Relatividad General – 2do. cuatrimestre de 2020

Hoja de fórmulas

Geometría

Simetrización y antisimetrización de tensores

$$T^{\alpha}_{(\mu_1\ldots\mu_n)\beta} = \frac{1}{n!}\left(T^{\alpha}_{\mu_1\ldots\mu_n\beta} + \text{ suma sobre permutaciones de } \mu_1\ldots\mu_n\right) ,$$
$$T^{\alpha}_{[\mu_1\ldots\mu_n]\beta} = \frac{1}{n!}\left(T^{\alpha}_{\mu_1\ldots\mu_n\beta} + \text{ suma alternada sobre permutaciones de } \mu_1\ldots\mu_n\right)$$

Producto wedge y derivada exterior

$$\tilde{p}\wedge\tilde{q} = (-1)^{pq}\tilde{q}\wedge\tilde{p} , \qquad \tilde{d}(\tilde{p}\wedge\tilde{q}) = \tilde{d}p\wedge\tilde{q} + (-1)^p\tilde{p}\wedge\tilde{d}\tilde{q}$$

$\tilde{p}$ es una $p-$forma y $\tilde{q}$ es una $q-$forma

Teorema de Stokes

$$\int_U \tilde{d}\tilde{\alpha} = \int_{\partial U}\tilde{\alpha}$$

U región de una variedad diferenciable de dimensión n, $\tilde{\alpha}$ es una $(n-1)-$forma

Símbolo de Levi-Civita

$$\epsilon^{\alpha\beta\ldots\gamma\delta} = \eta^{\alpha\mu}\eta^{\beta\nu}\ldots\eta^{\gamma\rho}\eta^{\delta\sigma}\epsilon_{\mu\nu\ldots\rho\sigma} ,$$
$$\epsilon_{\alpha_1\ldots\alpha_p\nu_1\ldots\nu_{n-p}}\epsilon^{\alpha_1\ldots\alpha_p\mu_1\ldots\mu_{n-p}} = \mathrm{sgn}(g)\, p!\,(n-p)!\,\delta^{[\mu_1}_{\nu_1}\ldots\delta^{\mu_{n-p}]}_{\nu_{n-p}}$$

n es la dimensión de la variedad

Dual de Hodge

$$(*\tilde{p})_{\nu_1\ldots\nu_{n-p}} = \frac{1}{p!}\sqrt{|g|}\epsilon_{\alpha_1\ldots\alpha_p\nu_1\ldots\nu_{n-p}}p^{\alpha_1\ldots\alpha_p} , \qquad **\tilde{p} = \mathrm{sgn}(g)(-1)^{p(n-p)}\tilde{p}$$

Derivada de Lie

$$(\mathcal{L}_{\vec{V}}T)^{\mu_1\ldots\mu_r}{}_{\nu_1\ldots\nu_s} = V^\alpha\partial_\alpha T^{\mu_1\ldots\mu_r}{}_{\nu_1\ldots\nu_s} - \sum_{i=1}^{r} T^{\mu_1\ldots\alpha\ldots\mu_r}{}_{\nu_1\ldots\nu_s}\partial_\alpha V^{\mu_i} + \sum_{j=1}^{s} T^{\mu_1\ldots\mu_r}{}_{\nu_1\ldots\alpha\ldots\nu_s}\partial_{\nu_j}V^\alpha$$

Derivada covariante

$$(\nabla_{\vec{V}}T)^{\mu_1\ldots\mu_r}{}_{\nu_1\ldots\nu_s} = V^\alpha \nabla_\alpha T^{\mu_1\ldots\mu_r}{}_{\nu_1\ldots\nu_s}\,,$$
$$\nabla_\alpha T^{\mu_1\ldots\mu_r}{}_{\nu_1\ldots\nu_s} = \partial_\alpha T^{\mu_1\ldots\mu_r}{}_{\nu_1\ldots\nu_s} + \sum_{i=1}^{r} \Gamma^{\mu_i}_{\beta\alpha} T^{\mu_1\ldots\beta\ldots\mu_r}{}_{\nu_1\ldots\nu_s} - \sum_{j=1}^{s} \Gamma^{\beta}_{\nu_j\alpha} T^{\mu_1\ldots\mu_r}{}_{\nu_1\ldots\beta\ldots\nu_s}$$

Torsión

$$T(\ ;\vec{U},\vec{V}) = \nabla_{\vec{U}}\vec{V} - \nabla_{\vec{V}}\vec{U} - [\vec{U},\vec{V}]\,, \qquad T^\alpha{}_{\mu\nu} = 2\Gamma^\alpha_{[\nu\mu]}$$

Tensor de Riemann

$$R(\ ;\vec{X},\vec{U},\vec{V}) = [\nabla_{\vec{U}},\nabla_{\vec{V}}]\vec{X} - \nabla_{[\vec{U},\vec{V}]}\vec{X}\,, \qquad R^\alpha{}_{\mu\beta\nu} = \Gamma^\alpha_{\mu\nu,\beta} - \Gamma^\alpha_{\mu\beta,\nu} + \Gamma^\lambda_{\mu\nu}\Gamma^\alpha_{\lambda\beta} - \Gamma^\lambda_{\mu\beta}\Gamma^\alpha_{\lambda\nu}$$

Propiedades del Riemann (sin torsión)

$$R^\alpha{}_{\mu\beta\nu} = -R^\alpha{}_{\mu\nu\beta}\,, \qquad R^\alpha{}_{[\mu\beta\nu]} = 0\,, \qquad R^\alpha{}_{\mu[\beta\nu;\lambda]} = 0$$

Conexión métrica

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}\left(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu}\right)\,, \qquad R_{\alpha\mu\beta\nu} = R_{\beta\nu\alpha\mu}$$

Relatividad

Fluido perfecto

$$T^{\mu\nu} = (\rho + p)U^\mu U^\nu + p g^{\mu\nu}$$

Tensor de Ricci linealizado

$$R_{\mu\nu} = -\frac{1}{2}\partial^\alpha\partial_\alpha h_{\mu\nu} + \partial_{(\mu}\partial^\alpha h_{\nu)\alpha} - \frac{1}{2}\partial_\mu\partial_\nu h$$

Gauge de Lorenz

$$\partial^\nu h_{\mu\nu} = \frac{1}{2}\partial_\mu h\,, \qquad \text{TT:} \quad h_{0\mu} = 0\,, \quad h = 0$$

Métrica de Schwarzschild

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Métrica de Robertson-Walker

$$ds^2 = -dt^2 + a^2(t)\left[d\chi^2 + r^2(\chi)(d\theta^2 + \sin^2\theta d\varphi^2)\right] , \qquad r(\chi) = \begin{cases} \chi & k = 0 \\ \sin\chi & k = 1 \\ \sinh\chi & k = -1 \end{cases}$$

Ecuaciones de Friedmann

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho\,, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Conservación (cosmología)

$$\dot{\rho} + 3H(\rho + p) = 0$$