

Relatividad General – 2do. cuatrimestre de 2020

Hoja de fórmulas

Geometría

Simetrización y antisimetrización de tensores

$$T_{(\mu_1 \dots \mu_n)\beta}^\alpha = \frac{1}{n!} (T_{\mu_1 \dots \mu_n \beta}^\alpha + \text{suma sobre permutaciones de } \mu_1 \dots \mu_n),$$
$$T_{[\mu_1 \dots \mu_n]\beta}^\alpha = \frac{1}{n!} (T_{\mu_1 \dots \mu_n \beta}^\alpha + \text{suma alternada sobre permutaciones de } \mu_1 \dots \mu_n)$$

Producto wedge y derivada exterior

$$\tilde{p} \wedge \tilde{q} = (-1)^{pq} \tilde{q} \wedge \tilde{p}, \quad \tilde{d}(\tilde{p} \wedge \tilde{q}) = \tilde{d}p \wedge \tilde{q} + (-1)^p \tilde{p} \wedge \tilde{d}q$$

\tilde{p} es una p -forma y \tilde{q} es una q -forma

Teorema de Stokes

$$\int_U \tilde{d}\tilde{\alpha} = \int_{\partial U} \tilde{\alpha}$$

U región de una variedad diferenciable de dimensión n , $\tilde{\alpha}$ es una $(n-1)$ -forma

Símbolo de Levi-Civita

$$\epsilon^{\alpha\beta\dots\gamma\delta} = \eta^{\alpha\mu}\eta^{\beta\nu} \dots \eta^{\gamma\rho}\eta^{\delta\sigma} \epsilon_{\mu\nu\dots\rho\sigma},$$
$$\epsilon_{\alpha_1\dots\alpha_p\nu_1\dots\nu_{n-p}} \epsilon^{\alpha_1\dots\alpha_p\mu_1\dots\mu_{n-p}} = \text{sgn}(\eta) p! (n-p)! \delta_{\nu_1}^{[\mu_1} \dots \delta_{\nu_{n-p}}^{\mu_{n-p}]}$$

n es la dimensión de la variedad

Dual de Hodge

$$(*\tilde{p})_{\nu_1\dots\nu_{n-p}} = \frac{1}{p!} \sqrt{|g|} \epsilon_{\alpha_1\dots\alpha_p\nu_1\dots\nu_{n-p}} p^{\alpha_1\dots\alpha_p}, \quad **\tilde{p} = \text{sgn}(g) (-1)^{p(n-p)} \tilde{p}$$

Derivada de Lie

$$(\mathcal{L}_{\vec{V}} T)^{\mu_1\dots\mu_r}_{\nu_1\dots\nu_s} = V^\alpha \partial_\alpha T^{\mu_1\dots\mu_r}_{\nu_1\dots\nu_s} - \sum_{i=1}^r T^{\mu_1\dots\alpha\dots\mu_r}_{\nu_1\dots\nu_s} \partial_\alpha V^{\mu_i} + \sum_{j=1}^s T^{\mu_1\dots\mu_r}_{\nu_1\dots\alpha\dots\nu_s} \partial_{\nu_j} V^\alpha$$

Derivada covariante

$$(\nabla_{\vec{V}} T)^{\mu_1 \dots \mu_r}_{\nu_1 \dots \nu_s} = V^\alpha \nabla_\alpha T^{\mu_1 \dots \mu_r}_{\nu_1 \dots \nu_s},$$

$$\nabla_\alpha T^{\mu_1 \dots \mu_r}_{\nu_1 \dots \nu_s} = \partial_\alpha T^{\mu_1 \dots \mu_r}_{\nu_1 \dots \nu_s} + \sum_{i=1}^r \Gamma_{\beta\alpha}^{\mu_i} T^{\mu_1 \dots \beta \dots \mu_r}_{\nu_1 \dots \nu_s} - \sum_{j=1}^s \Gamma_{\nu_j\alpha}^\beta T^{\mu_1 \dots \mu_r}_{\nu_1 \dots \beta \dots \nu_s}$$

Torsión

$$T(\ ; \vec{U}, \vec{V}) = \nabla_{\vec{U}} \vec{V} - \nabla_{\vec{V}} \vec{U} - [\vec{U}, \vec{V}], \quad T^\alpha_{\ \mu\nu} = 2\Gamma_{[\mu\nu]}^\alpha$$

Tensor de Riemann

$$R(\ ; \vec{X}, \vec{U}, \vec{V}) = [\nabla_{\vec{U}}, \nabla_{\vec{V}}] \vec{X} - \nabla_{[\vec{U}, \vec{V}]} \vec{X}, \quad R^\alpha_{\ \mu\beta\nu} = \Gamma_{\mu\nu,\beta}^\alpha - \Gamma_{\mu\beta,\nu}^\alpha + \Gamma_{\mu\nu}^\lambda \Gamma_{\lambda\beta}^\alpha - \Gamma_{\mu\beta}^\lambda \Gamma_{\lambda\nu}^\alpha$$

Propiedades del Riemann (sin torsión)

$$R^\alpha_{\ \mu\beta\nu} = -R^\alpha_{\ \nu\beta\mu}, \quad R^\alpha_{\ [\mu\beta\nu]} = 0, \quad R^\alpha_{\ \mu[\beta\nu;\lambda]} = 0$$

Conexión métrica

$$\Gamma_{\mu\nu}^\alpha = \frac{1}{2} g^{\alpha\beta} (\partial_\mu g_{\beta\nu} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu}), \quad R_{\alpha\mu\beta\nu} = R_{\beta\nu\alpha\mu}$$

Relatividad

Fluido perfecto

$$T^{\mu\nu} = (\rho + p)U^\mu U^\nu + pg^{\mu\nu}$$

Tensor de Ricci linealizado

$$R_{\mu\nu} = -\frac{1}{2} \partial^\alpha \partial_\alpha h_{\mu\nu} + \partial_{(\mu} \partial^\alpha h_{\nu)\alpha} - \frac{1}{2} \partial_\mu \partial_\nu h$$

Gauge de Lorenz

$$\partial^\nu h_{\mu\nu} = \frac{1}{2} \partial_\mu h, \quad \text{TT: } h_{0\mu} = 0, \quad h = 0$$

Métrica de Schwarzschild

$$ds^2 = - \left(1 - \frac{2GM}{r} \right) dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

Métrica de Robertson-Walker

$$ds^2 = -dt^2 + a^2(t) [d\chi^2 + r^2(\chi)(d\theta^2 + \sin^2 \theta d\varphi^2)] , \quad r(\chi) = \begin{cases} \chi & k = 0 \\ \sin \chi & k = 1 \\ \sinh \chi & k = -1 \end{cases}$$

Ecuaciones de Friedmann

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Conservación (cosmología)

$$\dot{\rho} + 3H(\rho + p) = 0$$