

Física de Semiconductores

Lección 2

Modelo de Drude

- Suponemos que en un metal o un semiconductor dopado existen electrones “libres” cuya interacción con el medio en el que se mueven está caracterizada por colisiones con probabilidad $1/\tau$ por unidad de tiempo, donde τ es una constante que se llama tiempo de relajación.
- Los electrones emergen de cada colisión con una velocidad dirigida al azar y con magnitud correspondiente a la temperatura local.

Ecuación de movimiento I

- Supongamos que una fuerza \mathbf{f} actúa sobre los electrones. El momentum es:

$$\mathbf{p}(t + dt) = \underbrace{\left(1 - \frac{dt}{\tau}\right)}_{\text{fracción que no sufre colisiones}} \underbrace{\left[\mathbf{p}(t) + \mathbf{f}(t) dt\right]}_{\text{fracción que no sufre colisiones}} + \underbrace{\left(\frac{dt}{\tau}\right)}_{\text{fracción que sufre colisiones}} \underbrace{\left[\mathbf{f}(t) dt\right]}_{\text{no memory!!}}$$

Ecuación de movimiento II

- Despreciando términos que van como $(dt)^2$

$$\frac{d\mathbf{p}(t)}{dt} = \mathbf{f}(t) - \frac{\mathbf{p}(t)}{\tau}$$

- Si $\mathbf{f}(t) = -e\mathbf{E}$

$$\mathbf{p}(t) = -e\mathbf{E}\tau(1 - e^{-t/\tau}) + \mathbf{p}(0)e^{-t/\tau}$$

Conductivity and resistivity

- La solución estacionaria es pues

$$\mathbf{p}_{\text{drift}} = -e\mathbf{E}\tau$$

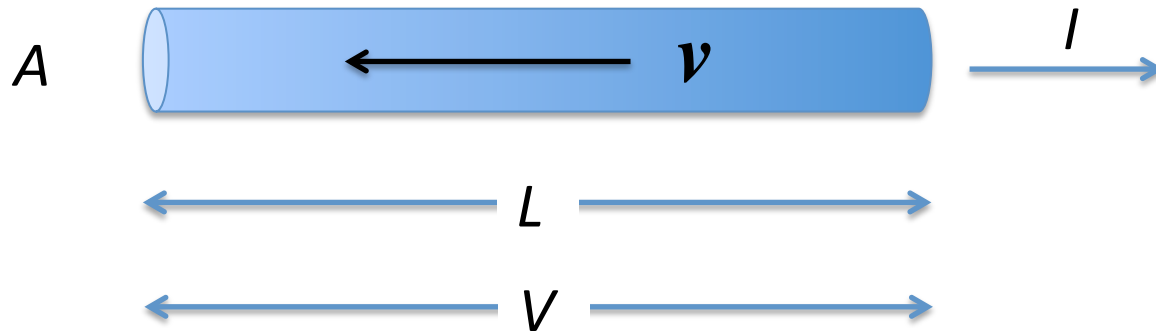
- o $\mathbf{v}_{\text{drift}} = -\frac{e\mathbf{E}\tau}{m}$

$$nev_{\text{drift}} = \mathbf{j} = -\frac{ne^2\mathbf{E}\tau}{m} = \sigma\mathbf{E} = \frac{1}{\rho}\mathbf{E}$$

- o sea

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$$

Resistividad



$$j = \frac{E}{\rho}$$

$$jA = \frac{EA}{\rho} = \frac{EAL}{\rho L}$$

$$I = \frac{V}{(\rho L/A)} = \frac{V}{R}$$

Tiempo de relajación

$$\tau = \frac{m}{\rho e^2 n}$$

e	$1.60219 \times 10^{-19} \text{ C}$
m	$9.109 \times 10^{-31} \text{ kg}$
ρ	$1.6 \times 10^{-8} \Omega\text{m}$ (copper)
n	$4/[(3.61 \times 10^{-10} \text{ m})^3]$ (copper)




$$\tau = 2.6 \times 10^{-14} \text{ C}$$

$$v_{\text{drift}} = \frac{eE\tau}{m} = \frac{(1.6 \times 10^{-19} \text{ C})(2.6 \times 10^{-14} \text{ s})}{(9.1 \times 10^{-31} \text{ kg})} \left(\frac{220 \text{ V}}{0.1 \text{ m}} \right)$$

$\sim 10 \text{ m/s}$

El error de Drude

- Drude supuso: $\frac{1}{2} m v_{\text{ave}}^2 = \frac{3}{2} k_B T$

 $v_{\text{ave}} \simeq 10^7 \text{ cm/s}$

$$10^7 \text{ cm/s} \times 2.6 \times 10^{-14} \text{ s} = 26 \text{ \AA}$$

Magnetic field effect

$$\begin{aligned}\frac{d\mathbf{p}(t)}{dt} &= -e \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{H} \right) - \frac{\mathbf{p}(t)}{\tau} \\ &= -e \left(\mathbf{E} + \frac{\mathbf{p}}{mc} \times \mathbf{H} \right) - \frac{\mathbf{p}(t)}{\tau}\end{aligned}$$

Steady state solution for $\mathbf{H} = (0, 0, H)$

$$0 = -eE_x - \frac{eH}{mc} p_y - p_x / \tau$$

$$0 = -eE_y + \frac{eH}{mc} p_x - p_y / \tau$$

$$0 = -eE_z - p_z / \tau$$

But $\mathbf{p} = (p_x, 0, 0)$ in the steady state, so

$$E_y = -\frac{H}{mc} p_x = -\frac{H}{c} v_x = -\frac{H}{nec} nev_x = -\frac{H}{nec} j_x$$

$$R_H = \frac{E_y}{j_x H} = -\frac{1}{nec}$$

Steady state solution for $\mathbf{H} = (0, 0, H)$

$$0 = -eE_x - \frac{eH}{mc} p_y - p_x / \tau$$

$$0 = -eE_y + \frac{eH}{mc} p_x - p_y / \tau$$

$$0 = -eE_z - p_z / \tau$$

$$E_x = \frac{\omega_c \tau}{\sigma_0} j_y + j_x / \sigma_0$$

$$E_y = -\frac{\omega_c \tau}{\sigma_0} j_x + j_y / \sigma_0$$

$$E_z = j_z / \sigma_0$$

$$\omega_c = \frac{eH}{mc}$$

$$\sigma_0 = \frac{ne^2 \tau}{m}$$

Forma matricial

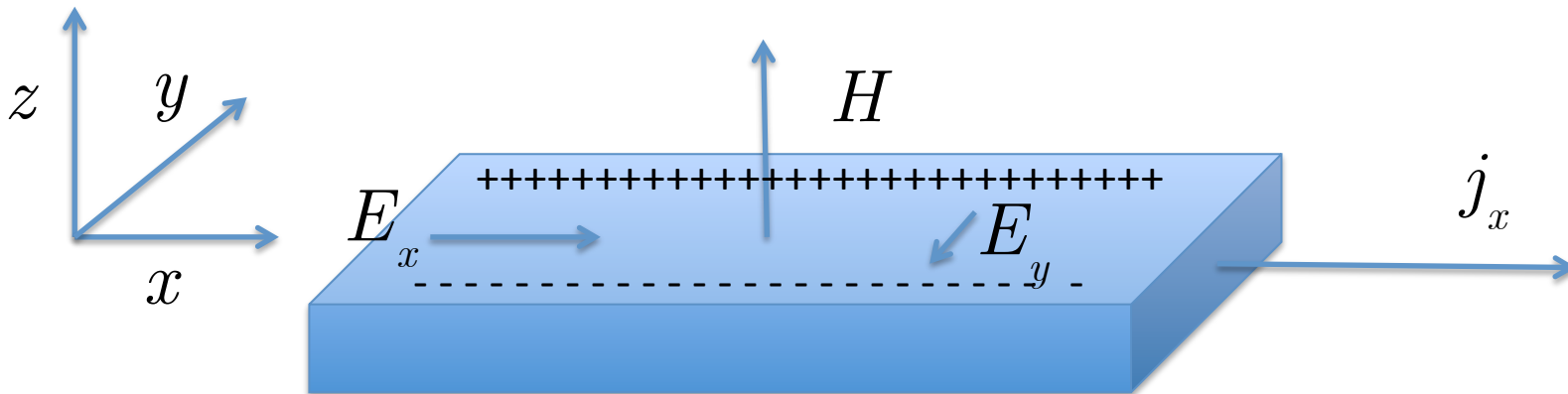
$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \frac{1}{\sigma_0} \begin{pmatrix} 1 & \omega_c \tau & 0 \\ -\omega_c \tau & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix}$$

La inversa es

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \frac{\sigma_0}{1 + (\omega_c \tau)^2} \begin{pmatrix} 1 & -\omega_c \tau & 0 \\ \omega_c \tau & 1 & 0 \\ 0 & 0 & 1 + (\omega_c \tau)^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

Geometría de Hall

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} & 0 \\ \rho_{yx} & \rho_{yy} & 0 \\ 0 & 0 & \rho_{zz} \end{pmatrix} \begin{pmatrix} j_x \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{aligned} E_x &= \rho_{xx} j_x \\ E_y &= \rho_{yx} j_x \\ E_z &= 0 \end{aligned}$$



Geometría de Hall

magnetoresistance $\rho_{xx} = \frac{1}{\sigma_0} = \frac{m}{ne^2\tau}$

Hall coef.

$$R_H = \frac{\rho_{yx}}{H} = -\frac{\omega_c \tau}{H \sigma_0} = -\frac{eH}{mc} \frac{1}{H} \tau \frac{m}{ne^2\tau} = -\frac{1}{nec}$$

Coeficiente de Hall

$$R_H = -\frac{\gamma}{nec} \rightarrow \text{Hall factor } \gamma$$

N	1.2
K	1.1
Rb	1.0
Cu	1.5
Ag	1.3
Al	-0.3

Campo dependiente del tiempo

- Supongamos $\mathbf{E}(t) = \text{Re}(\mathbf{E}(\omega)e^{-i\omega t})$
- Entonces, si $\mathbf{p}(t) = \text{Re}(\mathbf{p}(\omega)e^{-i\omega t})$

$$-i\omega\mathbf{p}(\omega) = -\frac{\mathbf{p}(\omega)}{\tau} - e\mathbf{E}(\omega)$$

$$\mathbf{j}(\omega) = \frac{-ne\mathbf{p}(\omega)}{m} = \frac{ne^2\tau/m}{1 - i\omega\tau}\mathbf{E}(\omega)$$

$$\mathbf{j}(\omega) = \sigma(\omega)\mathbf{E}(\omega) \text{ con } \sigma(\omega) = \frac{ne^2\tau/m}{1 - i\omega\tau}$$

Maxwell equations

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{j}$$

$$\nabla \times \nabla \times \mathbf{E} = -\nabla^2 \mathbf{E} = -\frac{1}{c} \nabla \times \frac{\partial \mathbf{H}}{\partial t} = \frac{i\omega}{c} \nabla \times \mathbf{H}$$

Aproximación local

- Si podemos escribir $\mathbf{j}(\mathbf{r}, \omega) = \sigma(\omega) \mathbf{E}(\mathbf{r}, \omega)$

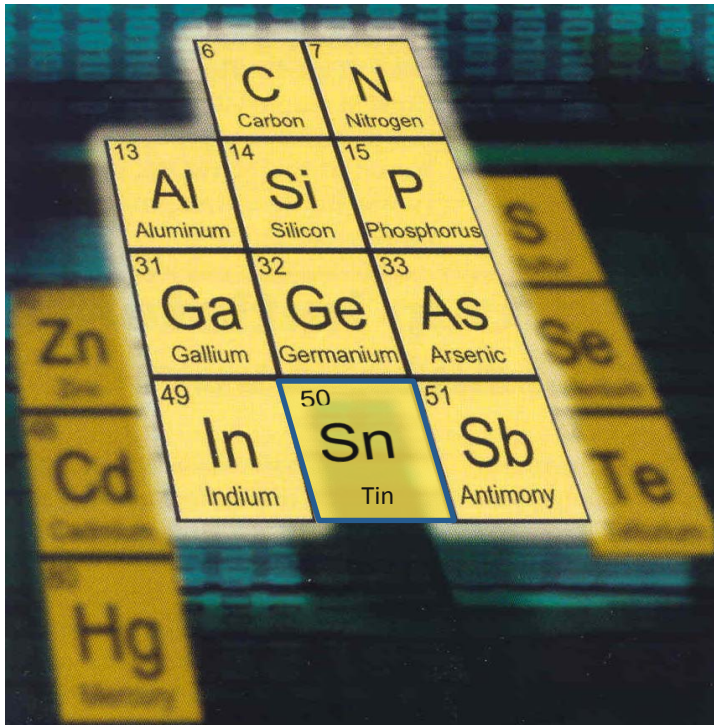
$$-\nabla^2 \mathbf{E} = \frac{i\omega}{c} \nabla \times \mathbf{H} = \frac{i\omega}{c} \left[\frac{4\pi\sigma}{c} \mathbf{E} - \frac{i\omega}{c} \mathbf{E} \right]$$

$$-\nabla^2 \mathbf{E} = \frac{\omega^2}{c^2} \left[1 + \frac{4\pi i\sigma}{\omega} \right] \mathbf{E} = \frac{\omega^2}{c^2} \varepsilon(\omega) \mathbf{E}$$

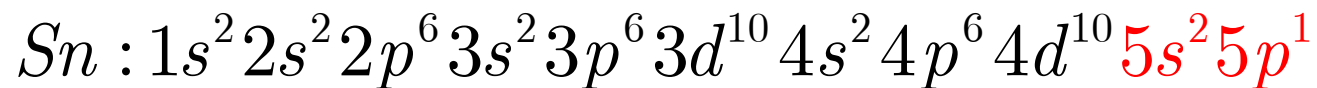
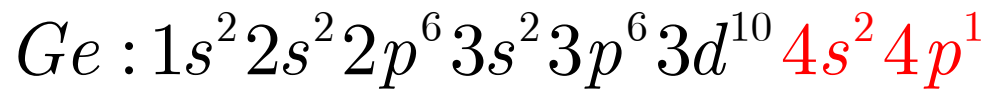
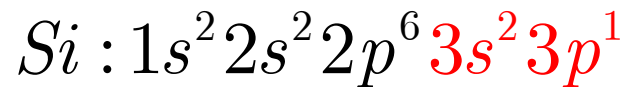
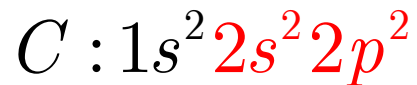
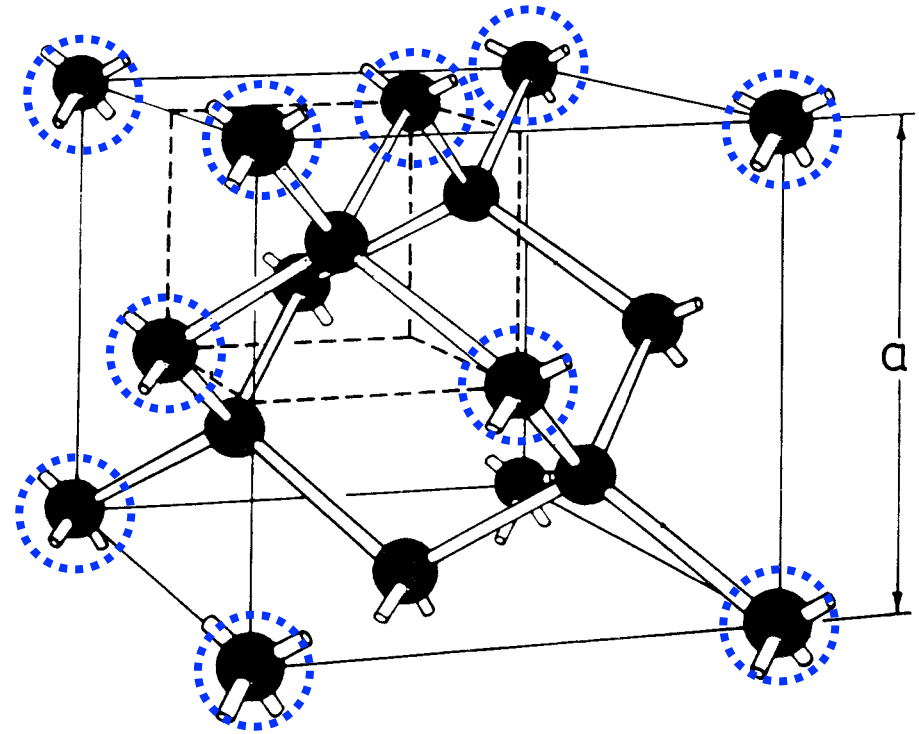
Función dieléctrica

$$\begin{aligned}\varepsilon(\omega) &= 1 + \frac{4\pi i\sigma}{\omega} \\ &= 1 + \frac{4\pi e^2 n \tau i}{m\omega(1 - i\omega\tau)} \\ &= 1 + \frac{4\pi}{\rho_0} \left[-\frac{\tau}{1 + \omega^2\tau^2} + \frac{i}{\omega(1 + \omega^2\tau^2)} \right] \\ \rho_0 &= \frac{m}{ne^2\tau} = \frac{1}{\sigma_0} \quad \sigma_0 = \frac{ne^2\tau}{m}\end{aligned}$$

Dopaje de semiconductores



6	C	7	N		
	Carbon		Nitrogen		
13	Al	14	Si	15	P
	Aluminum		Silicon		Phosphorus
31	Ga	32	Ge	33	As
	Gallium		Germanium		Arsenic
49	In	50	Sn	51	Sb
	Indium		Tin		Antimony



Spectroscopic Ellipsometry – 21

Experimental Details

Instrument: J.A. Woollam quartz-UV VASE with compensator, 0.74 to 6.6 eV Spot size: Several millimeters

Measurement time: 3-4 hours per site (with compensator).

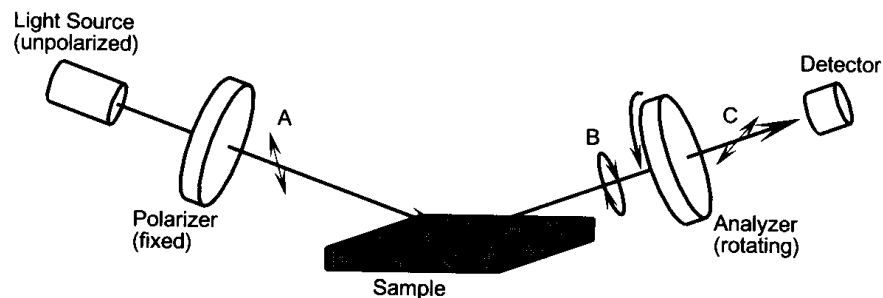
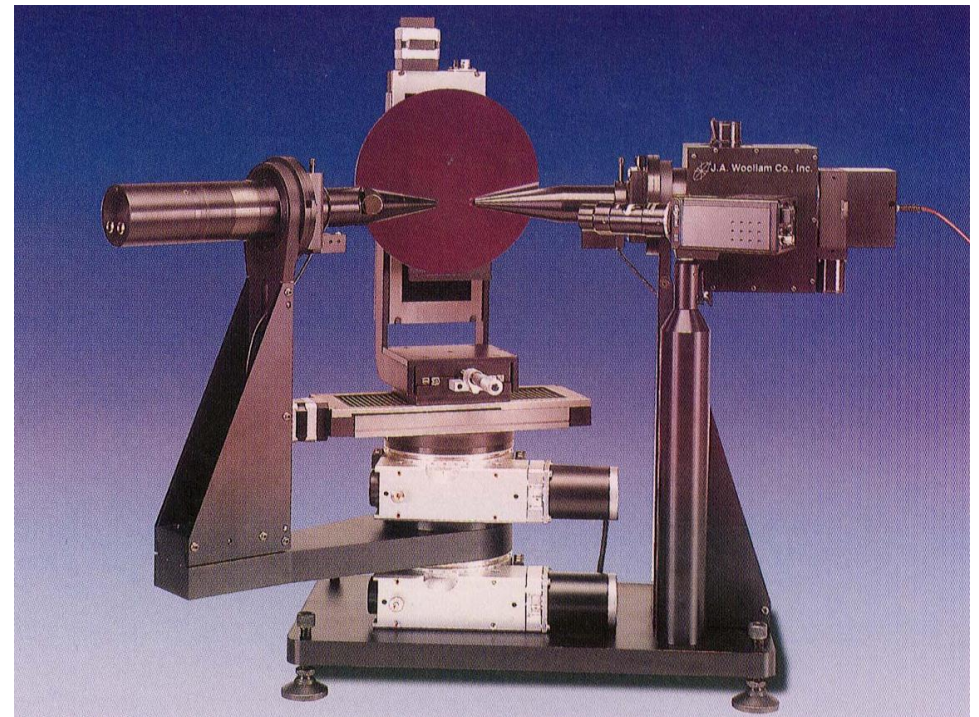
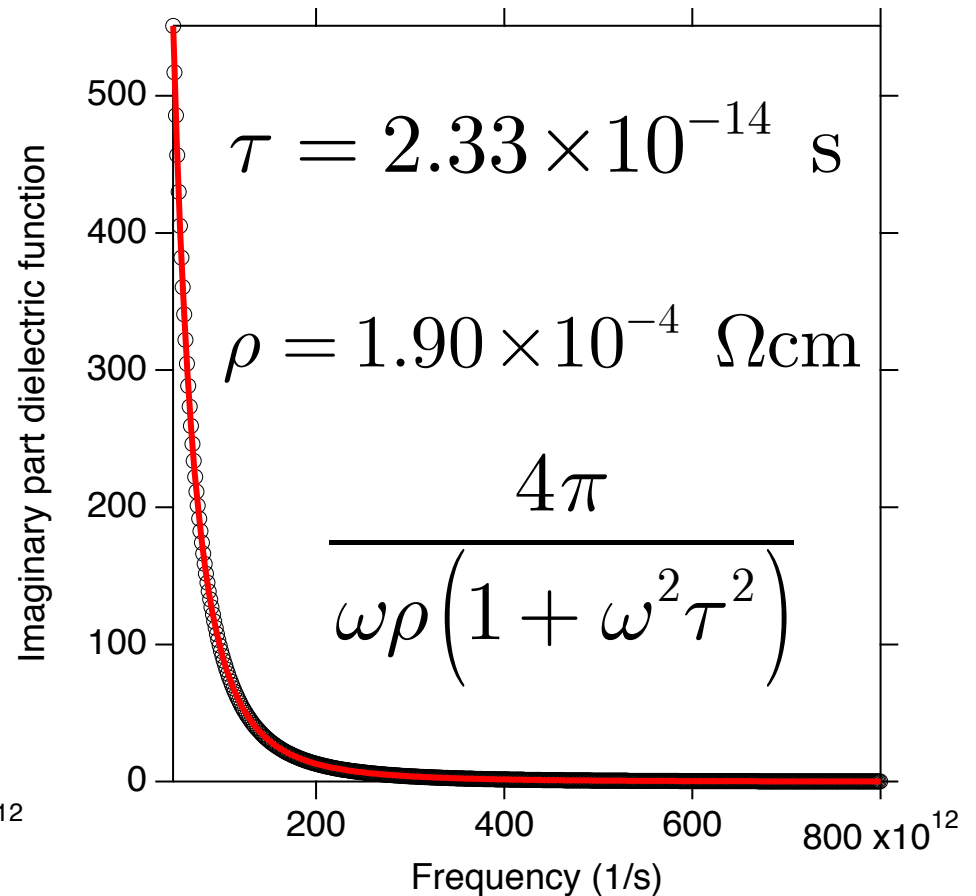
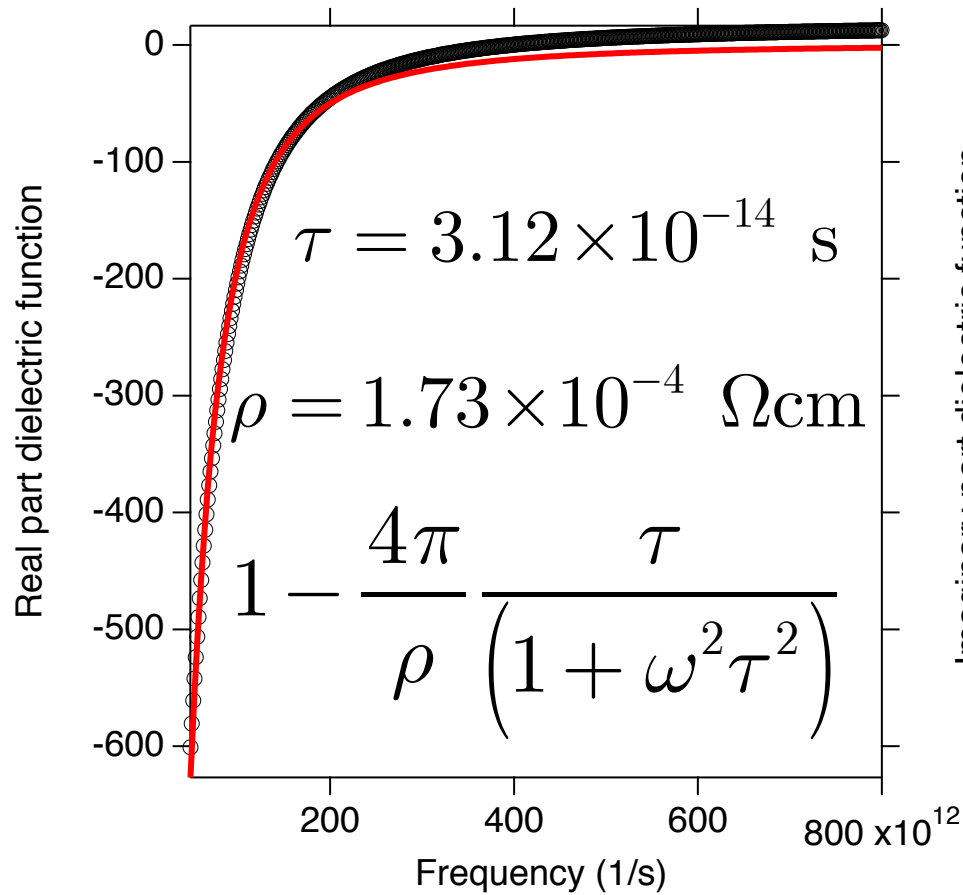


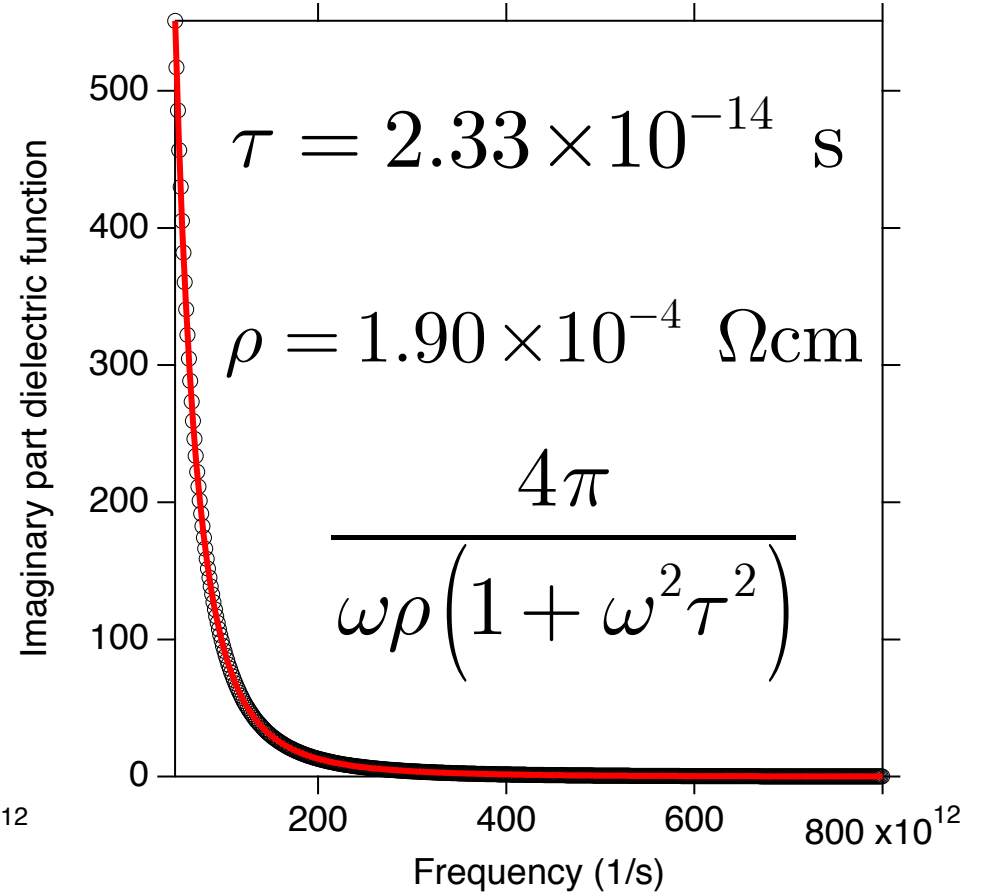
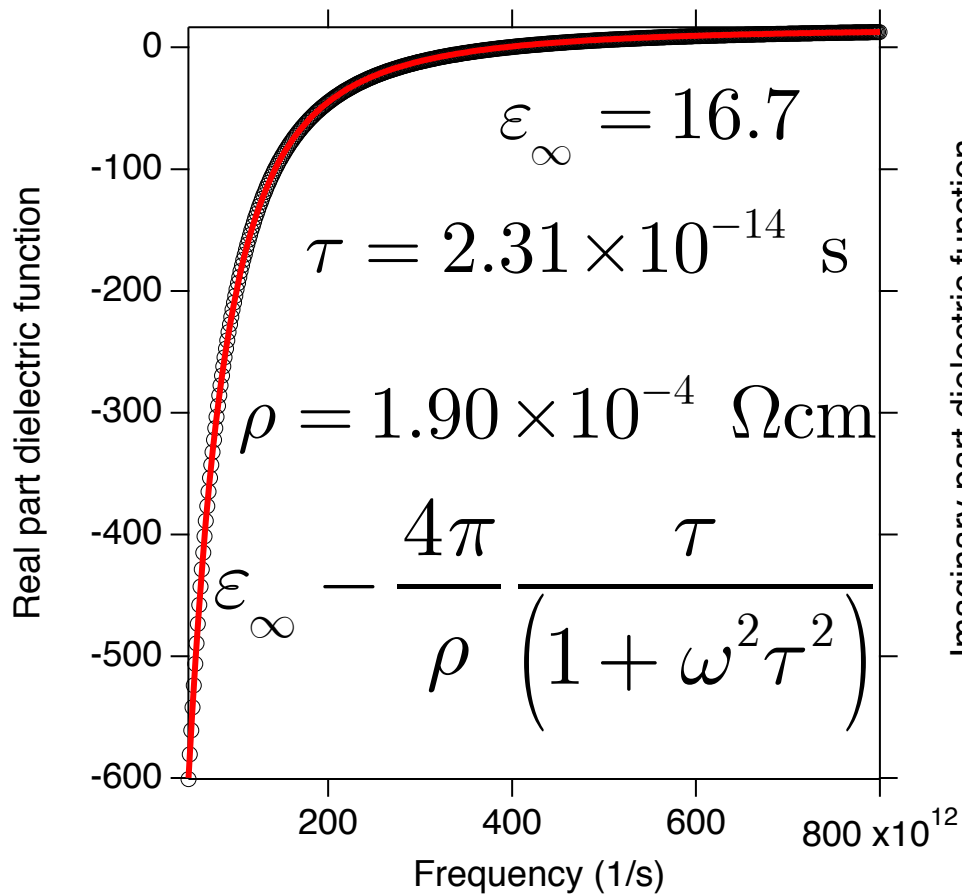
Figure 4.9 Schematic diagram of a rotating analyzer ellipsometer.



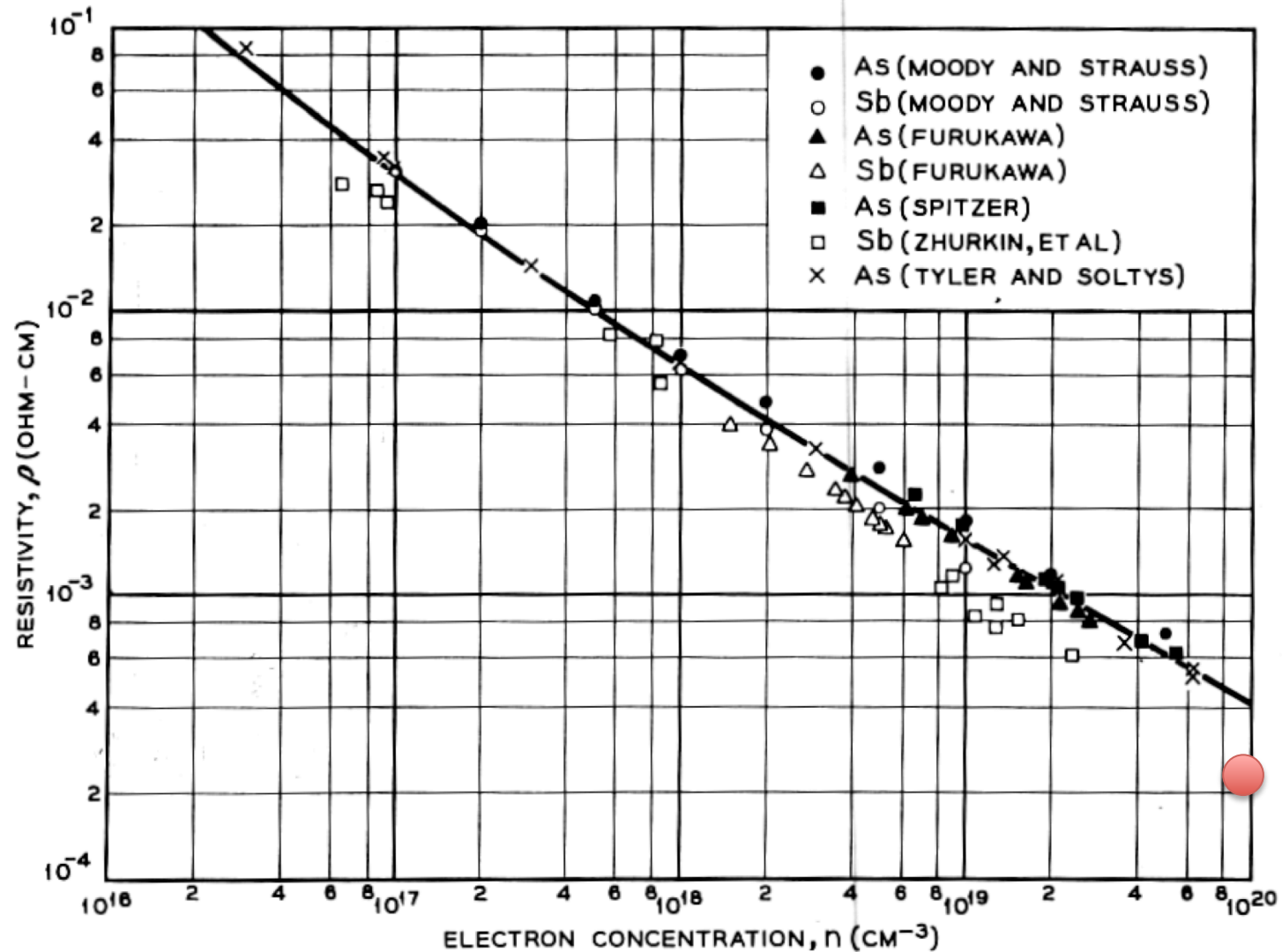
Fit germanio dopado $1.4 \times 10^{20} \text{ cm}^{-3}$



Mejorando el modelo...



Relación con otros métodos



D. B. Cuttriss, Bell Syst. Tech. J. **40** (2), 509 (1961).

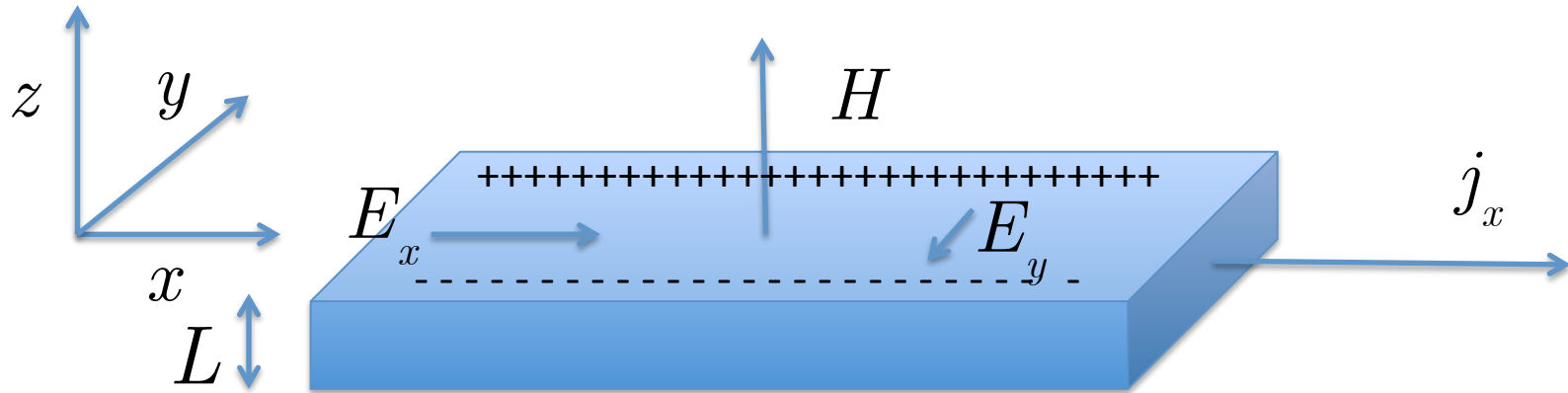
Cálculo de la densidad

$$\rho_0 = \frac{m}{ne^2\tau} \Rightarrow n = \frac{m}{\rho_0 e^2 \tau}$$

$$n = \frac{9.1095 \times 10^{-31} \text{ kg}}{\left(1.9 \times 10^{-6} \text{ } \Omega\text{m}\right) \left(1.602 \times 10^{-19} \text{ C}\right)^2 \left(2.31 \times 10^{-14} \text{ s}\right)}$$
$$= 8.1 \times 10^{20} \text{ cm}^{-3}$$

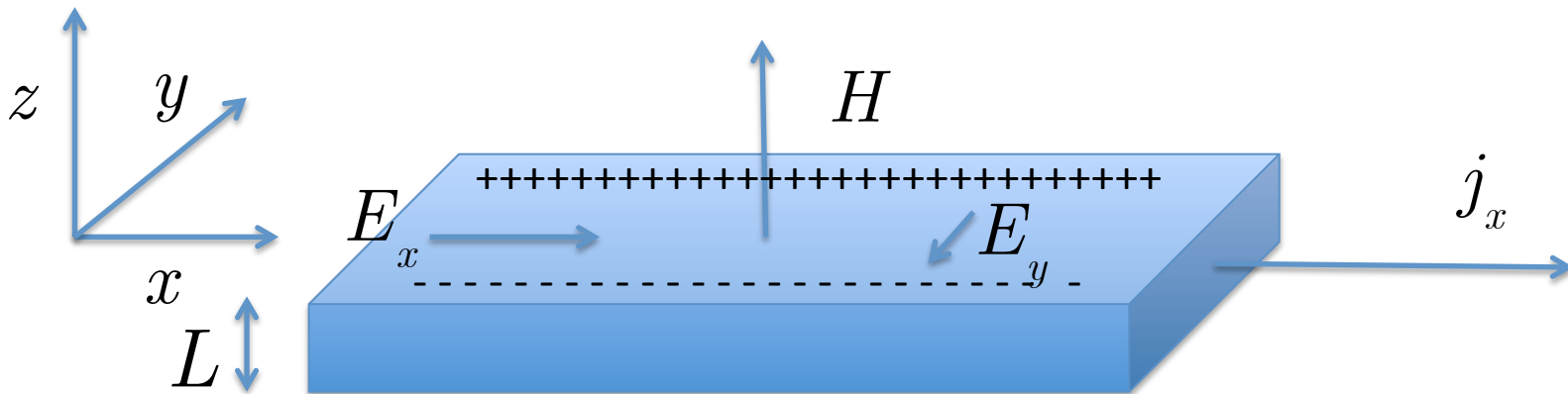
$$\frac{1.4 \times 10^{20}}{8.1 \times 10^{20}} = 0.17$$

Sistemas 2D I



$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{1}{\sigma_0 L} \begin{pmatrix} 1 & \omega_c \tau \\ -\omega_c \tau & 1 \end{pmatrix} \begin{pmatrix} j_x L \\ j_y L \end{pmatrix}$$

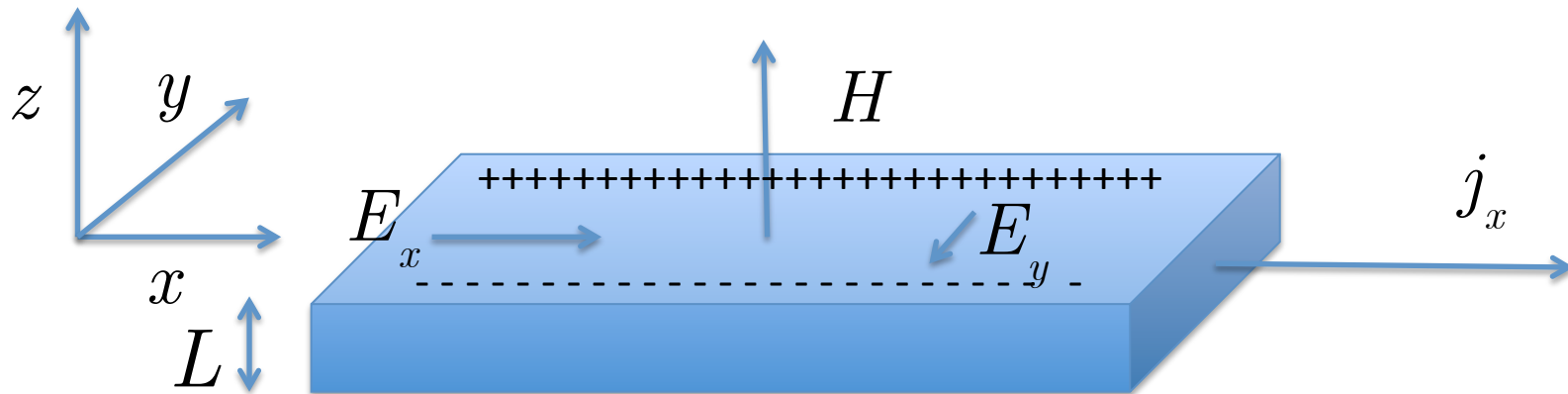
Sistemas 2D II



$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{1}{\sigma'_0} \begin{pmatrix} 1 & \omega_c \tau \\ -\omega_c \tau & 1 \end{pmatrix} \begin{pmatrix} j'_x \\ j'_y \end{pmatrix} \rightarrow \text{current per unit length.}$$

$$\sigma'_0 = \frac{nLe^2\tau}{m} = \frac{n'e^2\tau}{m} \quad n' \text{ electrones por unidad de area.}$$

Sistemas 2D III



- Podemos usar notación 3D (sin las primas) si acordamos que j es una corriente por unidad de longitud transversal, y que n es la cantidad de electrones por unidad de area.

Sistemas 2D IV

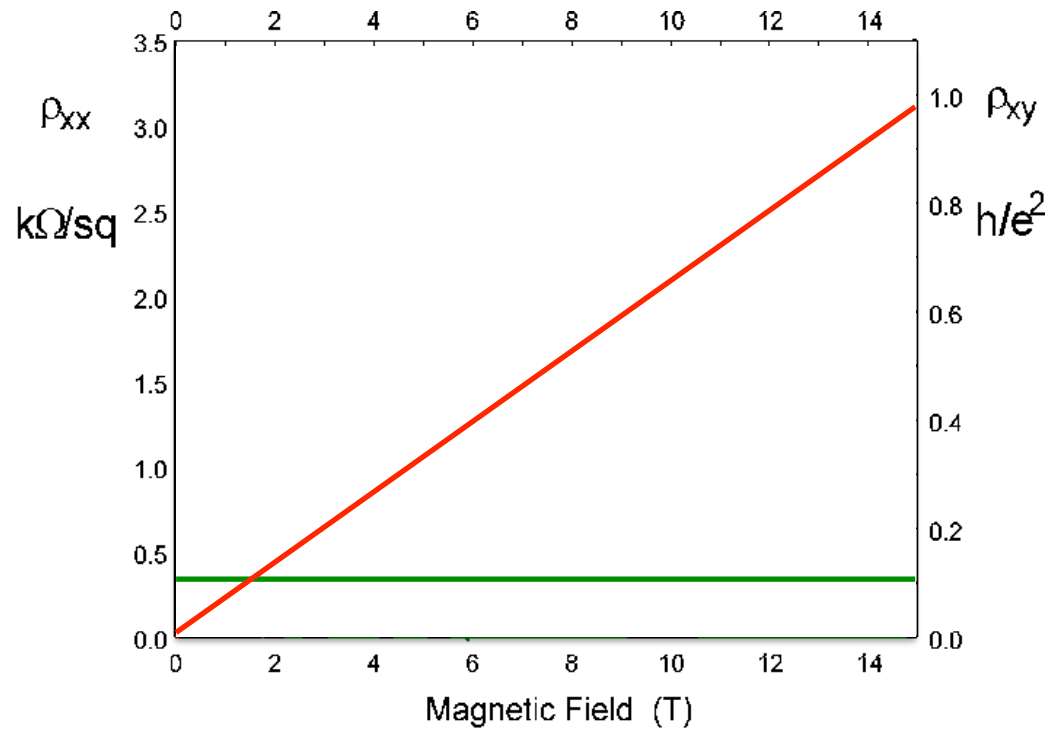
$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix} \quad \phi_0 = \frac{hc}{e} \text{ flux quantum}$$

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \quad \nu = \frac{N\phi_0}{\phi} \text{ filling factor}$$

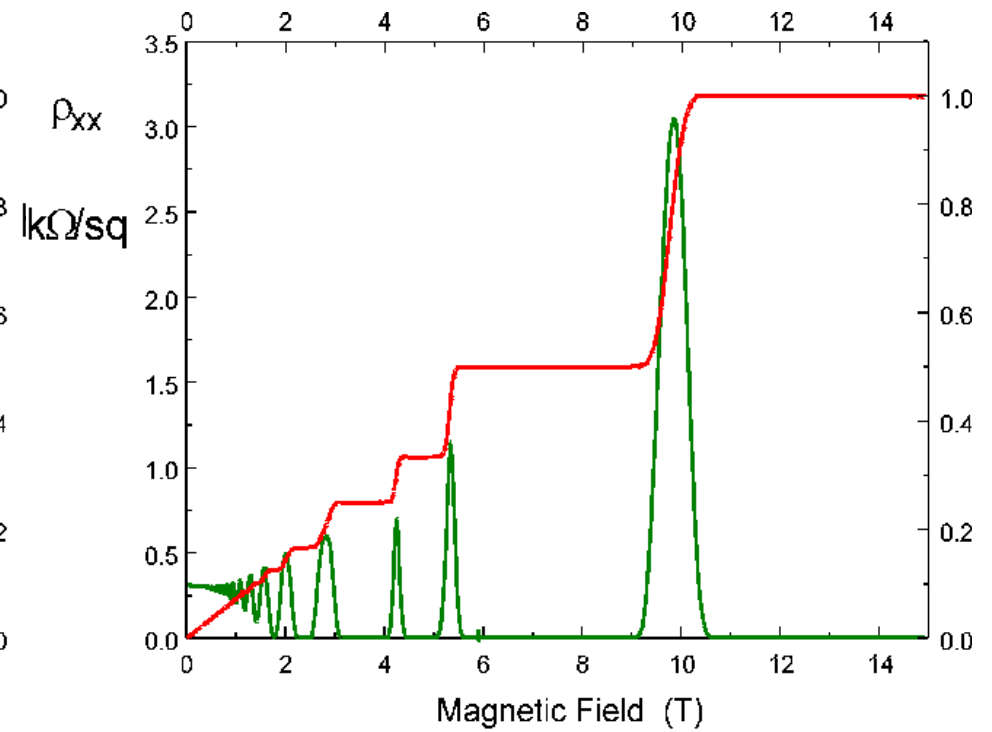
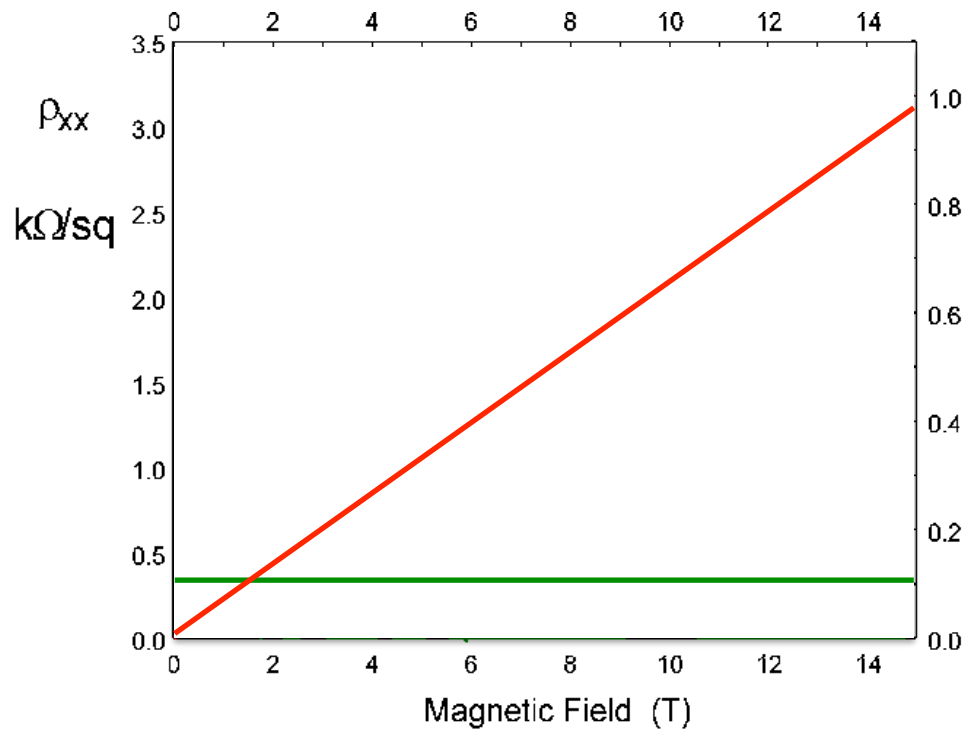
$$\rho_{xx} = \frac{1}{\sigma_0}$$

$$\rho_{xy} = \frac{HA}{nAec} = \frac{\phi}{Nec} = \frac{\phi h}{Nech} = \frac{\phi h e}{Ne^2 ch} = \frac{h\phi}{Ne^2 \phi_0} = \frac{1}{\nu} \frac{h}{e^2}$$

Predicción



Predicción+ experimento



Quantum Hall effect

Hamiltoniano

- En la formulación hamiltoniana, las ecuaciones de movimiento son

$$\frac{d\mathbf{p}}{dt} = -\frac{\partial H}{\partial \mathbf{r}} \quad \frac{d\mathbf{r}}{dt} = \frac{\partial H}{\partial \mathbf{p}}$$

- Proponemos que el hamiltoniano para un electrón en un campo es ($e > 0$)

$$H = \frac{1}{2m} \left[\mathbf{p} + \frac{e}{c} \mathbf{A}(\mathbf{r}) \right]^2 - e\varphi(\mathbf{r})$$