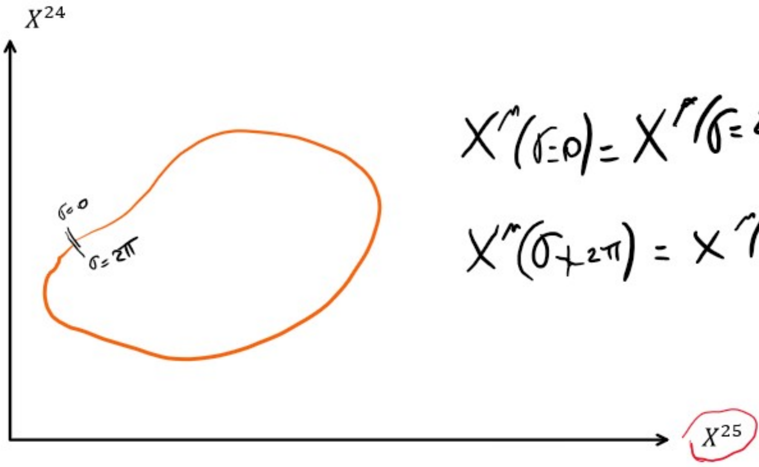
$(X^i \; con \; i < 24 \; constantes)$

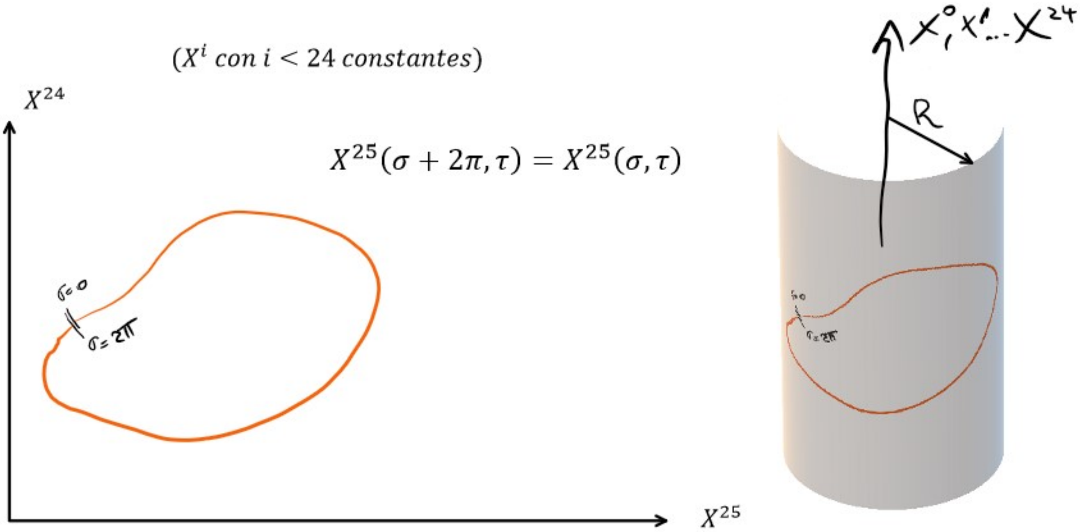


$$X^\mu(\sigma=0) = X^\mu(\sigma=2\pi)$$

$$X^\mu(\sigma+2\pi) = X^\mu(\sigma)$$

$(X^i \; con \; i < 24 \; constantes)$

$$X^{25}(\sigma + 2\pi, \tau) = X^{25}(\sigma, \tau)$$

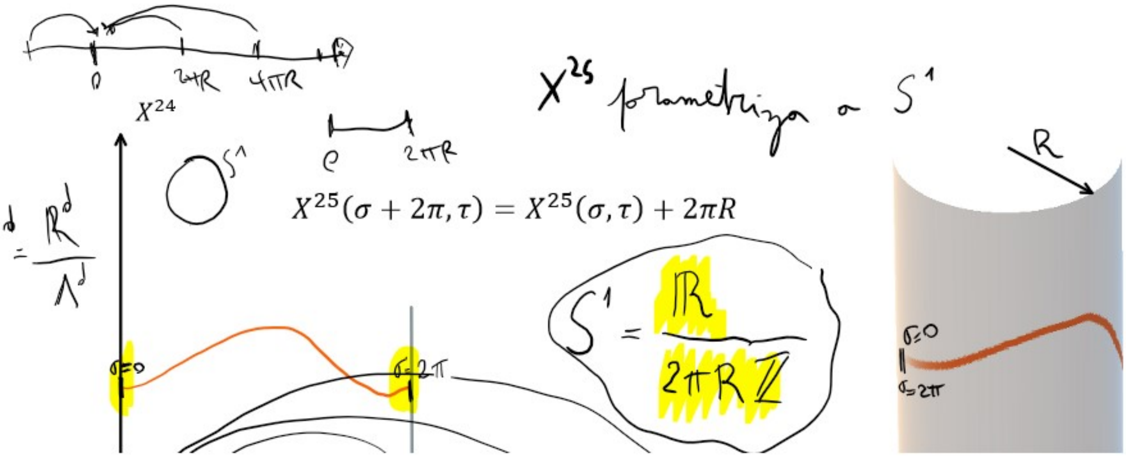


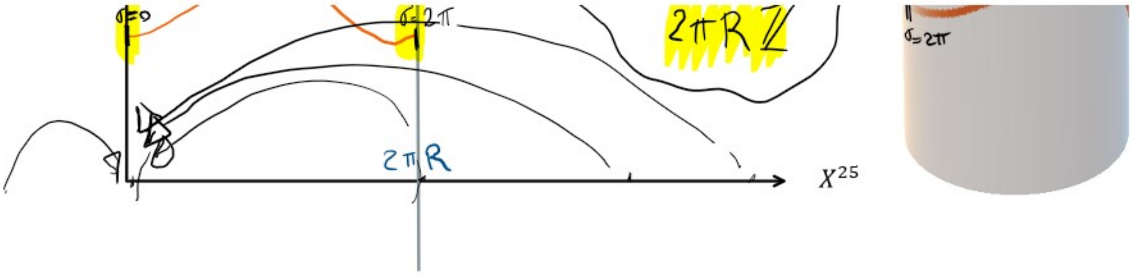
X^{25} parametriza a S^1

$$X^{25}(\sigma + 2\pi, \tau) = X^{25}(\sigma, \tau) + 2\pi R$$

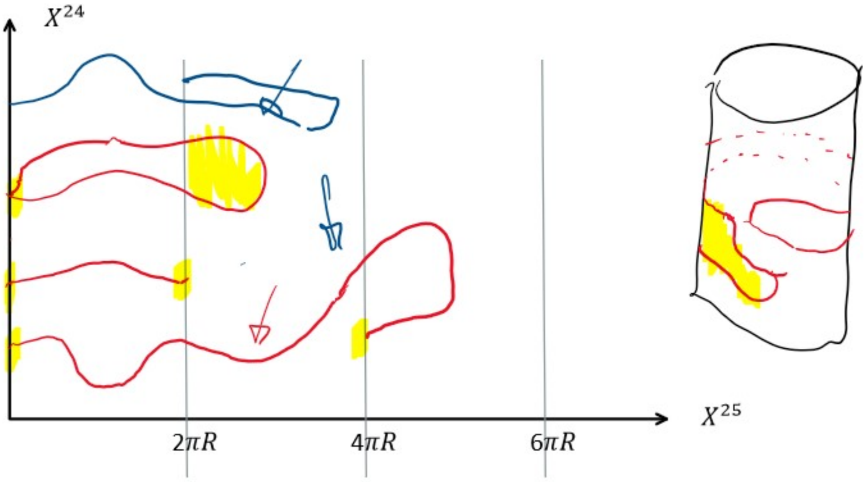
$$S^1 = \frac{\mathbb{R}}{2\pi R \mathbb{Z}}$$

$$T^d = \frac{\mathbb{R}^d}{\Lambda^d}$$





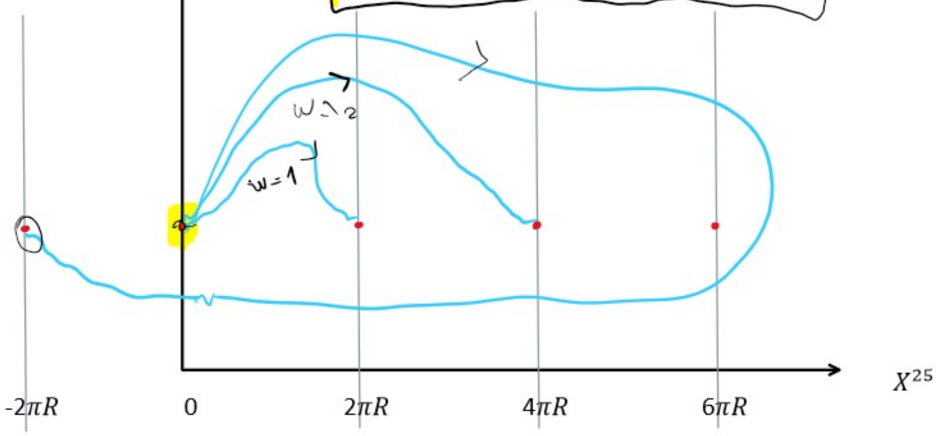
X^{25}parametriza al círculo S^1 (de radio R) $S^1 = \frac{\mathbb{R}}{2\pi R\mathbb{Z}}$



$$+(2)\,2\pi R$$

$$X^{25}(2\pi,\tau) = X^{25}(0,\tau) + (1)2\pi R \qquad X^{25}(2\pi,\tau) = X^{25}(0,\tau) - 2\pi R$$

$$\boxed{X^{25}(\sigma + 2\pi, \tau) = X^{25}(\sigma, \tau) + w2\pi R,} \quad w \in \mathbb{Z}$$



$0, \frac{1}{R}, \frac{2}{R}$

Los estados de la cuerda tienen que ser invariantes ante el cambio $x^{25} \to x^{25} + 2\pi R$

$$e^{i\Delta X_\mu p^\mu} \longrightarrow \underbrace{e^{i2\pi R\, p^{25}}}_{=1} \qquad p^{25} = \frac{n}{R},\ n \in \mathbb{Z}$$

$$X^{25}(\sigma,\tau) = x^{25} + \alpha' p^{25}\tau + \left(???\right) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^{25}e^{-in(\tau-\sigma)} + \bar{\alpha}_n^{25}e^{-in(\tau+\sigma)}\right)$$

$$p^{25} = \frac{n}{R} \qquad\qquad X^{25}(\sigma+2\pi,\tau) = X^{25}(\sigma,\tau) + w2\pi R$$

$$X^{25}(\sigma+2\pi,\tau) = x^{25} + \alpha'\frac{n}{R}\tau + Rw\sigma \; + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^{25}e^{-in(\tau-\sigma)} + \bar{\alpha}_n^{25}e^{-in(\tau+\sigma)}\right)$$

Separo en modos izquierdos y derechos: $X^{25}(\sigma,\tau) = X_L^{25}(\sigma+\tau) + X_R^{25}(\sigma-\tau)$

$$X_L^{25}(\tau+\sigma) = \tfrac{1}{2}x^{25} + \tfrac{\alpha'}{2}\left(\tfrac{n}{R} + \tfrac{wR}{\alpha'}\right)(\tau+\sigma) \quad + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\bar{\alpha}_n^{25}e^{-in(\tau+\sigma)}$$

$$X_R^{25}(\tau-\sigma) = \tfrac{1}{2}x^{25} + \tfrac{\alpha'}{2}\left(\tfrac{n}{R} - \tfrac{wR}{\alpha'}\right)(\tau-\sigma) \quad + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{25}e^{-in(\tau-\sigma)}$$

$$p_L^{25} = \tfrac{n}{R} + \tfrac{wR}{\alpha'} \qquad\qquad p_R^{25} = \tfrac{n}{R} - \tfrac{wR}{\alpha'}$$

$$p_L = \sqrt{\frac{2}{\alpha'}}\,\alpha_0$$

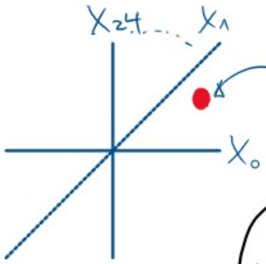
$$\partial X(z) = -i\sqrt{\frac{\alpha'}{2}}\sum_m \frac{\alpha_m}{z^{m+1}}$$

$$\Delta X = \oint dz\,\partial X - d\bar{z}\,\bar{\partial}X \propto \alpha_0 - \tilde{\alpha}_0$$

$$2\pi R w$$

$$\frac{n}{R} = p^{25} = \oint(\quad) + (\quad) \quad \alpha_0 + \tilde{\alpha}_0$$

$$p^{\mu} \longrightarrow \mu = 0, 1, \dots, 24$$



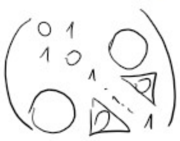
$p^\mu, \mu = 0, 1, \dots, 24$

$$\boxed{M^2 = -\sum_{\mu=0}^{24} p_\mu p^\mu}$$

CONSTRAINTS

$$\boxed{(\partial_+ X)^2 = (\partial_- X)^2 = 0} \,, \tag{2.11}$$

$$X^\pm = \sqrt{\frac{1}{2}}(X^0 \pm X^1) \,. \tag{2.12}$$



$$ds^2 = -2dX^+dX^- + \sum_{i=2}^{25} dX^i dX^i$$

$$X^+ = x^+ + \alpha' p^+ \tau \,. \tag{2.13}$$

$$X^- = X_L^-(\sigma^+) + X_R^-(\sigma^-)$$

We're still left with all the other constraints (2.11). Here we see the real benefit of working in lightcone gauge (which is actually what makes quantization possible at all): X^- is completely determined by these constraints. For example, the first of these reads

$$2\partial_+ X^- \partial_+ X^+ = \sum_{i=2}^{25} \partial_+ X^i \partial_+ X^i \tag{2.14}$$

which, using (2.13), simply becomes

$$\partial_+ X_L^- = \frac{1}{\alpha' p^+} \sum_{i=2}^{25} \partial_+ X^i \partial_+ X^i \; . \tag{2.15}$$

Similarly,

$$\partial_- X_R^- = \frac{1}{\alpha' p^+} \sum_{i=2}^{25} \partial_- X^i \partial_- X^i \; . \tag{2.16}$$

$$\frac{\alpha' p^-}{2} = \frac{1}{2p^+} \sum_{i=2}^{25} \left(\tfrac{1}{2}\, \alpha' p^i p^i + \sum_{n \neq 0} \alpha_n^i \alpha_{-n}^i \right) . \tag{2.18}$$

We also get another equation for p^- from the $\tilde{\alpha}_0^-$ equation arising from (2.15)

$$\frac{\alpha' p^-}{2} = \frac{1}{2p^+} \sum_{i=2}^{25} \left(\tfrac{1}{2}\, \alpha' p^i p^i + \sum_{n \neq 0} \tilde{\alpha}_n^i \tilde{\alpha}_{-n}^i \right) . \tag{2.19}$$

$$2p^+p^- = \sum_{i=2}^{25} \left(p^i p^i + \underbrace{\frac{2}{\alpha'} \sum_{n\neq 0} \alpha_n^i \alpha_{-n}^i}_{\frac{4}{\alpha'}\left(\underbrace{\sum_{n>0} \alpha_n^i \alpha_{-n}^i}_{N} - 1/24\right)} \right)$$

From these two equations, we can reconstruct the old, classical, level matching conditions (1.41). But now with a difference:

$$M^2 = 2p^+p^- - \sum_{i=2}^{25} p^i p^i = \frac{4}{\alpha'} \sum_{i=2}^{25} \sum_{n>0} \alpha_{-n}^i \alpha_n^i = \frac{4}{\alpha'} \sum_{i=2}^{25} \sum_{n>0} \tilde{\alpha}_{-n}^i \tilde{\alpha}_n^i \; . \tag{2.20}$$

EN NUESTRO CASO ES:

(INCLUYE OSC. EN DIR. COMPACTA)!!

$$m^2 = -\sum_{\mu=0}^{24} p_\mu p^\mu$$

$$\left\{ \begin{aligned} 2p^+p^- &= \left(\sum_{i=2}^{24} p^i p^i \right) + \underline{p_R^{25} p_R^{25}} + \underline{\frac{4}{\alpha'}\left(\sum_{n>0} \sum_{i=2}^{25} \alpha_n^i \alpha_{-n}^i - 1 \right)} \\ 2p^+p^- &= \left(\sum_{i=2}^{24} p^i p^i \right) + \underline{p_L^{25} p_L^{25}} + \underline{\frac{4}{\alpha'}\left(\sum_{n>0} \sum_{i=2}^{25} \tilde{\alpha}_n^i \tilde{\alpha}_{-n}^i - 1 \right)} \end{aligned} \right.$$

Antes era p_R^i=p_L^i siempre, pero en la dirección compacta no lo es cuando hay winding!

$$m^2 = 2p^+p^- - \cancel{\sum_{i=2}^{24} p^i p^i} = \left\{ \begin{aligned} & p_R^{25} p_R^{25} + \frac{4}{\alpha'}(N-1) \\ & p_L^{25} p_L^{25} + \frac{4}{\alpha'}(\tilde{N}-1) \end{aligned} \right.$$

$$\cancel{\left(\sum_{i=2}^{24} p^i p^i \right)} + p_R p_R + \frac{4}{\alpha'}(N-1)$$

$$L_0 = \tilde{L}_0 \qquad e^{L_0 - \tilde{L}_0} = 1$$

$$m_L^2 = \left(\frac{n}{R} + \frac{wR}{\alpha'} \right)^2 + \frac{4}{\alpha'}(\tilde{N}-1) \qquad m^2 = \frac{m_L^2 + m_R^2}{2}$$

$$m_R^2 = \left(\frac{n}{R} - \frac{wR}{\alpha'} \right)^2 + \frac{4}{\alpha'}(N-1) \qquad m_L^2 = m_R^2$$

$$m_R^2 = \left(\frac{n}{R} - \frac{wR}{\alpha'}\right)^2 + \frac{4}{\alpha'}(N-1) \qquad m_L^2 = m_R^2$$

Masa del estado con momento n, winding w, niveles N y $\tilde{N}$ y radio R:

$$m^2 = \frac{n^2}{R^2} + \frac{R^2 w^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2)$$

$$T \cdot \Delta X = T \cdot 2\pi R w$$

$$4\frac{n}{R}\cdot\frac{wR}{\alpha'} = \frac{4}{\alpha'}(N - \tilde{N})$$

$$T = \frac{1}{2\pi\alpha'} \qquad \frac{Rw}{\alpha'}$$

Condición de level-matching:
$N - \tilde{N} = nw$

Espectro de la teoría compactifacada:

$$N - \tilde{N} = nw \qquad \alpha' m^2 = \frac{\alpha'}{R^2} n^2 + \frac{R^2}{\alpha'} w^2 + 2(N + \tilde{N} - 2)$$

$n = w = 0$ $\qquad$ $n = w = 0$

-4

$$N - \tilde{N} = 0 \qquad \alpha' m^2 = 2(N + \tilde{N} - 2)$$

$|0, p^\mu\rangle$ $\mu \to 0, \dots, 24$

$N = \tilde{N} = 0 \quad |0, p^\mu; n = w = 0\rangle$ $\qquad$ TAQUIÓN $\qquad$ $\boxed{L_n \; \tilde{L}_n, \; n > 0}$

$\alpha' m^2 = -4$

$M \to 0, \dots, 25$

$$N = \tilde{N} = 1 \qquad \alpha_{-1}^M \tilde{\alpha}_{-1}^N \underbrace{|0, p^\mu; n = w = 0\rangle}_{|0\rangle}$$

$\alpha' m^2 = 0$

$$\alpha_{-1}^\mu \tilde{\alpha}_{-1}^\nu |0\rangle \to G^{\mu\nu} \text{ (en } 25-d)$$

$$\alpha'_{-1}\alpha_{-1}|0\rangle \qquad G^{\cdot\cdot}\ (\ln 25-d)$$

$$|V_1^\mu\rangle = \alpha_{-1}^{25}\tilde{\alpha}_{-1}^{\mu}|0\rangle \qquad |V_1^\mu\rangle + |V_2^\mu\rangle = A^\mu$$

$$|V_2^\mu\rangle = \alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{25}|0\rangle \qquad |V_1^\mu\rangle - |V_2^\mu\rangle = \bar{A}^\mu$$

$$\Phi = \alpha_{-1}^{25}\tilde{\alpha}_{-1}^{25}|0\rangle$$

$$\to \partial X^{25}(z)\,\bar{\partial} X^{25}(\bar{z})\, e^{ip_\mu X^\mu} \qquad G_{25,25}$$

$$\underbrace{:e^{i(p_L X_L^{25} + p_R X_R^{25})}:}\ \Rightarrow |0,n,w\rangle$$

$$\langle V_1^\mu(z,1), |0,n,w\rangle, |0,n,w\rangle\rangle \xrightarrow{|0,n,w\rangle} \quad |0\,n\,w\rangle$$

$$\propto (p_L^{25} + p_R^{25}) \propto n \quad \Big\} V_1^\mu$$

$$\langle V_2, |0,n,w\rangle, |0,n,w\rangle\rangle \propto (p_L^{25} - p_R^{25}) \propto w$$

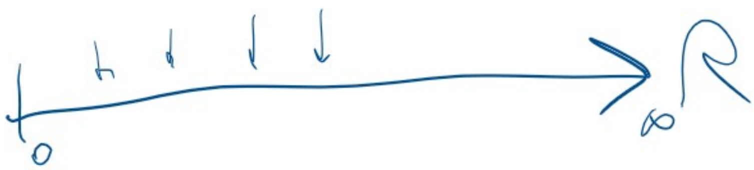
$$G^{MN} = \left(\begin{array}{c|c} G^{\mu\nu} & G^{\mu 25} \\ \hline G^{25\nu} & G^{25\,25} = \Phi \end{array}\right)$$

CARGADOS FRENTE A $G^{\mu 25}$

$$B^{MN} = \left(\begin{array}{c|c} B^{\mu\nu} & B^{\mu 25} \end{array}\right)$$

$w \to B^{\mu 25}$

$$B = \begin{pmatrix} B & 0 \\ B^{25} & 0 \end{pmatrix} \quad \to B$$



$N - \tilde{N} = nw \qquad \alpha' m^2 = \frac{\alpha'}{R^2} n^2 + \frac{R^2}{\alpha'} w^2 + 2(N + \tilde{N} - 2)$

$\boxed{\alpha' m^2 = 0}$

$$\begin{cases} 2(N - \tilde{N}) - 2wn = 0 \\ \frac{\alpha'}{R^2} n^2 + \frac{R^2}{\alpha'} w^2 + 2(N + \tilde{N} - 2) = 0 \end{cases}$$

$$\boxed{\begin{aligned} \left|\frac{\sqrt{\alpha'}}{R} n - \frac{R}{\sqrt{\alpha'}} w\right| &= 2\sqrt{1-N} \\ \left|\frac{\sqrt{\alpha'}}{R} n + \frac{R}{\sqrt{\alpha'}} w\right| &= 2\sqrt{1-\tilde{N}} \end{aligned}}$$