

Desigualdad de Chebyshev

$$P(|X - E(X)| > c) \leq \frac{\text{Var}(X)}{c^2}$$

$$\begin{aligned} \text{Var}(X) &= \int_{-\infty}^{\infty} (t - \mu)^2 f_X(t) dt \geq \int_{-\infty}^{\mu-c} (t - \mu)^2 f_X(t) dt + \int_{\mu+c}^{\infty} (t - \mu)^2 f_X(t) dt \\ &\geq \int_{-\infty}^{\mu-c} c^2 f_X(t) dt + \int_{\mu+c}^{\infty} c^2 f_X(t) dt \\ &= c^2 \left[\int_{-\infty}^{\mu-c} f_X(t) dt + \int_{\mu+c}^{\infty} f_X(t) dt \right] \\ &= c^2 \left[P(X < \mu - c) + P(X > \mu + c) \right] \\ &= c^2 \left[P(X - \mu < -c) + P(X - \mu > c) \right] = c^2 P(|x - \mu| > c) \end{aligned}$$

