Experimental Distinction between the Quantum and Classical Field Theoretic Predictions for the Photoelectric Effect*

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We have measured various coincidence rates between four photomultiplier tubes viewing cascade photons on opposite sides of dielectric beam splitters. This experimental configuration, we show, is sensitive to differences between the classical and quantum field-theoretic predictions for the photoelectric effect. The results, to a high degree of statistical accuracy, contradict the predictions by any classical or semiclassical theory in which the probability of photoemission is proportional to the classical intensity.

INTRODUCTION

It is commonly believed that experimental observations of the photoelectric effect establish the existence of uniquely quantum-mechanical properties of the electromagnetic field. Various classic experiments, coupled with the notion of microscopic energy conservation, are usually cited to establish this claim.[1] Unfortunately the insistence upon microscopic energy conservation amounts to an auxiliary criterion, which for a classical field theory (CFT) is inherently ambiguous. The quantum-mechanical energy of a photon, $h\nu$, is experimentally relevant to the photoelectric effect, determining the kinetic energy of the ejected electrons. This insistence, on the other hand, demands that the classical field energy $\int(E^2 + H^2)dV/8\pi$ be equal to $h\nu$ and be simultaneously conserved. The classical Maxwell equations contain no constraint that these energies be equal, as a quantum field theory (QFT) does.[2] This demand is, in fact, unreasonable for a classical field theory. It is therefore also unreasonable to use this constraint as a basis for an experimental distinction between the theories. With equal justification one might say that these experiments disprove microscopic energy conservation during the photoelectric process while upholding CFT. The above belief was finally shown to be totally unfounded when it was demonstrated that the above observations can be quantitatively accounted for by a semiclassical radiation theory

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in which the electromagnetic field is left unquantized.[3] The basic elements of this theory have since been used as a skeleton for the more recent and widely discussed neoclassical radiation theory (NCT) of Jaynes, Crisp, and Stroud.[4] In both of these theories it is hypothesized that the classical Maxwell equations describe the free electromagnetic field, and that this field never needs to be quantized to account for experimental observations. Previous experimental observations of the photoelectric effect, in and of themselves, are in agreement with this hypothesis, and do not appear to necessitate quantum-mechanical properties for the radiation field.

In 1955, following Schrödinger's suggestion, Ádám, Jánossy, and Varga[5] (AJV) searched for anomalous coincidences in a partially collimated beam of light.[5a] Jauch,[6] in his discussions of the foundations of quantum mechanics, has recently emphasized the importance of this experiment and an associated one performed by Jánossy and Náray[7] in establishing the existence of a wave-particle duality for photons. Moreover, the arguments of AJV and Jauch do not rely on energy conservation (although other assumptions are needed for their specific scheme) and as such are not subject to the above criticism. Attention is naturally called to this experiment by the recent discussions of semiclassical theories, in hopes that it might provide an additional aspect of the photoelectric effect upon which the predictions of CFT and QFT differ.

In this paper we will show that the actual values of the parameters for the arrangement of AJV (and subsequent similar experiments) unfortunately were insufficient to make that experiment conclusive. We then report new experimental results which are conclusive. Our measurements involved a comparison of various twofold coincidence rates between four photomultiplier tubes viewing cascade optical photons emitted by the same source through various beam splitters. We further show that this configuration is sensitive to differences between the QFT and CFT predictions for this effect without additional assumptions, such as those required by AJV. The results, to high statistical accuracy, contradict the predictions of any classical or semiclassical radiation theory in which the probability of photoemission is proportional to the classical field intensity. This includes, for example, NCT. Our experiment thus resurrects the photoelectric effect as a phenomenon requiring quantization of the electromagnetic field.

It is noteworthy that Aharonov et al.[8] presented a scheme similar to that of AJV as a *Gedanken-experiment*, while noting a paucity of actual experimental distinctions between CFT and QFT. The CFT prediction for our experiment follows reasoning similar to that by Titulaer and Glauber,[9] who discussed constraints applicable to GFT which demarcate a boundary between CFT and the more general QFT descriptions of the electromagnetic field.

In what follows we first contrast the CFT and QFT predictions for a single photon falling on a half-silvered mirror. We next discuss previous relevant experiments, contrast these with our own experimental scheme, and show that of these only ours provides the desired distinction. Finally we describe the apparatus and present the results.

PREDICTIONS FOR A SINGLE PHOTON FALLING ON A HALF-SILVERED MIRROR

In this section we review the arguments by AJV and Jauch. Consider the light emitted by a single atomic decay falling on a half-silvered mirror. During the decay a wave train (packet) of electromagnetic radiation is emitted. Suppose that it impinges upon a beam-splitting mirror, and that the two resultant wave trains are directed to two independent photomultipliers labelled γ_A and γ_B. We desire the QFT prediction for the $\gamma_A - \gamma_B$ coincidence rate. A simpler problem to consider first involves only the source atom and a second atom in one photocathode. We need the probability amplitude that, following deexcitation of the source atom, the second atom will become excited (or ionized). This has been obtained by Fermi[10] and Fano,[11] using the Wigner-Weisskopf approximation. The inclusion of a third atom in a second photocathode is then straightforward. Denote by S, A, and B, respectively, the ground states of the source atom and the two detector atoms, and by S^*, A^*, and B^* the corresponding excited or ionized states of these atoms. Initially the source atom is excited, and the two detector atoms are in their ground states; hence

$$|i\rangle = |S^*, A, B, 0_1, \ldots, 0_j, \ldots\rangle.$$

The remaining indices of the ket designate the state of the radiation field modes. The final state then has the form

$$\begin{aligned} |f\rangle = {} & U_A|S, A^*, B, 0_1, \ldots, 0_j, \ldots\rangle \\ & + U_B|S, A, B^*, 0_1, \ldots, 0_j, \ldots\rangle \\ & + U_S|S^*, A, B, 0_1, \ldots, 0_j, \ldots\rangle \\ & + \sum_j U_j|S, A, B, 0_1, \ldots, 1_j, \ldots\rangle. \end{aligned} \tag{1}$$

The various U_i can be evaluated from formulas found in Refs. 10 and 11. Thus QFT predicts that an observation will find at most one of the detector atoms ionized; i.e. coincident responses will occur only at the random accidental rate, induced by emissions from two different excited source atoms.[12]

Next we consider the same system from the CFT viewpoint. Our basic assumptions for this are twofold: (1) The electromagnetic field is described by the classical (unquantized) Maxwell equations, and (2) the probability of photoionization at a detector is proportional to the classical intensity of the incident radiation. These two assumptions alone are sufficient for our purposes, and they are in evident agreement with experiment.[13] Since ionizations at the γ_A and γ_B phototubes are independent, but are induced by nearly identical classical pulses of light, for a given split wave train both tubes will have roughly the same probability for registering a count. This independence implies that the probability that both will respond to the split wave train is simply the product of the probabilities that each will respond. The nonzero value of this product implies the existence of an anomalous coincidence rate above the accidental background. The CFT prediction is thus in marked contrast with QFT prediction, the latter requiring no coincidences above the background level.[12]

The above argument may be summarized very simply. Consider a radiation field quantum-mechanically with only one photon present. If we bring this into interaction with two separated atoms we will never get more than one photoelectron. If on the other hand we represent this field classically, we find that there is a nonvanishing probability for finding two photoelectrons. The classical Maxwell field has within it the possibility of providing with some probability any number of photons. Hence experiments of the above variety can distinquish between the two theories.

Such then is the argument of AJV and Jauch. Here we have also the basis for the usual particle interpretation of photons. A particle must be either transmitted or reflected. Both may be done simultaneously only by a wave. We then see how these macroscopic features of "particle-like" objects arise from the QFT formalism.

PREVIOUS EXPERIMENTAL RESULTS

That a photon is not split in two by a beam splitter is certainly "old hat", and it may seem surprising that we have gone to the effort to test this prediction experimentally. What is in fact much more surprising is that evidently no such experimental test has heretofore been performed, and such tests are clearly of great importance. Here we briefly review previous relevant experimental results and show that none provides the desired distinction.

Since the original work of AJV many two-photon coincidence experiments have been done, some involving light beams split by a half-silvered mirror. These all fall into two basic categories - atomic-cascade observations and Brown-Twiss-effect observations. Excellent reviews of these topics have been presented by Camhy-Val and Dumont[14] and Mandel and Wolf,[15] respectively. Cascade-photon observations in their usual configuration are not suitable for the above test, since in these, two different unsplit photons are observed.

The AJV experiment, although intended as a test of the above scheme, actually served as a fore-runner to the Brown-Twiss-effect experiments. Figure 1 reproduces a diagram of the experiment of AJV. In it they selected the light of a single spectral line with a monochromator, and focussed it through a beam splitter onto two photomultipliers whose outputs drove a coincidence circuit.

Let us evaluate the magnitude of the expected anomalous coincidence rate. The CFT predictions for one and two photodetectors sharing the same field were discussed earlier by Mandel[16] from the above fundamental assumptions. Denote by I(t) the instantaneous classical intensity incident simultaneously upon the γ_A and γ_B detectors due to their illumination by the whole source volume. The singles rates for the A and B detectors, averaged over their response time T, is given by

$$S_A = \alpha_A T^{-1} \int_{-T/2}^{T/2} \langle I(t+t')\rangle dt',$$

$$S_B = \gamma_B T^{-1} \int_{-T/2}^{T/2} \langle I(t+t')\rangle dt', \qquad (2)$$

where α_A and α_B are measures of the detector efficiencies, and the angular brackets denote an ensemble average over the emitted intensities. Similarly, the average coincidence rate as a function of event separation τ is given by

$$C_{AB}(\tau) = \alpha_A \alpha_B T^{-1} \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} \langle I(t+t')I(t+t''+\tau)\rangle dt'dt''. \qquad (3)$$

To obtain a model-independent prediction for the coincidence rate only from data on the singles rates does not appear possible, since (2) and (3) involve different averages of I(t). AJV thus had to make various assumptions (assump-

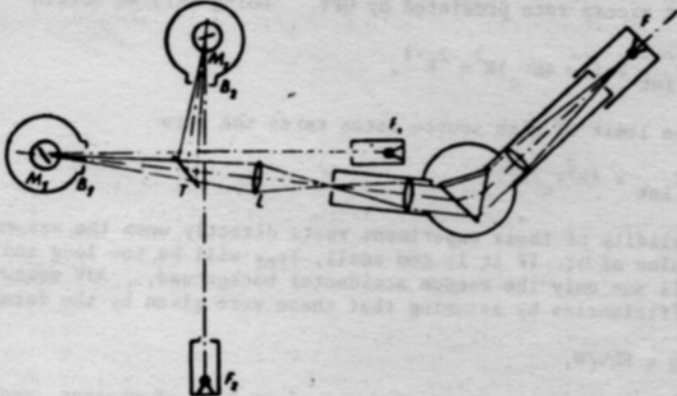

Fig.1. Experimental arrangement of Ádám, Jánossy, and Varga. Light from source F is focussed through a monochromator onto photomultipliers M_1 and M_2 via beam splitter T. (Figure after Ádám, Jánossy, and Varga.)

tions which were unnecessary in the case of our own experiment). They tacitly assumed that

$$\left(\int_{-T/2}^{T/2} \langle I(t+t')\rangle dt'\right)^2 \approx \int_{-T/2}^{T/2}\int_{-T/2}^{T/2} \langle (t+t')I(t+t''+\tau)\rangle dt'dt'' \tag{4}$$

holds for each decay, when τ is the order of the decaying state lifetime. If then E pulses per second are emitted per unit time by a source, and if η is the average probability that a photomultiplier will yield a count, given an atomic decay, the count rate at that detector is

$$S = E\eta. \tag{5}$$

The expected anomalous coincidence rate predicted by the AVJ assumptions is then given approximately by

$$C \approx \eta^2 E. \tag{6}$$

Assuming negligible detector dark rates, the accidental coincidence background rate from which C must be distinguished is

$$A \approx \eta^2 E^2 2\tau_c, \tag{7}$$

where τ_c is the resolving time of the coincidence system. One can now calculate the integration time required to measure to a precision of N standard deviations the difference between the excess coincidence rate given by Eq.(6) and the zero excess rate predicted by QFT. Doing this we obtain

$$T_{int} \approx (1 + 4E\tau_c)N^2\eta^{-2}E^{-1}, \tag{8}$$

which in the limit of high source rates takes the form

$$T_{int} \approx 4N^2\tau_c/\eta^2. \tag{9}$$

Thus the validity of their experiment rests directly upon the assumed or measured value of η: If it is too small, T_{int} will be too long and the experiment will see only the random accidental background. AJV measured their detector efficiencies by assuming that these were given by the formula

$$\eta = Rh\nu/W, \tag{10}$$

where R is the count rate obtained for a given beam of photons, and W is the power in the same beam measured bolometrically. They thus found $\eta = 1/300$. With a resolving time $\tau_c = 2.3$ μsec one calculates $T_{int} = 20.7$ sec for $N = 5$. From this reasoning AJV felt confident that they should have observed the anomalous coincidence rate, if it was present.

Let us reexamine from the CFT viewpoint the assumption tacitly contained in Eq.(10). Although the introductory arguments did not contain a requirement for energy conservation, AJV have unnecessarily reintroduced it with this assumption; this is in direct conflict with our fundamental assumptions for a CFT. In our derivation above, η is the probability for a detector response, given a source atom decay. Clearly a wavelike pulse emitted by a source atom will expand, in the worst case spherically, or at best with a radiation pattern having a preferred direction.[17] Much of this pulse will not enter the narrow acceptance solid angle subtended by the monochromator. Propagation will cause it to suffer an enormous decrease in intensity, commensurate with its expansion. Assuming macroscopic energy conservation on the average, the power W should then represent the total average power radiated by the source at the appropriate wavelength, not that which happens to be measured within the beam itself. The number calculated from Eq.(10) must be appropriately decreased by the fraction of the solid angle effectively subtended by the detectors. Other optical losses will decrease this number even further.

If we conservatively estimate from their diagram the solid-angle loss to be 1/400, their actual detector efficiency for spherically emitted wavelike pulses was undoubtedly less than 8×10^{-6} in which case the required integration time for even $N = 1$ becomes $T_{int} \approx 1.3 \times 10^5$ sec. This is an order of magnitude longer than the duration of their experiment. Thus the experiment of Ádám, Jánossy, and Varga appears to be inconclusive when reexamined in this light.

A similar analysis applies to the experiments of Givens and of Brannen and Ferguson.[5a] In the x-ray coincidence experiment of Givens, the source solid angle viewed by the detector pair was $\approx 3.5 \times 10^{-5}$ sr, smaller than that of AJV. Combining this with his ≈15% quantum efficiencies, we estimate the overall detector efficiencies to be $\approx 2.1 \times 10^{-7}$ (neglecting the appreciable loss due to the beam-splitting crystal). Givens employed a resolving time of $\approx 1.7 \times 10^{-4}$ sec. From Eq.(9), we find then that an integration time of nearly 500 yr is required for this apparatus to produce results with a confidence level corresponding to just one standard deviation. Similar reasoning finds the actual integration time of Brannen and Ferguson deficient by a factor $\approx 1.7 \times 10^5$. These experiments are thus likewise inconclusive for deciding the above question.

Finally let us consider experiments of the Brown-Twiss variety. These experiments have a configuration basically the same as that of AJV. Because of the nature of this effect, however, all existing data have been accumulated with detectors subtending extremely small solid angles, much smaller even than those of AJV. From Eq.(9) we see that the required integration time scales with the inverse square of the detector solid angles; hence it would be hopeless to try to search for the above anomalous coincidence rate with such arrangements. Furthermore, in these experiments, the Brown-Twiss effect itself would tend to mask the effect we seek. In summary, then, none of the above experiments can provide the desired distinction.

EXPERIMENTAL SCHEME REQUIRING NO ADDITIONAL ASSUMPTIONS

The above discussion indicates that an observation of the anomalous coincidences predicted by a CFT requires highly efficient photodetectors. However, even if AJV had had the required efficiency and integration time, their experimental arrangement necessitated assumptions concerning the various field averages, and hence assumed a basic model for the emission mechanism. Since no universally acceptable model is at hand, we have chosen to employ a scheme which renders our results model-independent. We did this by "splitting" simultaneously both the first and second photons of an atomic cascade. We viewed the light emitted on opposite sides of an assembly of excited atoms and focussed it separately into two beams. The wavelength λ_1 on one side was selected to correspond to that of the first transition of the cascade, and that on the other, λ_2, to the second. The two light beams impinged on beam splitters, thus creating a total of four beams. Four associated photomultipliers labelled γ_{1A}, γ_{1B}, γ_{2A}, and γ_{2B} detected them. We monitored the coincidence rates between the four combinations: $\gamma_{1A}-\gamma_{1B}$, $\gamma_{2A}-\gamma_{2B}$, $\gamma_{1A}-\gamma_{2B}$, and $\gamma_{2A}-\gamma_{1B}$. A diagram of the arrangement is shown in Fig.2.

Define $I_1(t)$ and $I_2(t)$ as the instantaneous intensity at the $\gamma_{1A}-\gamma_{1B}$ beam splitter with wavelength λ_1, and that at the $\gamma_{2A}-\gamma_{2B}$ beam splitter with wavelength λ_2, respectively. It follows directly from the Cauchy-Schwarz inequality that the following inequality holds:

$$\left[\int_{-T/2}^{T/2}\int_{-T/2}^{T/2}\langle I_1(t+t'+\tau_1)I_1(t+t''+\tau_1)\rangle dt'dt''\right]$$

$$\left[\int_{-T/2}^{T/2}\int_{-T/2}^{T/2}\langle I_2(t+t'+\tau_2)I_2(t+t''+\tau_2)\rangle dt'dt''\right]$$

$$\geq \left[\int_{-T/2}^{T/2}\int_{-T/2}^{T/2} I_1(t+t'+\tau_1)I_2(t+t''+\tau_2)\rangle dt'dt''\right]^2 \qquad (11)$$

Using (3), we can write this as

$$C_{1A-1B}(0)C_{2A-2B}(0) \geq C_{1A-2B}(\tau)C_{1B-2A}(\tau). \qquad (12)$$

Here we have ignored a possible polarization dependence of the detectors, and the finite photocathode areas, as well as the nonvanishing phototube dark rates. It can be shown that the inequality (12) may be summed over these contributions without change of form. Thus it is fully general and holds for these cases as well. The coincidence rates C_{1A-2B} and C_{2A-1B} here are the nonvanishing cascade rates. The product of these sets a lower bound to the product of the anomalous rates C_{1A-1B} and C_{2A-2B}. Thus, CFT predicts a large anomalous coincidence rate satisfying (12). The prediction of QFT significantly violates this inequality, requiring no coincidences except those due to two-atom excitations.

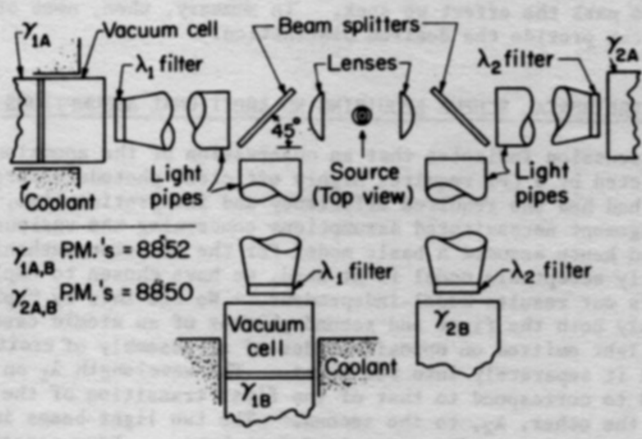

Fig.2 Schematic diagram of our apparatus.

APPARATUS AND RESULTS

Figure 2 is a diagram of the apparatus. The source contained ^{202}Hg atoms which were excited by electron bombardment. Light produced at $\lambda_1 = 5676$ Å and $\lambda_2 = 4358$ Å by the cascade 9 $^1P_1$ - 7^3S_1 - 6^3P_1 was used. It was made parallel by lenses (aspheric, f ≈ 1), and fell on TiO_2-coated glass beam splitters (transmission ≈ 63% and 35% for opposite linear polarizations, inclined at 45° to the incident beams). Each resulting beam was directed through an interference filter [transmission ≈ 50% at 5676 Å, full width at half maximum (FWHM) ≈ 50 Å for γ_{1A} and γ_{1B}; transmission ≈ 30% at 4358 Å, FWHM ≈ 100 Å for γ_{2A} and γ_{2B}] onto an appropriate photomultiplier tube [RCA 8852, quantum efficiency (QE) ≈ 15% at 5676 Å, dark current ≈ 50-300 Hz, operated at -80 °C for γ_{1A} and γ_{1B}; RCA 8850, QE ≈ 30% at 4358 Å, dark current ≈ 100 Hz, operated at 20 °C for γ_{2A} and γ_{2B}].

The source itself was patterned after a design by Holt, Nussbaum, and Pipkin,[18] and was made by using standard techniques. The electron gun was a standard 10-W cathode-ray tube gun obtained through the courtesy of the Raytheon Corporation. It was mounted with suitable deflecting electrodes and light masks in a quartz and Pyrex envelope, evacuated, and cleaned by baking and discharging; the metal parts were outgassed by induction heating, and the oxide cathode was activated. A few milligrams of 93%-pure ^{202}Hg were then distilled into the tube and the envelope sealed. The Hg vapour pressure was controlled by keeping a side arm immersed in ice water. A beam current of approximately 0.7μA traversed the cylindrical excitation region (length ≈ 2 mm, diam. ≈ 1 mm). The light output was stable. Photomultipliers operating in coincidence were separated from each other by more than 1.5 m to eliminate anomalous coincidences caused by cosmic rays. Light pipes minimized the light loss during transit. The interference filters were placed at the outer ends of the light pipes to minimize anomalous coincidences due to scintillations in the beam splitter and collimator lenses. These could be excited by cosmic rays and/or residual radioactivity therein. This configuration also effectively eliminated phototube cross talk induced by light emitted at the last dynodes. High-speed electronics with ≈ 1-nsec resolving time were used. The discriminators drove a time-to-amplitude converter whose output was fed to a pulse-height analyzer. External slow coincidence circuits gated the signals into one of the four analyzer memory quadrants, corresponding to the particular coincidence mode. The analyzer thus simultaneously accumulated the four different delayed coincidence spectra, i.e. the number of events pairs as a function of event separation time.

The results, shown in Fig.3(a)-3(d), represent more than 26 hours of integration. We find no evidence for an anomalous coincidence rate in either the γ_{1A}-γ_{1B} or the γ_{2A}-γ_{2B} mode, but the normal cascade mode is quite apparent. For a timing and sensitivity check, both tube pairs were excited through the beam splitters by short-duration "classical" light pulses from a barium-titanate source,[19] with approximately one photon per pulse. The resultant coincidence spectra are shown in Figs. 3(e) and 3(f). Finally, Fig.3(g) shows that our data severely violate (12) for a wide range of delays τ.

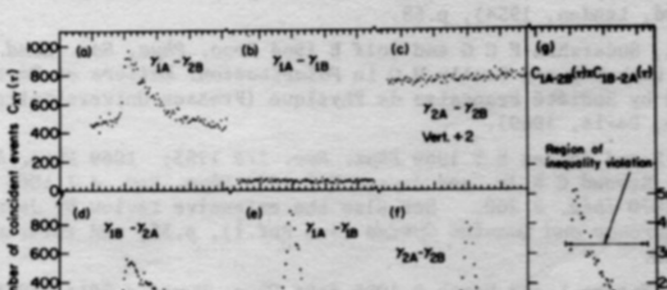

Fig.3. (a)-(d) Time-delay coincidence spectra of the four monitored channels: C_{1A-2B}, C_{1A-1B}, C_{2A-2B}, and C_{1B-2A}. (e)-(f) C_{1A-1B} and C_{2A-2B} coincidence spectra in response to short pulses of light incident upon beam splitters produced by a barium titanate source. (g) Product of C_{1A-2B} and C_{1B-2A} versus time delay. For small times this clearly exceeds the indicated value of the product C_{2A-2B} and C_{1A-1B} evaluated at zero delay.

DISCUSSION

The importance of experimentally demonstrating phenomena which require a quantization of the electromagnetic field has been emphasized recently by a number of suggestions that such a quantization is unnecessary. Many standard effects have thus been challenged as not providing definitive proof for the necessity of this quantization.[4,20] Several recent experiments testing the specific predictions of NCT and the Schrödinger interpretation have been performed[21] in this direction. The present experiment and others[20] have tested the quantum-mechanical aspects of Maxwell's equations. So far, none has uncovered any departure from the quantum-electrodynamic predictions, but severe departures from CFT predictions have been found. The classical (unquantized) Maxwell equations thus appear to have only limited validity.

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CHAPTER 4

Coherence Functions

4.1 CLASSICAL COHERENCE

In this Chapter we show how the classical notions of optical coherence can be generalized to describe correlations in quantized electromagnetic fields. We begin by reviewing elementary notions of coherence, show how higher order correlations can be described in classical theory and present an analysis of quantum coherence based on the analysis of Glauber. It is surprising how much of the classical formalism is found to remain useful.

We assume that the reader is familiar with the basic notions of classical coherence (e.g. Fowles, 1975; Hecht and Zajac 1974). In Young's two-slit experiment it is easy to demonstrate that the intensity at a point on the screen where the interference fringes are observed, is given in terms of the fields E_1 and E_2 at the slits 1 and 2 by

$$I = \langle|E_1|^2\rangle + \langle|E_2|^2\rangle + 2\,\mathrm{Re}\langle E_1E_2^*\rangle \tag{4.1}$$

If the difference in time for the light to travel to the observation point from the slits is τ, then the interference term $2\,\mathrm{Re}\langle E_1E_2^*\rangle$ may be re-written using the correlation function, or mutual coherence function, (Born and Wolf 1964; Mandel and Wolf 1965) as

$$\Gamma_{12}(\tau) = \langle E_1(t)E_2^*(t+\tau)\rangle. \tag{4.2}$$

The normalized form of the correlation function, or the degree of partial coherence, is

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)\,\Gamma_{22}(0)}} = \frac{\Gamma_{12}(\tau)}{\sqrt{I_1\,I_2}} \tag{4.3}$$

and we see that

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\;\mathrm{Re}\,\gamma_{12}(\tau) \tag{4.4}$$

If Rayleigh's definition of fringe visibility is now used

$$V = (I_{max} - I_{min})/(I_{max} + I_{min})$$

we see at once that $V = 2\sqrt{I_1I_2}\,|\gamma_{12}|/(I_1 + I_2)$ because