TIME-DEPENDENT STATISTICAL PROPERTIES OF THE LASER RADIATION*

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(Received 16 October 1967)

The statistical properties of a stationary laser system have been studied theoretically to quite a great extent,[1] and recently it has been possible to perform measurements of ensemble distributions and time correlations in the threshold region with high stability,[2] by using a method of "preparation of the initial state" before taking each sample of an ensemble.

By joint use of a Q-switched gas laser and of the linear method for photon counting[3] we have studied a nonstationary statistical ensemble, measuring the time evolution of a laser field during a fast build up. The importance of the transient statistics lies in the fact that this is the way of measuring quantities which, though having zero stationary average, bring useful information on the dynamics of a system (such as the expectation values of odd powers of field operators).

The experiment is carried out in the following way. We put a Kerr cell with end faces at the Brewster angle within a single-mode 6328-Å He-Ne laser cavity. The cavity is 45 cm long, with a nearly confocal configuration, and the mode position is controlled by a piezoelectric tuner as in Ref. 2. The cell is filled with ortho-dichlorobenzene, which is more transparent than the usual nitrobenzene at 6328 Å. The small residual insertion losses are helpful to allow no more than one mode oscillating over a wide range of pumping conditions. The Kerr constant of the ortho-dichlorobenzene is much smaller than that of the nitrobenzene; however the laser action is easily cut off by a field of 10 kV/cm (corresponding to a phase retardation of only a few degrees between the two orthogonal components of the light field in the cell).

Starting with some preset pump and cavity parameters, but with the optical shutter closed, the Kerr cell is switched on in a time shorter than 5 nsec at the instant $t=0$. The laser field undergoes a transient build up from an initial statistical distribution, corresponding to the equilibrium between gain and losses far below threshold, up to an asymptotic condition above threshold. At the instant $t=\tau$ we perform photocount measurements for a measuring interval T of 50 nsec, very small compared with the build-up time which is, in our case, of the order of some microseconds. Once a steady-state condition has been reached, an amplitude stabilizing operation is performed by sampling the laser output and comparing this with a standard reference signal, following a procedure already described.[2] This is equivalent to "preparing" an identical initial state for a successive measuring cycle. After the sampling, the shutter is switched off for about 10 msec.[4] At the end of this interval the shutter is again switched on and the above described cycle of operations is repeated. This way we collect an ensemble distribution of macroscopically identical events. By successively varying τ, we obtain the time evolution of the photocount distribution $p(n, T, \tau)$. A set of experimental results is shown in Fig. 1.

The measured distributions must be corrected for some unavoidable background light through the mirrors of the cavity.[5] Furthermore, to avoid dead-time problems on the small measuring interval T, we use the linear method,[3] that is, the current pulses from the photomultiplier are not standardized, but sent directly to an integrating capacitor. The statistical charge distribution measured on the capacitor is the convolution of the photocount distribution and of the amplitude distribution associated with the single-photoelectron response.[6] Both the background light and the single-photoelectron amplitude distribution are evaluated from independent measurements with the same apparatus. The moments of the photocount distributions are computed (a) by assuming that the background light is uncorrelated with the laser light and (b) by using the formulas given in Ref. 3 for the linear method.

The average photocount number $\langle n\rangle$ and the associated variance $\Delta n^2=\langle n^2\rangle-\langle n\rangle^2$ are reported as functions of the time delay in Figs. 2

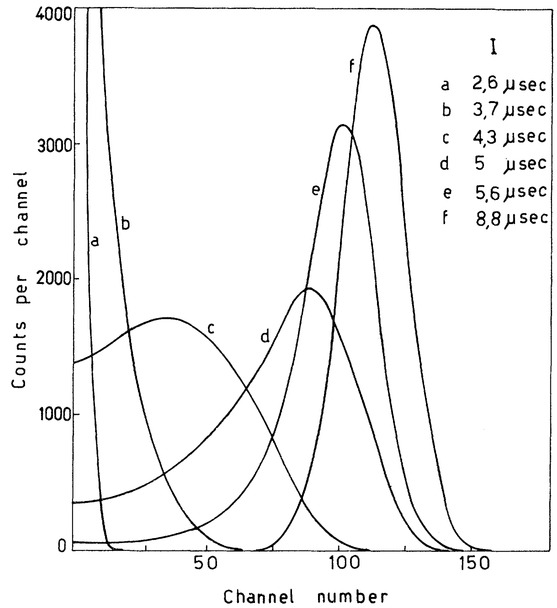

FIG. 1. Experimental statistical distributions with different time delays obtained on a laser transient. The solid lines connect the experimental points which are not shown to make the figure clearer. All distributions are normalized to the same area.

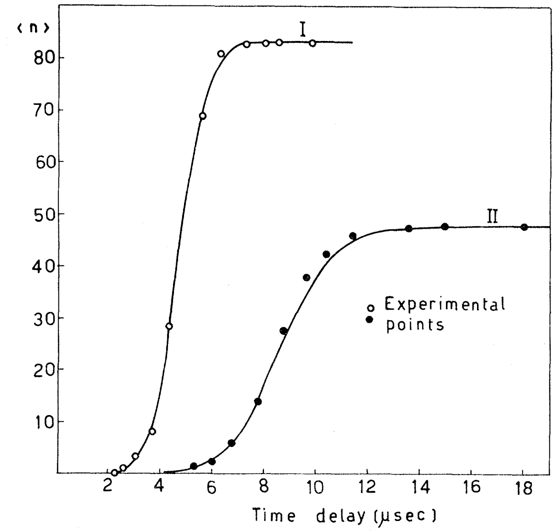

FIG. 2. Evolution of the mean value $\langle n\rangle$ of the statistical distribution $p(n, T, \tau)$. The solid lines represent the theoretical curves which best fit the experimental points. The experimental points related to curve I have been obtained by the statistical distributions of Fig. 1.

and 3 for two different pumping conditions I and II. We may distinguish a first region, where $\langle n\rangle$ increases rapidly because of the stimulated amplification process, and the variance grows much larger than if going slowly through a succession of stationary conditions (as shown in Fig. 1 of Arecchi, Giglio, and Sona[2]). This is explained as an enhancement of the initial spread in the photon distribution, due to linear amplification by stimulated emission.[7] Once a great amount of electromagnetic energy has been built inside the cavity, the field-atom interaction is no longer a linear process; hence the curve of $\langle n\rangle$ has a point of inflection and reaches eventually a saturation value, and the variance goes through a maximum and then decreases down to an asymptotic value appropriate to the stationary distribution.

The experimental results can be interpreted by the following semiclassical arguments. The complex light amplitude of a laser obeys a van der Pol equation with a Gaussian-Markoffian noise source $\Gamma(t)$ having zero average and autocorrelation function given by[8]

$$\langle\Gamma^*(t)\Gamma(t+\tau)\rangle = 4q\delta(\tau). \qquad (1)$$

Writing a corresponding equation for the photon number n and taking an ensemble average, one obtains

$$(d\langle n\rangle/d\tau) - 2\beta d\langle n\rangle + 2\beta\langle n^2\rangle = 0, \qquad (2)$$

where d is proportional to the difference between gain and losses and β is the saturation parameter. Quite above threshold, the small nonlinear contribution $2\beta\langle n^2\rangle$ (which becomes important only for long times) can be reasonably ap-

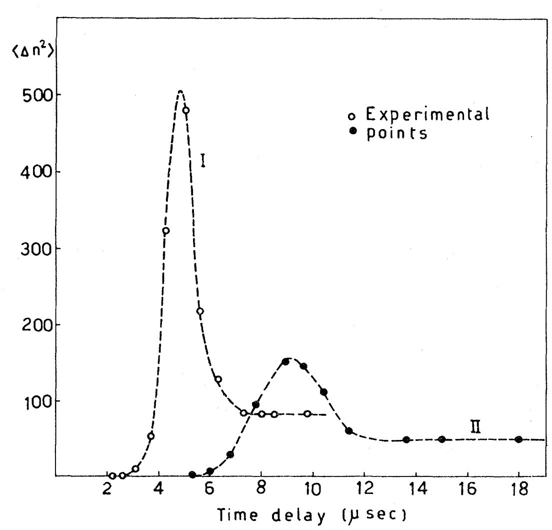

FIG. 3. Evolution of the variance $\langle\Delta n^2\rangle = \langle n^2\rangle - \langle n\rangle^2$ of the statistical distribution $p(n, T, \tau)$. Dashed lines represent an interpolation of the experimental points.

Table I. Numerical values of the laser parameters corresponding to the curves *I* and *II* of Figs. 2 and 3.[a]

	$\langle n_0\rangle$ [b]	$\beta=\frac{1}{2}B$ (sec^{-1})	$10^5 d=10^5\langle n\rangle_\infty$ [b]	$q=\frac{1}{4}A$ ($10^6\ \text{sec}^{-1}$)	$\lvert a_0\rvert$ [c]	a	$C=4q-2\beta d$ ($10^6\ \text{sec}^{-1}$)	$\langle n\rangle_{\text{thr}}$ [b]
I	82	1.09	9.96	1.1	24.5	993	2.23	1130
II	90	0.98	5.76	0.6	17	738	1.27	880

[a]q, β, d, and a are the parameters used by Risken (Ref. 1); A, B, and C are the parameters used by Scully and Lamb (Ref 1).
[b]Average photon number inside the cavity, obtained from the measured photocount number in 50 nsec, by multiplying by 1.2×10^4 (reciprocal of the attenuation factor due to the cavity mirror, filters, and quantum efficiency of the photocathode).
[c]$a_0<0$ is the pump parameter of the laser before the switching-on operation (below threshold).

proximated by $2\beta\langle n\rangle^2$. Using this approximation, Eq. (2) can be integrated for an initial condition n_0 giving

$$\langle n(\tau)\rangle_{n_0}=\frac{n_0 d}{n_0[1-\exp(-2\beta d\tau)]+d\exp(-2\beta d\tau)}, \quad (3)$$

which is the solution one would obtain from Lamb's semiclassical theory.[9] In fact, n_0 is a random variable, with a probability distribution $p(n_0)$ corresponding to the laser condition before switching on. Therefore, the measured photon number is given by

$$\langle n(\tau)\rangle=\int_0^\infty\langle n(\tau)\rangle_{n_0}p(n_0)dn_0. \quad (4)$$

We have done a best fit of the experimental points of Fig. 2 with Eq. (4) leaving β, d, and $\langle n_0\rangle$ as floating parameters, and taking $p(n_0)$ as a geometrical distribution (corresponding to a Gaussian field), since we start from far below threshold.[10] The agreement between the theoretical curves and the experimental points (see Fig. 2) is satisfactory.

In Table I we give the numerical values of the three fitted parameters in the two reported experimental conditions. Furthermore, we calculate the value of q using the experimental steady-state values $\langle n\rangle_\infty$ and $\langle\Delta n^2\rangle_\infty$, through the relation[8] $2q=\beta d\langle\Delta n^2\rangle_\infty/\langle n\rangle_\infty$. We give also the pump parameter $a=(\beta/q)^{1/2}d$ before the switching on ($a_0<0$) and after, and for comparison we calculate what the average photon number at threshold $\langle n\rangle_{\text{thr}}=1.13(q/\beta)^{1/2}$ would be in a stationary operation. Finally, we think it useful to give the numerical values of the parameters A, B, and C used by Scully and Lamb.[1] The evolution of the laser-field distribution from the vacuum to the steady state can be computed by solving the time-dependent statistical equations used in the theory,[11] and this has been done in two recent contributions.[11,12] Our results agree qualitatively with the theoretical ones, but we could not make a quantitative comparison, because the available calculations refer to much smaller photon numbers[11] or pump parameters[12] than those considered by us.

We thank B. Polli for the careful construction of the Brewster-window Kerr cell, C. Cottini for designing the electronics, and A. Sona for useful discussions.

*This work was partially supported by the Consiglio Nazionale delle Ricerche, Rome, Italy (Italian National Research Council).

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[3]F. T. Arecchi, Phys. Rev. Letters 15, 912 (1965); F. T. Arecchi, A. Berné, A. Sona, and P. Burlamacchi, IEEE J. Quant. Electron. 2, 341 (1966).

[4]This time is long enough to let the oscillation decay completely, but short compared with the time scale of the slow drifts in the cavity length or in the atomic pumping.

[5]Background light is due to laser modes which are below threshold in the "on" operation. In the operating condition it gives an average number of about 0.20 photoelectrons in 50 nsec. This background light depends on the mode spacing; therefore it becomes negligible with a shorter cavity (see Ref. 2).

[6]In the chosen operating conditions, the contribution from thermal electrons, either from the photocathode or from the dynodes of the multiplier, is negligible.

[7]The statistical spread associated with a linear amplification process has been recently analyzed by B. R. Mollow and R. J. Glauber in a paper on the parametric amplification [Phys. Rev. 160, 1076 (1967)].

[8]We use the notation of Risken, Ref. 1.

[9]W. E. Lamb, Phys. Rev. 134, A1429 (1964).

[10]B. Pariser and T. C. Marshall [Appl. Phys. Letters 6, 232 (1965)] have observed the single-mode laser transient with a Q-switching system different from ours and found a satisfactory accord with our Eq. (3) for the single transient. They report that in each switching-on operation the shape of the transient intensity is about the same, but there is a random jitter in time. We observed a similar behavior, which can be well explained by the statistical fluctuations of n_0.

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THEORY OF SELF-TRAPPED FILAMENTS OF LIGHT

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(Received 30 August 1967)

We present a calculation modeled after the theory of phase transitions to explain the observations on self-trapped filaments of laser light in liquids. The resulting state is shown to be similar to the Abrikosov vortex state in superconductors.

Self-focusing and self-trapping of intense light beams have recently become one of the most important and interesting subjects in nonlinear optics. While self-focusing as a result of intensity-dependent changes of the refractive index is now more or less understood both theoretically[1] and experimentally,[2] the formation of intense filaments arising from self-trapping[3] still remains a mystery. It is believed that the filament formation is also a consequence of the change of refractive index with intensity.[3] However, experimental results indicate that the change in the refractive index of a filament, calculated from the observed intensity in the filament under Kerr effect assumptions, is not sufficient to account for the observed filament size.[4] In addition, a number of other experimental facts have received no satisfactory explanation.

In this paper, we present a calculation which enables us to explain most of the experimental observations on self-trapped filaments. The calculation is based on the assumption of a field-induced phase transition in the medium and is similar to that of vortex formation in Type-II superconductors. Preliminary results of the calculation yield the following predictions: (1) The splitting of an intense beam into small-scale circular filaments is energetically favorable; (2) aside from fluctuations, all filaments have the same size and the same power density; (3) the filament size and the power contained in each filament are characteristics of the medium independent of the input beam intensity. In the calculation, we will assume that a critical field exists and that aside from the intensity-dependent dielectric constant $\epsilon(\omega)=\epsilon_0(\omega)+\epsilon_2(\omega)|E(\omega)|^2$, to produce this field, other nonlinear optical processes can be neglected before the filaments are formed.

Grob and Wagner[5] have also suggested the analog of vortex lines in superconductors to the filaments in this problem. However, they assume that the filament formation is a result of coupling between light fields and density fluctuations in the medium. Their results are essentially the same as those obtained by Chiao, Gamire, and Townes.[3]

Our calculation is modeled after the theory of phase transitions and the theory of vortex formation in superconductivity.[6] We assume that the molecules in a liquid are correlated, and at temperature T, the state of the liquid can be described by a dielectric function. We further assume that in the presence of an intense optical field greater than the critical field E_C, the molecular interactions in the liquid can be changed, and the system can experience a phase transition. (Field-induced phase transitions have been observed in ferroelectrics.)

We shall begin by discussing the energy of an arbitrary two-phase configuration of the liquid and then go on to discuss a liquid with trapped light filaments. In both cases we as-