.

We could further track the motion of the plate, but we can already draw conclusions from what we have seen so far. During a quarter of the spin period of $\hat{\boldsymbol{x}}_1$, the tip of $\hat{\boldsymbol{x}}_3$ has traced half of its circle, and therefore it is moving with double the frequency of vectors $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$. The frequency of wobbling is twice as high as the frequency of spin.

The reasoning in this section is simple and qualitative. It is straightforward to use vector algebra to show that if the tips of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ move uniformly on their circles, then the tip of $\hat{\boldsymbol{x}}_3$ moves uniformly on its small circle with twice the frequency.[8]

IV. WHY CIRCLES?

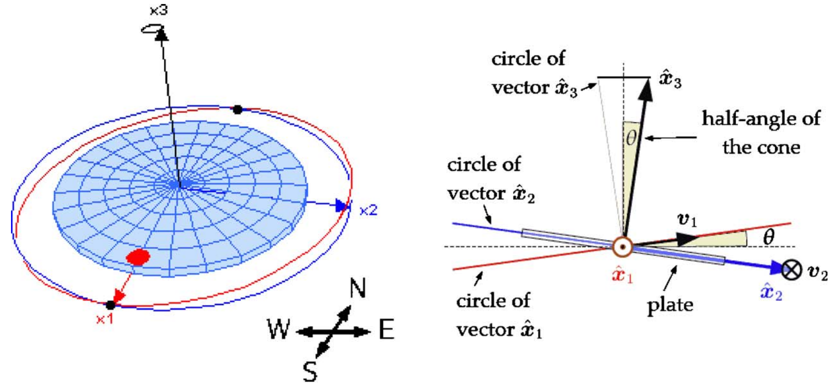
Thus far we have discovered that if the tips of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ move on the circles we have described, then the plate wobbles twice as fast as it spins. But we haven't explained why the tips of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ must move on such circles. Our strategy will be to investigate the accelerations of the tips of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ to show that when the angle of wobble is small, their tangential components vanish. Consequently the tips of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ move uniformly on circles. To derive this result we start with Newton's laws of motion.

A. Acceleration balance

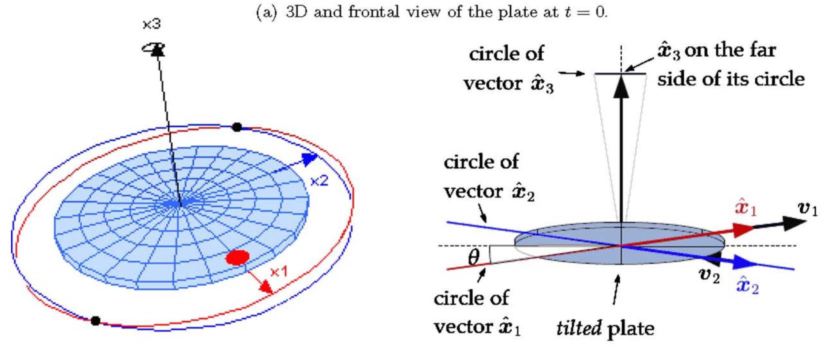
Let the mass of the plate be M and its radius R. Each mass element dm of the plate with position vector $\boldsymbol{r}$ experiences a net force $d\boldsymbol{F}$ exerted on it by neighboring particles and gravity. The relation between the force and the corresponding acceleration $\boldsymbol{a}=\ddot{\boldsymbol{r}}$ determined by this force is given by Newton's second law:

$$d\boldsymbol{F}=\ddot{\boldsymbol{r}}dm. \tag{1}$$

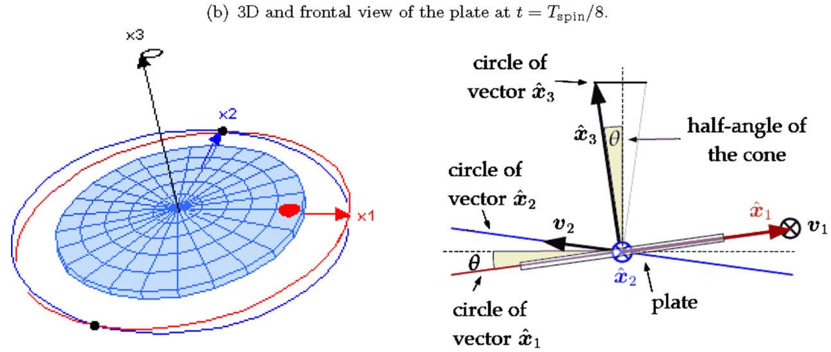
We know that because the external gravitational forces acting on the mass elements of the plate can be replaced by a



(a) 3D and frontal view of the plate at $t = 0$.



(b) 3D and frontal view of the plate at $t = T_{\rm spin}/8$.



(c) 3D and frontal view of the plate at $t = T_{\rm spin}/4$.

Fig. 2. Left: Three sequential snapshots of the plate separated by an eighth-period of its spin (a quarter-period of its wobble). The two black dots represent the front and back intersection points of the orbits followed by the tips of the vectors $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$. These orbits are circles. Because $\hat{\boldsymbol{x}}_3$ is perpendicular to $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$, it must follow a circular path with twice the frequency. The directions NSEW are used in the text description. Right: Frontal views of the plate corresponding to the snapshots on the left. The vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ represent the velocities of the tips.

net gravitational force acting at its center of mass, there is zero torque acting on the plate relative to the center of mass:

$$\boldsymbol{\tau} = \int\int_{\text{all the mass elements}} \boldsymbol{r} \times d\boldsymbol{F}$$

$$= \int\int_{\text{all the mass elements}} \boldsymbol{r} \times \ddot{\boldsymbol{r}}\, dm = \boldsymbol{0}. \quad (2)$$

The position vector of the mass element dm shown in Fig. 3 can be expressed as

$$\boldsymbol{r} = r \cos \alpha\, \hat{\boldsymbol{x}}_1 + r \sin \alpha\, \hat{\boldsymbol{x}}_2. \quad (3)$$

If we differentiate Eq. (3) twice with respect to time, we obtain an expression for the acceleration of the mass element

$$\ddot{\boldsymbol{r}} = r \cos \alpha\, \ddot{\hat{\boldsymbol{x}}}_1 + r \sin \alpha\, \ddot{\hat{\boldsymbol{x}}}_2. \quad (4)$$

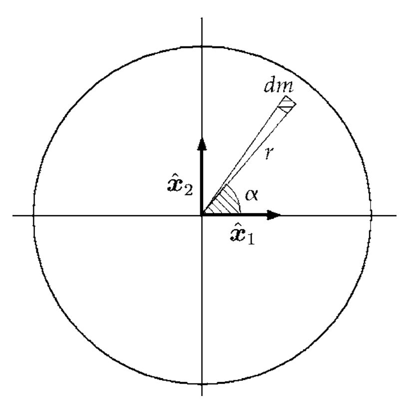


Fig. 3. The plate in the coordinate system defined by $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ fixed in the plane of the plate. The position of the mass element $dm = (M/\pi R^2) r d\alpha dr$ can be determined either by its position vector $\boldsymbol{r}$ or by polar coordinates r and α.

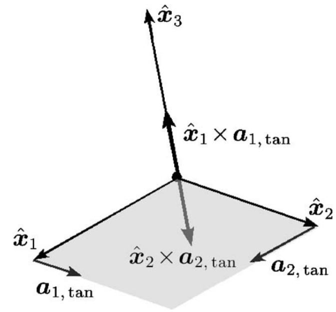


Fig. 4. There are two ways that the vector products in Eq. (7) can balance each other. The first one is shown in the figure. The $\boldsymbol{a}_{1,\tan}$ vector is parallel to $\hat{\boldsymbol{x}}_2$ and $\boldsymbol{a}_{2,\tan}$ is parallel to $\hat{\boldsymbol{x}}_1$. The second possibility differs only in the opposite directions of $\boldsymbol{a}_{1,\tan}$ and $\boldsymbol{a}_{2,\tan}$ and consequently the opposite directions of the vector products $\hat{\boldsymbol{x}}_1 \times \boldsymbol{a}_{1,\tan}$ and $\hat{\boldsymbol{x}}_2 \times \boldsymbol{a}_{2,\tan}$.

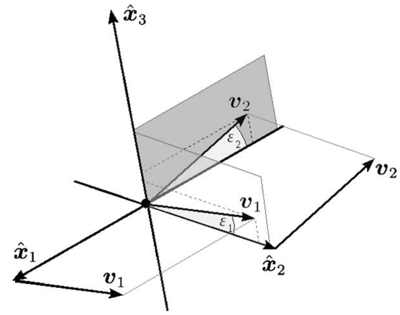


Fig. 5. The vectors $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ and the velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ when the wobble is small. The case in which the angle ε_1 between $\boldsymbol{v}_1$ and $\hat{\boldsymbol{x}}_2$ has its maximum value, $\varepsilon_1 = 2\theta$, is depicted by the frontal view in Fig. 2(a). The other extreme is shown in Fig. 2(c) corresponding to the situation in which the angle ε_2 between $\boldsymbol{v}_2$ and $-\hat{\boldsymbol{x}}_1$ assumes its maximum value, $\varepsilon_2 = 2\theta$. In the general case shown here and in Fig. 2(b) the values of the angles are between 0 and 2θ.

After the substitution of Eqs. (3) and (4) into Eq. (2), the vector product $\boldsymbol{r} \times \ddot{\boldsymbol{r}}$ contains four terms, and therefore the torque can be written as the sum of four integrals.[9] We integrate over r and α and find that two of the four integrals vanish and the expression for the net torque acting on the plate takes the simple form

$$\boldsymbol{\tau} = \frac{MR^2}{4}[(\hat{\boldsymbol{x}}_1 \times \ddot{\hat{\boldsymbol{x}}}_1) + (\hat{\boldsymbol{x}}_2 \times \ddot{\hat{\boldsymbol{x}}}_2)]. \tag{5}$$

Because the net torque has to be zero, we obtain an elegant condition for $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ and their accelerations $\ddot{\hat{\boldsymbol{x}}}_1 \equiv \boldsymbol{a}_1$ and $\ddot{\hat{\boldsymbol{x}}}_2 \equiv \boldsymbol{a}_2$,

$$(\hat{\boldsymbol{x}}_1 \times \boldsymbol{a}_1) + (\hat{\boldsymbol{x}}_2 \times \boldsymbol{a}_2) = \boldsymbol{0}, \tag{6}$$

which might be what Feynman called the acceleration balance. This condition can be further simplified. The radial components of the accelerations, $\boldsymbol{a}_{1,\mathrm{rad}}$ and $\boldsymbol{a}_{2,\mathrm{rad}}$, are parallel to $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ and therefore do not contribute to the vector products in Eq. (6), and the vector products can be expressed in terms of the tangential accelerations $\boldsymbol{a}_{1,\tan}$ and $\boldsymbol{a}_{2,\tan}$. If we write the accelerations as $\boldsymbol{a}_1 = \boldsymbol{a}_{1,\mathrm{rad}} + \boldsymbol{a}_{1,\tan}$ and $\boldsymbol{a}_2 = \boldsymbol{a}_{2,\mathrm{rad}} + \boldsymbol{a}_{2,\tan}$ and substitute this form into Eq. (6), we obtain the simplified acceleration balance condition

$$(\hat{\boldsymbol{x}}_1 \times \boldsymbol{a}_{1,\tan}) + (\hat{\boldsymbol{x}}_2 \times \boldsymbol{a}_{2,\tan}) = \boldsymbol{0}. \tag{7}$$

B. Directions and magnitudes of tangential accelerations

We will show that the acceleration balance condition expressed by Eq. (7) can be satisfied only if the $\boldsymbol{a}_{1,\tan}$ and $\boldsymbol{a}_{2,\tan}$ vectors lie in the plane specified by $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$. It follows from Eq. (7) that the vector products $\hat{\boldsymbol{x}}_1 \times \boldsymbol{a}_{1,\tan}$ and $\hat{\boldsymbol{x}}_2 \times \boldsymbol{a}_{2,\tan}$ are opposite vectors and therefore must lie along a single direction; call it $\boldsymbol{p}$. The vector $\hat{\boldsymbol{x}}_1 \times \boldsymbol{a}_{1,\tan}$ is perpendicular to $\hat{\boldsymbol{x}}_1$, and the vector $\hat{\boldsymbol{x}}_2 \times \boldsymbol{a}_{2,\tan}$ is perpendicular to $\hat{\boldsymbol{x}}_2$. Therefore $\boldsymbol{p}$ must be perpendicular to both $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$. Thus $\boldsymbol{p}$ must be parallel to $\hat{\boldsymbol{x}}_3$. Consequently either $\boldsymbol{a}_{1,\tan}$ must be parallel to $\hat{\boldsymbol{x}}_2$ and $\boldsymbol{a}_{2,\tan}$ must be parallel to $\hat{\boldsymbol{x}}_1$, as shown in Fig. 4, or $\boldsymbol{a}_{1,\tan}$ must be antiparallel to $\hat{\boldsymbol{x}}_2$ and $\boldsymbol{a}_{2,\tan}$ must be antiparallel to $\hat{\boldsymbol{x}}_1$.

Because $\hat{\boldsymbol{x}}_1$ is a unit vector perpendicular to $\boldsymbol{a}_{1,\tan}$, the magnitude of the vector $\hat{\boldsymbol{x}}_1 \times \boldsymbol{a}_{1,\tan}$ is $|\boldsymbol{a}_{1,\tan}|$. For similar reasons the magnitude of $\hat{\boldsymbol{x}}_2 \times \boldsymbol{a}_{2,\tan}$ is $|\boldsymbol{a}_{2,\tan}|$. Because the vector products are opposite vectors, it follows from Eq. (7) that the magnitude of both tangential accelerations must be equal, that is, $|\boldsymbol{a}_{1,\tan}| = |\boldsymbol{a}_{2,\tan}| \equiv a_{\tan}$.

The fact that $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ are perpendicular has important implications for our analysis. It can be expressed by the condition

$$\hat{\boldsymbol{x}}_1 \cdot \hat{\boldsymbol{x}}_2 = 0. \tag{8}$$

If we differentiate Eq. (8) twice with respect to time, we obtain a constraint on the accelerations and velocities of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$:

$$\boldsymbol{a}_1 \cdot \hat{\boldsymbol{x}}_2 + \boldsymbol{a}_2 \cdot \hat{\boldsymbol{x}}_1 + 2\boldsymbol{v}_1 \cdot \boldsymbol{v}_2 = 0. \tag{9}$$

If we again decompose the accelerations into their radial and tangential components, Eq. (9) simplifies to

$$a_{\tan} = |\boldsymbol{v}_1 \cdot \boldsymbol{v}_2|, \tag{10}$$

which is valid for an arbitrary motion of the plate, enabling us to determine the magnitude of the tangential accelerations.

C. Consequences for small-angle wobble

We know that the velocity vector $\boldsymbol{v}_1$ is perpendicular to $\hat{\boldsymbol{x}}_1$. If it were not, $\hat{\boldsymbol{x}}_1$ would change its magnitude in time. Therefore $\boldsymbol{v}_1$ can be expressed as a linear combination of $\hat{\boldsymbol{x}}_2$ and $\hat{\boldsymbol{x}}_3$. Similarly, the velocity $\boldsymbol{v}_2$ is perpendicular to $\hat{\boldsymbol{x}}_2$ and can be expressed as a linear combination of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_3$. Moreover, when the wobble is small ($\theta \ll 1$), the angles ε_1 and ε_2 in Fig. 5 are both also small.

To see what Eq. (10) and Fig. 5 predict for the magnitudes of the tangential accelerations, we express the velocity vectors as linear combinations of vectors fixed in the plate:

$$\boldsymbol{v}_1 = |\boldsymbol{v}_1| \cos \varepsilon_1 \, \hat{\boldsymbol{x}}_2 + |\boldsymbol{v}_1| \sin \varepsilon_1 \, \hat{\boldsymbol{x}}_3, \tag{11a}$$

$$\boldsymbol{v}_2 = -|\boldsymbol{v}_2| \cos \varepsilon_2 \, \hat{\boldsymbol{x}}_1 + |\boldsymbol{v}_2| \sin \varepsilon_2 \, \hat{\boldsymbol{x}}_3, \tag{11b}$$

and substitute them into Eq. (10). Three of the four scalar products in the resulting expression are zero because the vectors are perpendicular. The remaining term gives the magnitude of the tangential acceleration

$$a_{\tan} = |\boldsymbol{v}_1||\boldsymbol{v}_2| \sin \varepsilon_1 \sin \varepsilon_2 \approx |\boldsymbol{v}_1||\boldsymbol{v}_2| \varepsilon_1 \varepsilon_2. \tag{12}$$

If the wobble is small, that is, $\theta \to 0$, then $\varepsilon_1, \varepsilon_2 \to 0$ and Eq. (12) implies that the tangential acceleration will vanish as a small quantity of second order. Therefore when the wobble angle is small, the tips of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ experience practically no tangential acceleration. The direct consequence of this fact is that $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ move uniformly on their unit circles centered on the center of mass of the plate and lying in the planes specified by their initial position and velocity vectors. It follows that the circles of $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$ will coincide if and only if $\boldsymbol{v}_1$ is parallel or antiparallel to $\hat{\boldsymbol{x}}_2$, and $\boldsymbol{v}_2$ is parallel or antiparallel to $\hat{\boldsymbol{x}}_1$. Otherwise, the circles will lie in slightly different planes intersecting in a line going through the center of mass of the plate, as shown in Fig. 2.

V. DISCUSSION

The problem of the torque-free motion of a rigid thin disc has an exact solution based on Euler's equations that is well known and described in many textbooks on classical mechanics.[10] It is interesting to see how the results obtained by our simple analysis compare with those obtained by the exact solution. Because the comparison is much more specialized than the presentation above, it can be found as extra material in Ref. 6.

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[1]Richard P. Feynman, *Surely, You Are Joking, Mr. Feynman!* (Norton, New York, 1997), pp. 173–174.

[2]James Gleick, *Genius: The Life and Science of Richard Feynman* (Vintage, New York, 1992), pp. 227–228.

[3]The original Feynman's story, as told in Ref. 1, has a twist: Feynman states the plate spins twice as fast as it wobbles, while in truth it wobbles twice as fast as it spins. Whether his error was a mere slip in memory, or another practical joke meant for those who do physics without experimenting, is not known. The argument based on Euler's equations of rigid body motion proving that Feynman was wrong is given by Benjamin Fong Chao, "Feynman's dining hall dynamics," Phys. Today **42**(2), 15 (1989). Similar reasoning can be found in Jagdish Mehra, *The Beat of a Different Drum: The Life and Science of Richard Feynman* (Oxford U.P., New York, 1994), pp. 179–180.

[4]One treatment of the plate based on the analysis of motion of its representative particles was provided by J. C. Martinez, "Force-free precession of a spinning plate," Eur. J. Phys. **13**, 142–144 (1992). Very interesting insights to the problem of particles motion can be found in P. L. Edwards, "A physical explanation of the gyroscope effect," Am. J. Phys. **45**, 1194–1195 (1977).

[5]The differential equations obtained from the Lagrangian for the plate can be found in the material mentioned in Ref. 6.

[6]See EPAPS Document No. E-AJPIAS-75-001702 for the computer model of the plate (Java applet), simple tutorials for students' active explorations in the form of exercises where students need to interact with the applet that provides direct experience and support in better understanding of concepts and related behavior. The materials and the paper assume only knowledge at an introductory mechanics level. Advanced technical materials meant for teachers or students familiar with advanced mechanics subject, the theory of rigid body, provide a connection between our elementary explanation and the standard approach of the rigid body theory based on Euler's equations. This document can be reached via a direct link in the online article's HTML reference section or via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html).

[7]The initial orientation of the plate in the applet is given by the Euler angles θ, ϕ, and ψ, as defined in Herbert Goldstein, Charles Poole, and John Safko, *Classical Mechanics*, 3rd ed. (Addison-Wesley, New York, 2002), Sec. 4.4. Correspondingly, the initial motion of the plate is represented by rates of change $\dot{\theta}$, $\dot{\phi}$, and $\dot{\psi}$.

[8]The alternative quantitative proof based on vector algebra is presented as a tutorial in Ref. 6.

[9]Calculation of the integrals is straightforward and is included in the problems given in Ref. 6.

[10]See for example David J. Morin, *There once was a classical theory…*, *Introductory Classical Mechanics with Problems and Solutions*, Sec. 8.6. available at ⟨http://isites.harvard.edu/fs/docs/icb.topic58975.files/old-book/ch8.pdf⟩, or L. N. Hand and J. D. Finch, *Analytical Mechanics* (Cambridge U.P., Cambridge, UK, 1998), p. 296, or John R. Taylor, *Classical Mechanics* (University Science Books, Sausalito, CA, 2004), Sec. 10.8.