Física de Semiconductores

Lección 3

Repaso

- Para electrones en x-y con un campo magnético en z

$$\sigma_0 = \frac{ne^2\tau}{m}; \quad \omega_c = \frac{eH}{mc}$$

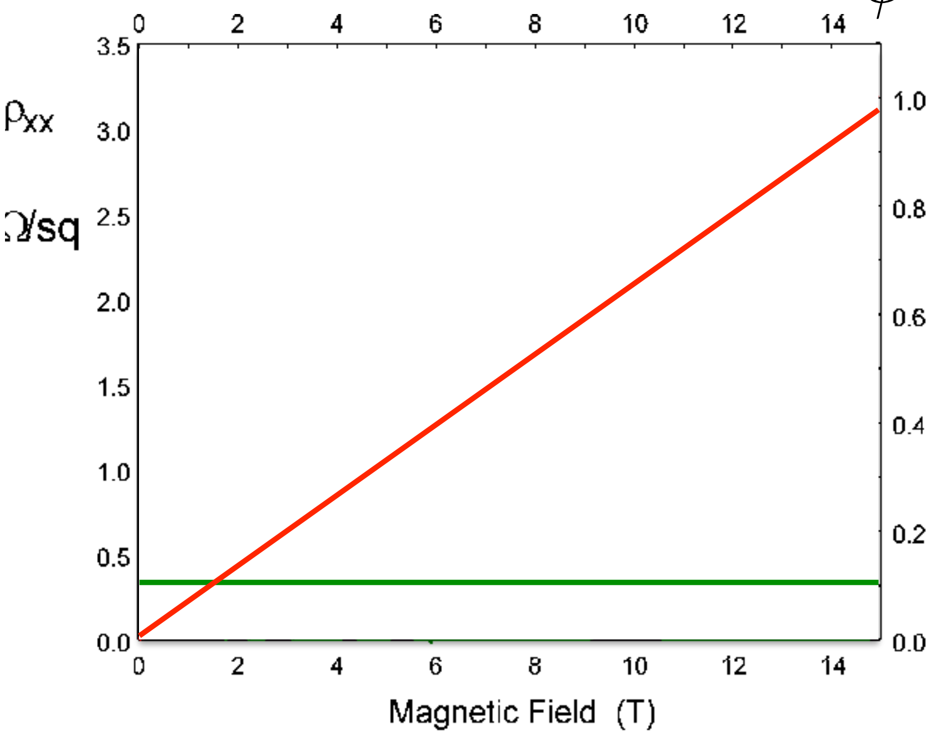
$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \frac{\sigma_0}{1+\omega_c^2\tau^2} \begin{pmatrix} 1 & -\omega_c\tau \\ \omega_c\tau & 1 \end{pmatrix}; \sigma_0 = \frac{ne^2\tau}{m}$$

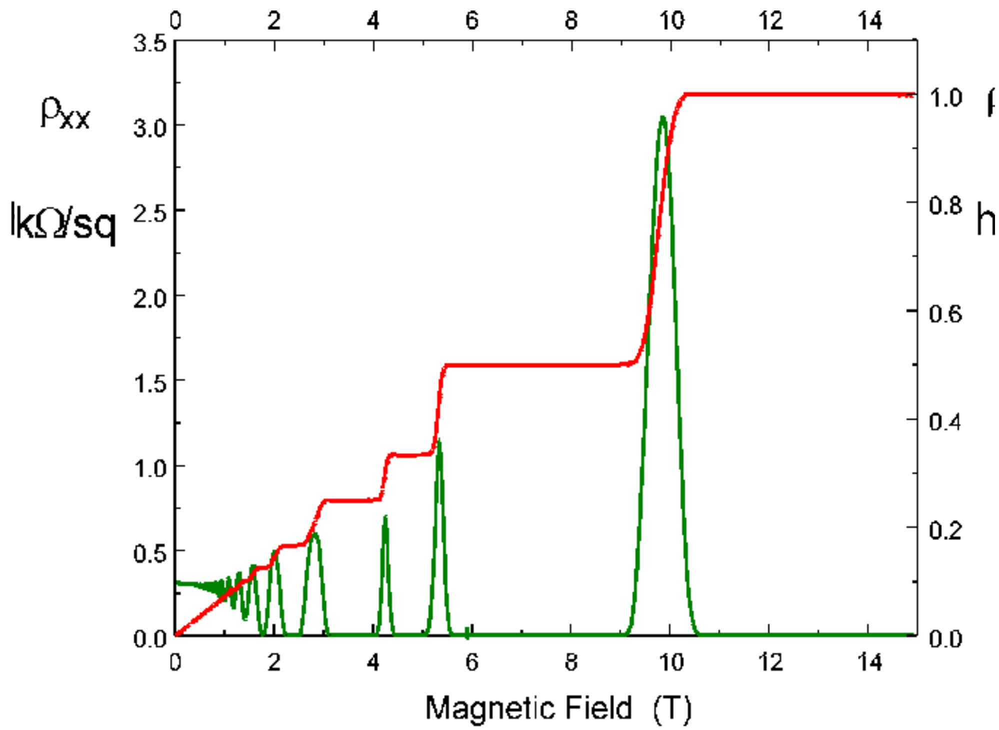
$$\rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} = \sigma^{-1} = \frac{1}{\sigma_0} \begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix}$$

Predicción vs. experimento

$$\rho_{xx} = \frac{m}{ne^2\tau}; \qquad \rho_{xy} = \frac{eH\tau}{mc}\frac{m}{ne^2\tau} = \frac{H}{nec} = \frac{1}{\nu}\frac{h}{e^2}$$

$$\nu = \frac{N\phi_0}{\phi}; \qquad \phi_0 = \frac{hc}{e}$$





Hamiltoniano

- En la formulación hamiltoniana, las ecuaciones de movimiento son

$$\frac{d\boldsymbol{p}}{dt} = -\frac{\partial H}{\partial \boldsymbol{r}} \qquad \frac{d\boldsymbol{r}}{dt} = \frac{\partial H}{\partial \boldsymbol{p}}$$

- Proponemos que el hamiltoniano para un electrón en un campo es $\left(e > 0\right)$

$$H = \frac{1}{2m}\left[\boldsymbol{p} + \frac{e}{c}\boldsymbol{A}\left(\boldsymbol{r}\right)\right]^2 - e\varphi\left(\boldsymbol{r}\right)$$

Verificación I

- En componentes, $H = \frac{1}{2m}\sum_{\alpha}\left(p_{\alpha} + \frac{e}{c}A_{\alpha}\right)^{2} - e\varphi$

$$\frac{dr_{\alpha}}{dt} = \frac{\partial H}{\partial p_{\alpha}} = \frac{1}{m}\left[p_{\alpha} + \frac{e}{c}A_{\alpha}\left(\boldsymbol{r}\right)\right]$$

$$\frac{dp_{\alpha}}{dt} = -\frac{\partial H}{\partial r_{\alpha}} = -\frac{e}{cm}\sum_{\beta}\left(p_{\beta} + \frac{e}{c}A_{\beta}\right)\frac{\partial A_{\beta}}{\partial r_{\alpha}} + e\frac{\partial \varphi}{\partial r_{\alpha}}$$

$$= -\frac{e}{c}\sum_{\beta}\frac{dr_{\beta}}{dt}\frac{\partial A_{\beta}}{\partial r_{\alpha}} + e\frac{\partial \varphi}{\partial r_{\alpha}}$$

Verificación II

$$\frac{dp_\alpha}{dt} = -\frac{e}{c}\sum_\beta \frac{dr_\beta}{dt}\frac{\partial A_\beta}{\partial r_\alpha} + e\frac{\partial \phi}{\partial r_\alpha}$$

- También

$$\frac{e}{c}\frac{dA_\alpha}{dt} = \frac{e}{c}\left(\sum_\beta \frac{\partial A_\alpha}{\partial r_\beta}\frac{dr_\beta}{dt} + \frac{\partial A_\alpha}{\partial t}\right)$$

- Sumando:

$$\frac{d}{dt}\left(p_\alpha + \frac{e}{c}\frac{dA_\alpha}{dt}\right) = \frac{e}{c}\sum_\beta \frac{dr_\beta}{dt}\left(\frac{\partial A_\alpha}{\partial r_\beta} - \frac{\partial A_\beta}{\partial r_\alpha}\right) - e\frac{\partial \phi}{\partial r_\alpha} + \frac{e}{c}\frac{\partial A_\alpha}{\partial t}$$

Verificación III

$$m\frac{d^2 r_\alpha}{dt^2} = \frac{e}{c}\sum_\beta \frac{dr_\beta}{dt}\left(\frac{\partial A_\alpha}{\partial r_\beta} - \frac{\partial A_\beta}{\partial r_\alpha}\right) - e\frac{\partial \phi}{\partial r_\alpha} + \frac{e}{c}\frac{\partial A_\alpha}{\partial t}$$

Pero $\displaystyle\sum_\beta \frac{dr_\beta}{dt}\left(\frac{\partial A_\alpha}{\partial r_\beta} - \frac{\partial A_\beta}{\partial r_\alpha}\right) = \left[\left(\boldsymbol{\nabla}\times\mathbf{A}\right)\times\boldsymbol{v}\right]_\alpha$

o sea

$$m\frac{d^2\boldsymbol{r}}{dt^2} = -\frac{e}{c}\boldsymbol{v}\times\left(\boldsymbol{\nabla}\times\boldsymbol{A}\right) - e\left(\boldsymbol{\nabla}\phi - \frac{1}{c}\frac{\partial\boldsymbol{A}}{\partial t}\right)$$

$$m\frac{d^2\boldsymbol{r}}{dt^2} = -\frac{e}{c}\boldsymbol{v}\times\boldsymbol{H} - e\boldsymbol{E}$$

Landau gauge

$$\boldsymbol{A} = \left(0, Hx, 0\right)$$

$$\begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & Hx & 0 \end{vmatrix} = \hat{\boldsymbol{k}}\left(\frac{\partial\left(Hx\right)}{\partial x} - 0\right) = \hat{\boldsymbol{k}}H$$

Ecuación de Schrödinger for $\varphi\left(\boldsymbol{r}\right)=0$

$$H=\frac{1}{2m}\left[\boldsymbol{p}+\frac{e}{c}\boldsymbol{A}\left(\boldsymbol{r}\right)\right]^2$$

$$=\frac{1}{2m}\left[-i\hbar\nabla+\frac{e}{c}\boldsymbol{A}\left(\boldsymbol{r}\right)\right]^2$$

$$=\frac{1}{2m}\left[-\hbar^2\nabla^2-\frac{i\hbar e}{c}\left(\nabla\cdot\boldsymbol{A}+\boldsymbol{A}\cdot\nabla\right)+\left(\frac{e}{c}\right)^2A^2\right]$$

$$=\frac{1}{2m}\left[-\hbar^2\nabla^2-\frac{i\hbar eHx}{c}\frac{\partial}{\partial y}+\left(\frac{e}{c}\right)^2H^2x^2\right]$$

Ecuación de Schrödinger (cont 1)

$$= \frac{1}{2m}\left[-\hbar^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)-\frac{i\hbar eHx}{c}\frac{\partial}{\partial y}+\left(\frac{e}{c}\right)^2 H^2x^2\right]$$

$$= \frac{1}{2m}\left[-\hbar^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)-\hbar^2\frac{\partial^2}{\partial y^2}-\frac{i\hbar eHx}{c}\frac{\partial}{\partial y}+\left(\frac{e}{c}\right)^2 H^2x^2\right]$$

$$= \frac{1}{2m}\left[-\hbar^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)-\left(\hbar\frac{\partial}{\partial y}+i\frac{eHx}{c}\right)^2\right]$$

Ecuación de Schrödinger (cont 2)

Finalmente:

$$\frac{1}{2m}\left[-\hbar^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)-\left(\hbar\frac{\partial}{\partial y}+i\frac{eHx}{c}\right)^2\right]\psi\left(x,y,z\right)$$

$$=E\psi\left(x,y,z\right)$$

Ecuación de Schrödinger solución

$$\frac{1}{2m}\left[-\hbar^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)-\left(\hbar\frac{\partial}{\partial y}+i\frac{eHx}{c}\right)^2\right]\psi\left(x,y,z\right)$$
$$=E\psi\left(x,y,z\right)$$

Proponemos $\psi\left(x,y,z\right)=f\left(z\right)g\left(x,y\right)$ Entonces

$$\frac{1}{2m}\left[-\hbar^2\left(f\frac{\partial^2 g}{\partial x^2}+g\frac{d^2 f}{dz^2}\right)-f\left(\hbar\frac{\partial}{\partial y}+i\frac{eHx}{c}\right)^2 g\right]=Efg$$

Schrödinger (sol. 2)

Dividing by *fg*

$$\frac{1}{2m}\left[-\hbar^2\left(\frac{1}{g}\frac{\partial^2 g}{\partial x^2}+\frac{1}{f}\frac{d^2 f}{dz^2}\right)-\frac{1}{g}\left(\hbar\frac{\partial}{\partial y}+i\frac{eHx}{c}\right)^2 g\right]=E$$

This can only be true if

$$\frac{-\hbar^2}{2m}\frac{1}{f}\frac{d^2 f}{dz^2}=E_z=\text{constant}$$

$$\frac{d^2 f}{dz^2}+\frac{2mE_z}{\hbar^2}f=0$$

Schrödinger (sol. 3)

For f to be normalizable

$$f\left(z\right) = e^{ik_z z}$$

So that

$$-k_z^2 + \frac{E_z}{\hbar^2} = 0 \Rightarrow E_z = \frac{\hbar^2 k_z^2}{2m}$$

Schrödinger (sol. 4)

We can then rewrite the original equation as

$$\frac{1}{2m}\left[-\frac{\hbar^2}{g}\frac{\partial^2 g}{\partial x^2}-\frac{1}{g}\left(\hbar\frac{\partial}{\partial y}+i\frac{eHx}{c}\right)^2 g\right]=E-E_z$$

$$\frac{1}{2m}\left[-\hbar^2\frac{\partial^2 g}{\partial x^2}-\left(\hbar\frac{\partial}{\partial y}+i\frac{eHx}{c}\right)^2 g\right]=\left(E-E_z\right)g$$

Schrödinger (sol. 5)

Proponemos otra separación de variables

$$g(x,y) = u(x)e^{ik_y y}$$

$$\frac{1}{2m}\left[-\hbar^2 e^{ik_y y}\frac{d^2u}{dx^2} - \left(\hbar\frac{\partial}{\partial y} + i\frac{eHx}{c}\right)^2 e^{ik_y y}u\right] = \left(E - E_z\right)e^{ik_y y}u$$

$$\frac{1}{2m}\left[-\hbar^2 e^{ik_y y}\frac{d^2u}{dx^2} - \left(\hbar^2\frac{\partial^2}{\partial y^2} + i\frac{2e\hbar Hx}{c}\frac{\partial}{\partial y} - \left(\frac{eHx}{c}\right)^2\right)e^{ik_y y}u\right]$$

$$= \left(E - E_z\right)e^{ik_y y}u$$

Schrödinger (sol. 6)

$$\frac{1}{2m}\left[-\hbar^2 e^{ik_y y}\frac{d^2u}{dx^2}-\left(-\hbar^2k_y^2e^{ik_y y}u-\frac{2eHxk_y}{c}e^{ik_y y}u-\left(\frac{eHx}{c}\right)^2e^{ik_y y}u\right)\right]$$

$$=\left(E-E_z\right)e^{ik_y y}u$$

$$\frac{1}{2m}\left[-\hbar^2\frac{d^2u}{dx^2}-\left(-\hbar^2k_y^2-\frac{2e\hbar Hxk_y}{c}-\left(\frac{eHx}{c}\right)^2\right)u\right]=\left(E-E_z\right)u$$

$$\frac{1}{2m}\left[-\hbar^2\frac{d^2u}{dx^2}+\left(\hbar^2k_y^2+\frac{2e\hbar Hxk_y}{c}+\left(\frac{eHx}{c}\right)^2\right)u\right]=\left(E-E_z\right)u$$

Schrödinger (sol. 7)

$$\frac{1}{2m}\left[-\hbar^2\frac{d^2u}{dx^2}+\left(\hbar^2k_y^2+\frac{2e\hbar Hxk_y}{c}+\left(\frac{eHx}{c}\right)^2\right)u\right]=\left(E-E_z\right)u$$

$$\frac{1}{2m}\left[-\hbar^2\frac{d^2u}{dx^2}+\left(\hbar k_y+\frac{eHx}{c}\right)^2u\right]=\left(E-E_z\right)u$$

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2}+\frac{m}{2}\left(\frac{\hbar k_y}{m}+\frac{eHx}{cm}\right)^2u=\left(E-E_z\right)u$$

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2}+\frac{m}{2}\left(\frac{\hbar k_y}{m}+\omega_c x\right)^2u=\left(E-E_z\right)u$$

Schrödinger (sol. 8)

Define $x_0 = -\frac{\hbar k_y}{m\omega_c} = -\frac{\hbar c}{eH} k_y = -\ell_H^2 k_y$

Then $$-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2} + \frac{m}{2}\omega_c^2\left(x - x_0\right)^2 u = \left(E - E_z\right)u$$

Let $$x' = x - x_0$$

Then

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx'^2} + \frac{m}{2}\omega_c^2 x'^2 u = \left(E - E_z\right)u$$

Harmonic oscillator

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx'^2}+\frac{m}{2}\omega_c^2 x'^2 u=\left(E-E_z\right)u$$

This means

$$E-E_z=\left(n-\tfrac{1}{2}\right)\hbar\omega_c \qquad n=1,2,\ldots.$$

These are the **Landau levels**

Densidad de estados

- Algunas propiedades dependen de los autovalores a través de la energía:

$$\sum_{\alpha} f(\alpha) = \sum_{\alpha} f\left[\varepsilon(\alpha)\right]$$

- Entonces

$$\sum_{\alpha} f(\alpha) = \sum_{\alpha} f\left[\varepsilon(\alpha)\right] = \int d\varepsilon \sum_{\alpha} f(\varepsilon)\,\delta\left(\varepsilon - \varepsilon(\alpha)\right)$$

$$= \int d\varepsilon f(\varepsilon) \sum_{\alpha} \delta\left(\varepsilon - \varepsilon(\alpha)\right) = \int d\varepsilon f(\varepsilon)\, g(\varepsilon)$$

$$g(\varepsilon) = \sum_{\alpha} \delta\left(\varepsilon - \varepsilon(\alpha)\right)$$

Ejemplo: partícula libre

- Partícula libre $\psi\left(\boldsymbol{r}\right) = \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}}}{\sqrt{V}}$ $\quad V = L_x L_y L_z$

$$\psi\left(x + L_x, y, z\right) = \psi\left(x, y, z\right)$$

- Born von-Karman

$$\psi\left(x, y + L_y, z\right) = \psi\left(x, y, z\right)$$

$$\psi\left(x, y, z + L_z\right) = \psi\left(x, y, z\right)$$

$$\Delta k_x \Delta k_y \Delta k_z = \frac{\left(2\pi\right)^3}{V} \qquad \sum_{\boldsymbol{k}} = \frac{V}{\left(2\pi\right)^3} \sum_k \Delta\boldsymbol{k} \simeq \frac{V}{\left(2\pi\right)^3} \int d\boldsymbol{k}$$

$$g\left(\varepsilon\right) = \frac{V}{\left(2\pi\right)^3} \int d\boldsymbol{k}\,\delta\left(\varepsilon - \varepsilon\left(\boldsymbol{k}\right)\right)$$

Densidad de estados partícula libre

$$\varepsilon\left(\boldsymbol{k}\right)=\frac{\hbar^2k^2}{2m}$$

3D

$$d\boldsymbol{k}=4\pi k^2dk$$

2D

$$d\boldsymbol{k}=2\pi kdk$$

1D

$$d\boldsymbol{k}=dk$$

$$g_{3D}\left(\varepsilon\right)=\frac{V}{\left(2\pi\right)^3}\int dk\left(4\pi k^2\right)\delta\left(\varepsilon-\frac{\hbar^2k^2}{2m}\right)=\frac{1}{\sqrt{2}\pi^2}\left(\frac{m}{\hbar^2}\right)^{3/2}\varepsilon^{1/2}$$

$$g_{2D}\left(\varepsilon\right)=\frac{A}{\left(2\pi\right)^2}\int dk\left(2\pi k\right)\delta\left(\varepsilon-\frac{\hbar^2k^2}{2m}\right)=\frac{A}{\left(2\pi\right)^2}\left(\frac{m}{\hbar^2}\right)$$

$$g_{1D}\left(\varepsilon\right)=\frac{L}{2\pi}\int dk\delta\left(\varepsilon-\frac{\hbar^2k^2}{2m}\right)=\frac{L}{2\pi}\left(\frac{m}{2\hbar^2}\right)^{1/2}\varepsilon^{-1/2}$$

Density of states Landau levels

$$E_{n,k_y,k_z} = \left(n - \tfrac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m}$$

- We can define a density of states per Landau level as $g\left(\varepsilon\right) = \sum_n g_n\left(\varepsilon\right)$ $\quad g_n\left(\varepsilon\right) = \sum_{k_y} g_{n,k_y}\left(\varepsilon\right)$

$$g_{n,k_y}\left(\varepsilon\right) = \sum_{k_z} \delta\left(\varepsilon - \varepsilon_{n,k_y} - \frac{\hbar^2 k_z^2}{2m}\right) = \frac{L_z}{2\pi}\left(\frac{m}{2\hbar^2}\right)^{1/2} \frac{1}{\sqrt{\varepsilon - \varepsilon_{n,k_y}}}$$

$$= \frac{L_z}{2\pi}\left(\frac{m}{2\hbar^2}\right)^{1/2} \frac{1}{\sqrt{\varepsilon - \varepsilon_n}}$$

Sumando sobre k_y

$$\sum_{k_y} g_{n,k_y}(\varepsilon) = N(k_y)\frac{L_z}{2\pi}\left(\frac{m}{2\hbar^2}\right)^{1/2}\frac{1}{\sqrt{\varepsilon-\varepsilon_n}}$$

$$x_0 = \frac{\hbar k_y}{m\omega_c} = \frac{\hbar c}{eH}k_y = \ell_H^2 k_y \Rightarrow \Delta x_0 = \ell_H^2 \Delta k_y$$

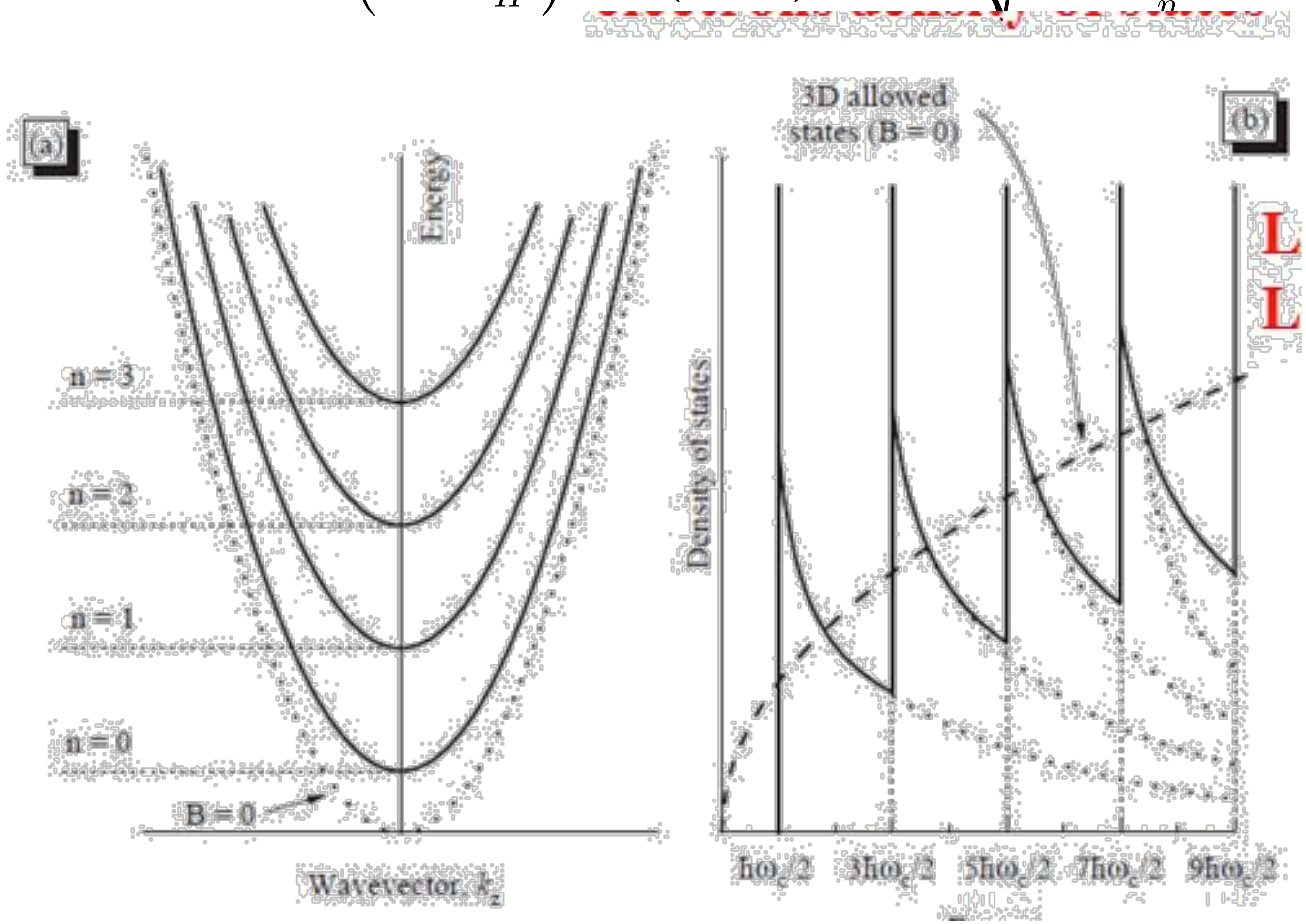
$$N(k_y) = \frac{L_x}{\Delta x_0} = \frac{L_x}{\ell_H^2 \Delta k_y}$$

- Apply periodic boundary conditions along *y*:

$$k_y = \frac{2\pi m}{L_y} \quad \Rightarrow \Delta k_y = \frac{2\pi}{L_y} \Rightarrow N(k_y) = \frac{L_x}{\ell_H^2 \Delta k_y} = \frac{L_x L_y}{2\pi \ell_H^2}$$

Densidad total

$$g\left(\varepsilon\right)=\left(\frac{L_xL_y}{2\pi\ell_H^2}\right)\frac{L_z}{2\pi}\left(\frac{m}{2\hbar^2}\right)^{1/2}\sum_n\frac{1}{\sqrt{\varepsilon-\varepsilon_n}}$$



Electrones en 2D

- Si confinamos los electrones en 2D, tenemos para el estado fundamental

$$g(\varepsilon) = \left(\frac{L_x L_y}{2\pi \ell_H^2}\right) \sum_n \delta(\varepsilon - \varepsilon_n)$$

Nivel de degeneración

Filling factor

- Si tenemos *N* electrones, podemos definir el filling factor como

$$\nu = \frac{N}{A \big/ 2\pi\ell_H^2} = 2\pi\ell_H^2 n = 2\pi\frac{\hbar c}{eH} n = \left(\frac{hc}{e}\right)\frac{nA}{HA} = \frac{\phi_0 N}{\phi}$$

- O sea que los plateaus ocurren cuando llenamos exactamente un nivel de Landau…

Agregando un campo eléctrico

$$H = \frac{1}{2m}\left[-\hbar^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) - \left(\hbar\frac{\partial}{\partial y} + i\frac{eHx}{c}\right)^2\right] + eEx$$

Proponemos $\psi(x,y,z) = f(z)g(x,y)$ Thus

$$\frac{1}{2m}\left[-\hbar^2\left(f\frac{\partial^2 g}{\partial x^2} + g\frac{d^2 f}{dz^2}\right) - f\left(\hbar\frac{\partial}{\partial y} + i\frac{eHx}{c}\right)^2 g\right] + eExfg$$

$$= Efg$$

Solución para *u*

La ecuación final para *u* es

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2}+\frac{m}{2}\left(\frac{\hbar k_y}{m}+v_d+\omega_c x\right)^2 u=$$

$$\left(E-E_z+\hbar k_y v_d+\frac{1}{2}mv_d^2\right)u$$

Corriente

- Since

$$\boldsymbol{v} = \boldsymbol{p} + \frac{e}{c}\boldsymbol{A} \qquad \boldsymbol{A} = \left(0, Hx, 0\right)$$

$$\boldsymbol{j} = -ne\left\langle \boldsymbol{v} \right\rangle = -ne\frac{\sum \boldsymbol{v}}{nA} = -e\frac{\sum \boldsymbol{v}}{A}$$

$$= -\frac{e}{mA}\sum_{n=1}^{\nu}\sum_{k}\left\langle \psi_{nk} \right| -i\hbar\boldsymbol{\nabla} + e\boldsymbol{A}\left| \psi_{nk} \right\rangle$$

$$j_x = -\frac{e}{mA}\sum_{n=1}^{\infty}\sum_{k}\left\langle \psi_{nk} \right| -i\hbar\frac{\partial}{\partial x}\left| \psi_{nk} \right\rangle = 0$$

$$j_y = -\frac{e}{mA}\sum_{n=1}^{\infty}\sum_{k}\left\langle \psi_{nk} \right| -i\hbar\frac{\partial}{\partial y} + \frac{e}{c}Hx\left| \psi_{nk} \right\rangle$$

Corriente en la dirección *y*

$$j_y = -\frac{e}{mA}\sum_{n=1}^{\nu}\sum_{k}\left\langle \psi_{nk}\middle| -i\hbar\frac{\partial}{\partial y} + \frac{e}{c}Hx \middle| \psi_{nk}\right\rangle$$

$$j_y = -\frac{e}{mA}\sum_{n=1}^{\nu}\sum_{k}\left(\hbar k + \frac{eH}{c}\langle x\rangle_{nk}\right)$$

But $$\langle x\rangle_{nk} = \frac{-\hbar k_y}{m\omega_c} - \frac{v_d}{\omega_c}$$

$$j_y = -\frac{e}{mA}\sum_{n=1}^{\nu}\sum_{k}\left(\hbar k + \frac{eH}{c}\left(\frac{-\hbar k}{m\omega_c} - \frac{v_d}{\omega_c}\right)\right)$$

$$= -\frac{e}{mA}\sum_{n=1}^{\nu}\sum_{k}\left(\hbar k + -\hbar k + mv_d\right) = -\frac{ev_d}{mA}\sum_{n=1}^{\nu}\sum_{k}1 = -\frac{e\nu v_d}{mA}N$$

$$= -e\nu v_d\frac{\phi}{A\phi_0} = -e\nu\frac{cE}{H}\frac{AH}{Ahc/e} = -\nu\frac{e^2}{h}E$$

Resistivity

We just calculated
$$j_x = 0$$
$$j_y = -\nu e^2 A/\hbar$$

with
$$E_x = E$$
$$E_y = 0$$

But
$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{xx} \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$$

$$\Rightarrow \begin{cases} \rho_{xx} = 0 \\ \rho_{xy} = -\dfrac{\hbar}{\nu e^2} \end{cases}$$

Método alternativo

$$\boldsymbol{v} = \frac{1}{m}\left(\boldsymbol{p} + \frac{e}{c}\boldsymbol{A}\right) \qquad H = \frac{1}{2m}\left[\boldsymbol{p} + \frac{e}{c}\boldsymbol{A}\left(\boldsymbol{r}\right)\right]^2$$

$$\boldsymbol{v} = \frac{c}{e}\frac{\partial H}{\partial \boldsymbol{A}}$$

Podemos entonces agregar un potencial vector ficticio:

$$H = \frac{1}{2m}\left[\boldsymbol{p} + \frac{e}{c}\left(\boldsymbol{A}\left(\boldsymbol{r}\right) - \boldsymbol{a}\left(\boldsymbol{r}\right)\right)\right]^2$$

$$\boldsymbol{a}\left(\boldsymbol{r}\right) = q\frac{\phi_0}{L_y}\hat{\boldsymbol{j}} \qquad \nabla \times \boldsymbol{a}\left(\boldsymbol{r}\right) = 0$$

Transformación de gauge

- Hagamos

$$\boldsymbol{A} = (0, Hx, 0) \rightarrow (0, Hx - q\phi_0 / L_y, 0)$$

$$\frac{\partial H}{\partial q} = \frac{\partial}{\partial q}\left[\frac{1}{2m}\left(\boldsymbol{p} + \frac{e}{c}\boldsymbol{A} - \frac{e}{c}\frac{q\phi_0}{L_y}\hat{\boldsymbol{j}}\right)^2\right]$$

$$= \frac{1}{m}\left(\boldsymbol{p} + \frac{e}{c}\boldsymbol{A} - \frac{e}{c}\frac{q\phi_0}{L_y}\hat{\boldsymbol{j}}\right)\left(-\frac{e}{c}\frac{q\phi_0}{L_y}\hat{\boldsymbol{j}}\right)$$

$$= -\frac{e}{mc}\frac{q\phi_0}{L_y}\left(p_y + \frac{e}{c}A_y - \frac{e}{c}\frac{q\phi_0}{L_y}\right) = \frac{-e\phi_0}{L_y c}v_y$$

Corriente como derivada

$$j_y = \frac{-ev_y}{L_x L_y} = \frac{-e}{L_x L_y}\left(-\frac{L_y c}{e\phi_0}\right)\frac{\partial H}{\partial q} = \frac{c}{L_x \phi_0}\frac{\partial H}{\partial q} = \frac{e}{L_x h}\frac{\partial H}{\partial q}$$

Efecto del parámetro *q*

Then the equation for *u* becomes

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2}+\frac{m}{2}\left(\frac{\hbar\left(k_y-2\pi q/L_y\right)}{m}+\omega_c x\right)^2 u=\left(E-E_z\right)u$$

$$x_0=-\frac{\hbar k_y}{m\omega_c}+\frac{2\pi\hbar}{L_y m\omega_c}q$$

Notice that for $q = 1$ the solutions are identical.

Flujo extra en el caso *E*≠0

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2}+\frac{m}{2}\left(\frac{\hbar\left(k_y-2\pi q/L_y\right)}{m}+v_d+\omega_c x\right)^2 u=$$

$$\left(E-E_z+\hbar\left(k_y-2\pi q/L_y\right)v_d+\frac{1}{2}mv_d^2\right)u$$

Notice that for *q* = 1 the solutions are identical but energy changes. So this must be motion of charge to the right. Therefore

$$\hbar\left(\frac{2\pi}{L_y}v_d\right)=-\nu' eE_xL_x\Rightarrow E_x=\frac{1}{\nu' eL_x}\hbar\left(\frac{2\pi}{L_y}v_d\right)=\frac{hv_d}{\nu' eL_xL_y}$$

Corriente

- But $$j_y = \frac{e}{L_x h}\frac{\partial H}{\partial q} = \frac{e}{L_x h}\left(\frac{-2\pi\hbar}{L_y}\right)v_d = -\frac{ev_d}{L_x L_y}$$

Therefore

$$\sigma_{yx} = \frac{j_y}{E_x} = \frac{-\frac{ev_d}{L_x L_y}}{\frac{hv_d}{\nu' e L_x L_y}} = -\frac{\nu' e^2}{h}$$