Negative dispersion using pairs of prisms

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We show that pairs of prisms can have negative group-velocity dispersion in the absence of any negative material dispersion. A prism arrangement is described that limits losses to Brewster-surface reflections, avoids transverse displacement of the temporally dispersed rays, permits continuous adjustment of the dispersion through zero, and yields a transmitted beam collinear with the incident beam.

Negative group-velocity dispersion is important in ultrashort-pulse generation.[1] The use of diffraction gratings to generate negative dispersion,[2] however, introduces relatively large losses and does not provide a dispersion easily adjusted through zero value. We describe the use of prism pairs to provide negative group-velocity dispersion that is both low loss and easily adjusted from negative through positive values. Additional advantages are an absence of transverse displacement of the temporally dispersed rays (such displacement can broaden ultrashort pulses) and a transmitted beam collinear with the incident beam. The latter feature is useful, for example, in introducing negative group-velocity dispersion in existing optical devices, such as a colliding-pulse laser.[3] Recent related papers are those of Gordon and Fork,[4] which describes a resonator with net negative group-velocity dispersion, Martinez *et al.*,[5] which describes the advantages of tunable group-velocity dispersion in a passively mode-locked laser, and Martinez *et al.*,[6] which discusses the general relationship between angular dispersion and negative group-velocity dispersion.

Although a number of prism arrangements can be devised,[3] one arrangement is especially advantageous (Fig. 1). We give principal attention to that case of four identical prisms used at minimum deviation and Brewster's angle incidence at each surface. The entrance face of prism II is parallel to the exit face of prism I, the exit face of prism II is parallel to the entrance of prism I, etc., and the prisms have been cut so that the angle of minimum deviation is also Brewster's angle. The plane MM' normal to the rays between prisms II and III and midway between the two prisms is a plane of symmetry.

We calculate the dispersion constant

$$D = -L^{-1}\frac{\mathrm{d}T}{\mathrm{d}\lambda} = \left(\frac{\lambda}{cL}\right)\frac{\mathrm{d}^2P}{\mathrm{d}\lambda^2} \qquad (1)$$

after the manner of Gordon and Fork.[4] Here L is the physical length of the light path, P is the optical path length, λ is the optical wavelength in air, and T is the time for the light pulse to transverse L. To obtain $\mathrm{d}^2P/\mathrm{d}\lambda^2$, consider the rays that propagate near the apices of the prisms. Let the extreme ray that passes from apex to apex be a reference ray, and define its slant length between prisms I and II (and by symmetry also between prisms III and IV) as l.

The optical path length of a ray that propagates at an angle β with respect to the reference ray is calculated by using an optical construction similar to that employed by Gordon and Fork.[4] Consider Fig. 2(a), where CB is the reference ray from the apex of prism I to the apex of prism II. We seek the optical path length of the ray indicated by the path CDE. The path AB equals CDE because AC and BE are both possible wave fronts. The path CJ in Fig. 2(b) is equal and parallel to AB by construction and hence also equal to CDE. It follows that the optical path of the ray CDE is

$$P = l\cos\beta. \qquad (2)$$

The optical path lengths EFG and BH are equal because BE and GH are both possible wave fronts and FG and BH are parallel. (The rays FG and BH are parallel to the incident ray and hence to each other because the initial prism pair is equivalent in its effect on the ray direction to a slab having plane-parallel faces.) The section of path indicated by rays EFG and BH therefore makes no contribution to $\mathrm{d}^2P/\mathrm{d}\lambda^2$. By symmetry, the

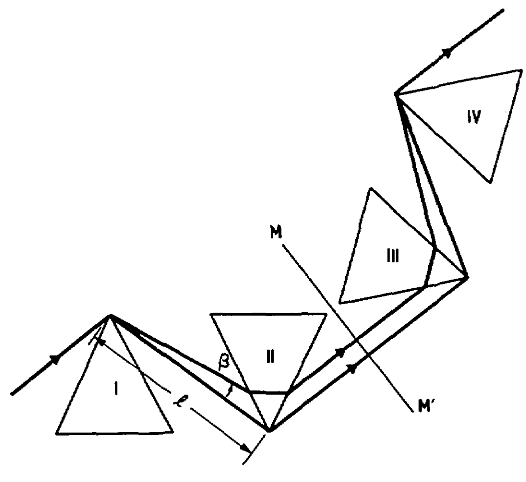

Fig. 1. Four-prism sequence having negative dispersion. The prisms are used at minimum deviation and oriented so that the rays enter and leave at Brewster's angle. The arrangement is symmetric about the plane MM'.

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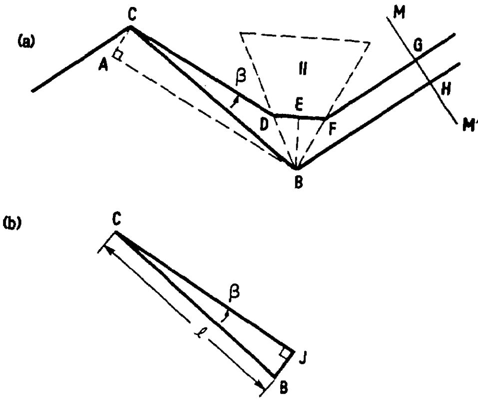

Fig. 2. Construction for calculating the paths *CDE*, *EFG*, and *BH*. (a) Path *CDE* and path *AB* are equal because *AC* and *BE* are both possible wave fronts. (b) Path *CJ* is both parallel and equal to *AB* by construction. It follows that the optical path length of *CDE* is equal to $l \cos\beta$.

second pair of prisms introduces the same amount of dispersion as the first pair but reverses the transverse displacement of the rays. The final rays are therefore collinear with each other and collinear with the path of the incident rays. The total optical path that contributes to the dispersion is then

$$P = 2l\cos\beta, \tag{3}$$

and

$$dP/d\beta = -2l\sin\beta, \tag{4}$$

$$d^2P/d\beta^2 = -2l\cos\beta. \tag{5}$$

Employing the chain rule for derivatives, we have

$$\frac{d^2P}{d\lambda^2} = \left[\frac{d^2n}{d\lambda^2}\frac{d\beta}{dn} + \left(\frac{dn}{d\lambda}\right)^2\frac{d^2\beta}{dn^2}\right]\frac{dP}{d\beta} + \left(\frac{dn}{d\lambda}\right)^2\left(\frac{d\beta}{dn}\right)^2\frac{d^2P}{d\beta^2}. \tag{6}$$

The evaluation of these derivatives differs from the case treated by Gordon and Fork.[4] In the present case the angle of incidence of the beam at prism I, ϕ_1, is taken as fixed, and the angle that the transmitted beam makes with the normal to the prism face, ϕ_2, is allowed to vary. Let the respective interior angles be ϕ_1' and ϕ_2' (i.e., $n \sin\phi_1' = \sin\phi_1$ and $n\sin\phi_2' = \sin\phi_2$; see Ref. 7 for a typical figure using this notation). For prism index n and apex angle α we use Snell's law and the relation $\alpha = \phi_1' + \phi_2'$ to obtain

$$d\phi_2/dn = (\cos\phi_2)^{-1}[\sin(\phi_2') + \cos(\phi_2')\tan(\phi_1')], \tag{7}$$

$$\frac{d^2\phi_2}{dn^2} = \tan\phi_2\left(\frac{d\phi_2}{dn}\right)^2 - \frac{\tan^2\phi_1'}{n}\left(\frac{d\phi_2}{dn}\right). \tag{8}$$

For minimum deviation and Brewster-angle incidence, $\phi_1' = \phi_2'$ and $\tan\phi_2 = n$. By inspection, $d\beta/dn = -(d\phi_2/dn)$ and $d^2\beta/dn^2 = -(d^2\phi_2/dn^2)$, which yields

$$d\beta/dn = -2, \tag{9}$$

$$d^2\beta/dn^2 = -4 + 2/n^3. \tag{10}$$

Inserting these relations and Eqs. (4) and (5) into Eq. (6) yields

$$\frac{d^2P}{d\lambda^2} = 4l\left\{\left[\frac{d^2n}{d\lambda^2} + \left(2n - \frac{1}{n^3}\right)\left(\frac{dn}{d\lambda}\right)^2\right]\sin\beta - 2\left(\frac{dn}{d\lambda}\right)^2\cos\beta\right\}. \tag{11}$$

In general, β is of the order of the angular deviation of the ray bundle, so $\sin\beta \ll \cos\beta$. This prism arrangement therefore has negative dispersion for sufficiently large values of l, provided that $d^2n/d\lambda^2$ is not excessively large compared with $(dn/d\lambda^2)$. We can evaluate Eq. (11) for typical values for quartz at 0.620 μm, which are $n = 1.457$, $dn/d\lambda = -0.03059\ \mu m^{-1}$, and $d^2n/d\lambda^2 = 0.1267\ \mu m^{-2}$.[8] The term $l\sin\beta$ need only be of the order of twice the spot size, or ~2 mm,[9] and $\cos\beta$ can be approximated by unity, so

$$d^2P/d\lambda^2 = 1.0354 - l(7.48 \times 10^{-3}), \tag{12}$$

where P and l are measured in millimeters and λ in micrometers. For $l \gtrsim 138.4$ mm, for example, four quartz prisms oriented as shown in Fig. 1 have negative dispersion. The magnitude of this negative dispersion can be compared with the positive dispersion of quartz, e.g., by dividing by $d^2n/d\lambda^2$ given above for quartz. For example, for $l = 250$ mm Eq. (1) predicts a negative dispersion adequate to compensate 6.6 mm of quartz.

An arrangement with a negative dispersion equivalent to that for this four-prism case can be obtained by using only two prisms and placing a flat mirror at the symmetry plane MM'.[6] The temporally dispersed beam is then collinear with the incident beam but oppositely directed. Beam separation can be achieved, for example, by offsetting the return beam in the direction normal to the plane of the figure before retroreflection. Multiple-pass arrangements could also be devised to increase the net negative dispersion further. When transverse displacement of the dispersed rays is not a problem, negative dispersion can also be obtained by using only two prisms in transmission, such as prisms I and II in Fig. 1.

We have encountered a popular notion that prisms (at least those having positive material dispersion) cannot produce negative dispersion. It is important in

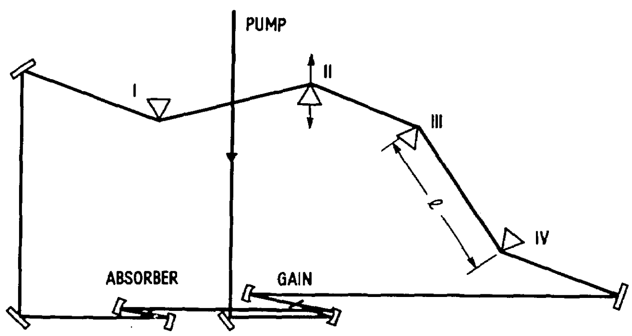

Fig. 3. Colliding-pulse laser with adjustable negative dispersion. Each prism can be translated along a line normal to its base, as indicated, for example, for prism II.

this regard to recognize that it is the second derivative of the path length with respect to wavelength that determines the group-velocity dispersion; see Eq. (1) above.[4] The phase delay for the blue components is larger than that for the red components since $dP/d\lambda$ is negative. The group velocity, however, is determined by $d^2P/d\lambda^2$. Thus the blue components traverse the prism sequence in a shorter time than do the red components, despite the negative value of $dP/d\lambda$.

As a way of testing this calculation we introduced this four-prism sequence in a colliding-pulse laser[3] in the manner indicated in Fig. 3. The resulting increase in laser threshold typically amounts to less than 0.5-W pump power. For $l = 250$ mm we can adjust the laser through the minimum-pulse-width condition by translating any one of the prisms along an axis normal to its base. This motion introduces a positive dispersion of variable magnitude[10] without altering the ray directions or the negative dispersion that is due to the geometry of the ray paths. The amount of positive material dispersion that must be introduced to obtain the minimum pulse width, approximately a 1-cm path in quartz, is consistent with negative dispersion that is due to the geometric path through the prism sequence, as predicted by Eq. (11).

In this preliminary investigation we have not observed significant pulse shortening below that already obtained by adjusting the mirror spectra in the resonator (~65 fsec).[1] This observation agrees with the earlier assumption that the intracavity dispersion has already been adjusted close to zero dispersion simply by selecting mirror spectra that yield the shortest pulse.[1] We expect that future improvements, such as introduction of appropriately adjusted amounts of self-phase modulation and negative dispersion, would yield shorter pulses.[5]

The third derivative $d^3P/d\lambda^3$ can be evaluated in a manner similar to that used above. Although the resulting expression is rather complex, it is well approximated for typical materials, such as quartz, by

$$\frac{d^3P}{d\lambda^3} \cong 4l\left(\frac{d^3n}{d\lambda^3}\sin\beta - 6\frac{dn}{d\lambda}\frac{d^2n}{d\lambda^2}\cos\beta\right). \qquad (13)$$

The variation in $d^2P/d\lambda^2$ over the bandwidth of a pulse is relatively small for typical cases of interest, for example, 5% for a 60-fsec pulse and quartz prisms. For significantly shorter pulses these third-derivative terms could become important.

In summary, this prism arrangement provides negative group-velocity dispersion with low insertion loss, no transverse displacement of the temporally dispersed rays, a magnitude of dispersion easily adjusted through zero, and a transmitted beam collinear with the incident beam. The negative dispersion is also appropriate to compensate for the amount of positive dispersion, for example, in the 0–4 cm of quartz frequently encountered in work with femtosecond pulses. The possibility of extending these techniques to smaller structures, such as semiconductor devices,[6] is suggested by the quadratic dependence of the dispersion on $dn/d\lambda$ and the order-of-magnitude increase in $dn/d\lambda$ for semiconductor materials compared with quartz.

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References

1. R. L. Fork, C. V. Shank, R. Yen, and C. A. Hirlimann, IEEE J. Quantum Electron. **QE-19**, 500 (1983).
2. E. B. Treacy, IEEE J. Quantum Electron. **QE-5**, 454 (1969).
3. R. L. Fork, B. I. Greene, and C. V. Shank, Appl. Phys. Lett. 38, 671 (1981).
4. J. P. Gordon and R. L. Fork, Opt. Lett. **9**, 153 (1984).
5. O. E. Martinez, R. L. Fork, and J. P. Gordon, Opt. Lett. (to be published).
6. O. E. Martinez, J. P. Gordon, and R. L. Fork, J. Opt. Soc. Am. B (to be published).
7. F. A. Jenkins and H. E. White, *Fundamentals of Optics* (McGraw-Hill, New York, 1957), p. 21.
8. D. Marcuse, Appl. Opt. **19**, 1653 (1980).
9. For our idealized example, $l \sin\beta$ need only be of the order of the beam diameter; however, actual systems require that the incident beam also pass at least a beam diameter inside the apex of the first prism. We account for this by taking $l \sin\beta$ as twice the beam diameter.
10. The use of a prism translation of this type to adjust the amount of positive material dispersion in a laser was previously reported. See, e.g., W. Dietel, J. J. Fontaine, and J. C. Diels, Opt. Lett. **8**, 4 (1983).