Resultados de BCS : Gap y Tc

$$(u_s - u_n)(T = 0) = -\frac{G(\varepsilon_F)}{2}\Delta^2(0)$$

$$\Delta(0) = 2\hbar\omega_D \, e^{-\frac{1}{VG(\varepsilon_F)}}.$$

$$\Delta(T) \rightarrow \Delta(T_c) = 0 \rightarrow \quad kT_c = 1.13 \; \hbar\omega_D \, e^{-\frac{1}{VG(\varepsilon_F)}}.$$

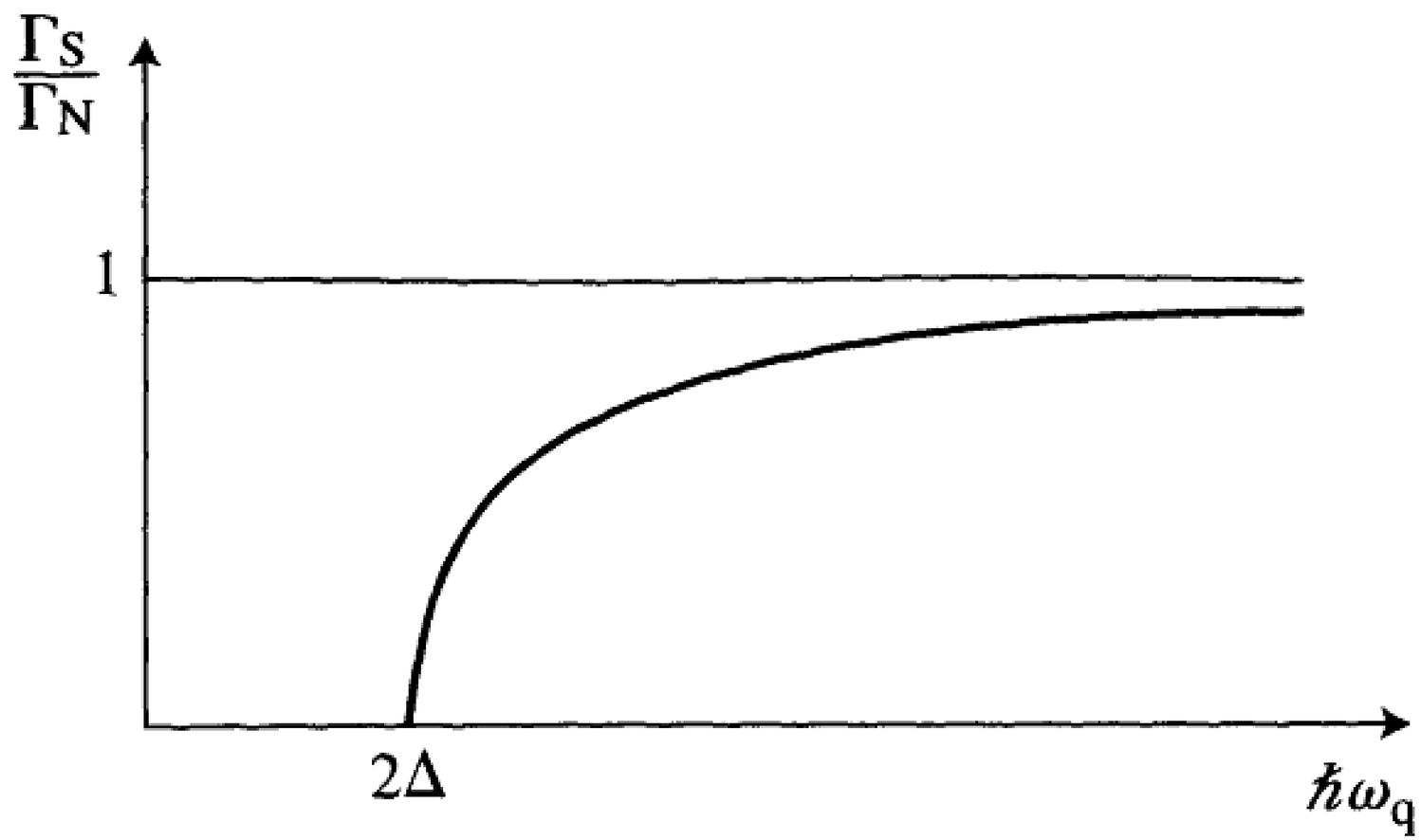
$$2\Delta(0) = 3.52 k_B T_c$$

Valores típicos:

$$\varepsilon_F \approx 5 \text{ eV} \quad ; \hbar\omega_D \approx 20 \text{ meV} \quad ; \quad G(\varepsilon_F)V \approx 0.25 \quad ; \quad \Delta(0) \approx 0.75 \text{ meV}$$

$$T_c \sim 5 \text{ K}$$

La absorción por debajo del gap es nula: infrarrojo lejano y microondas



Cociente entre la absorción en el estado superconductor y el normal

Resultados de BCS : Gap y Tc

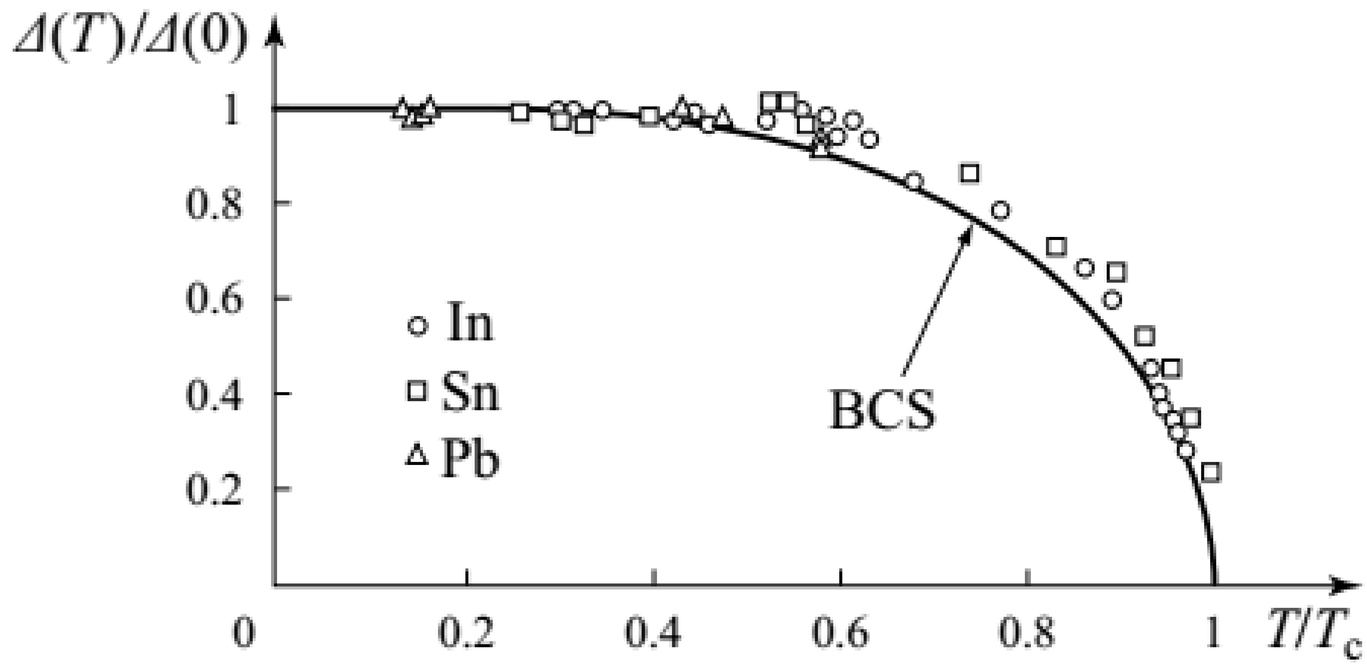
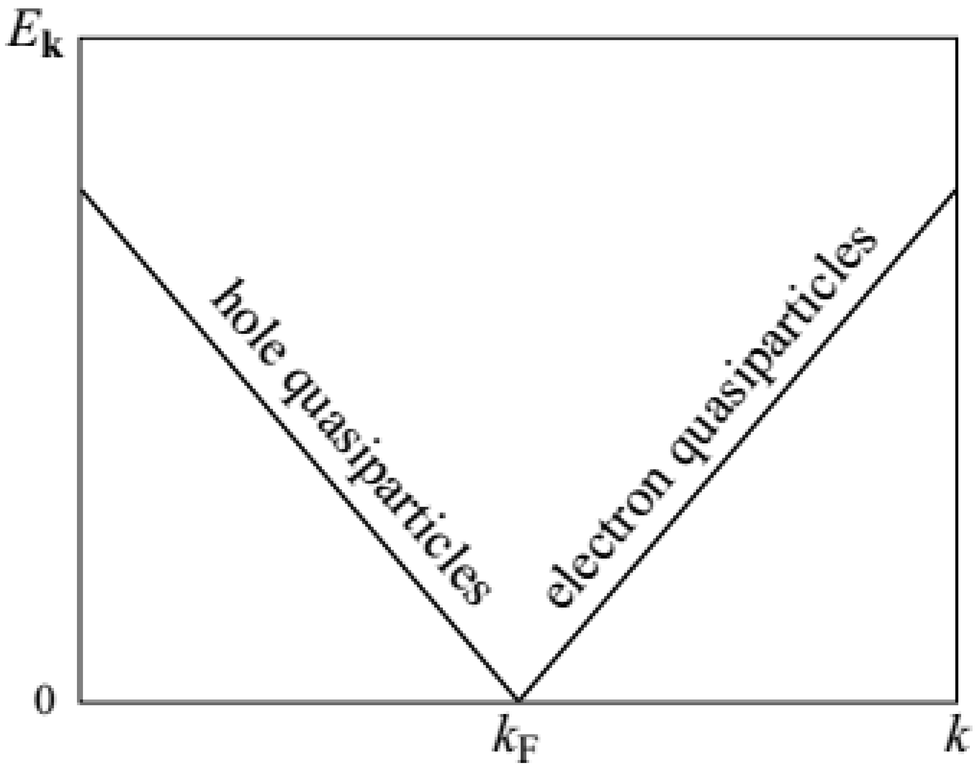


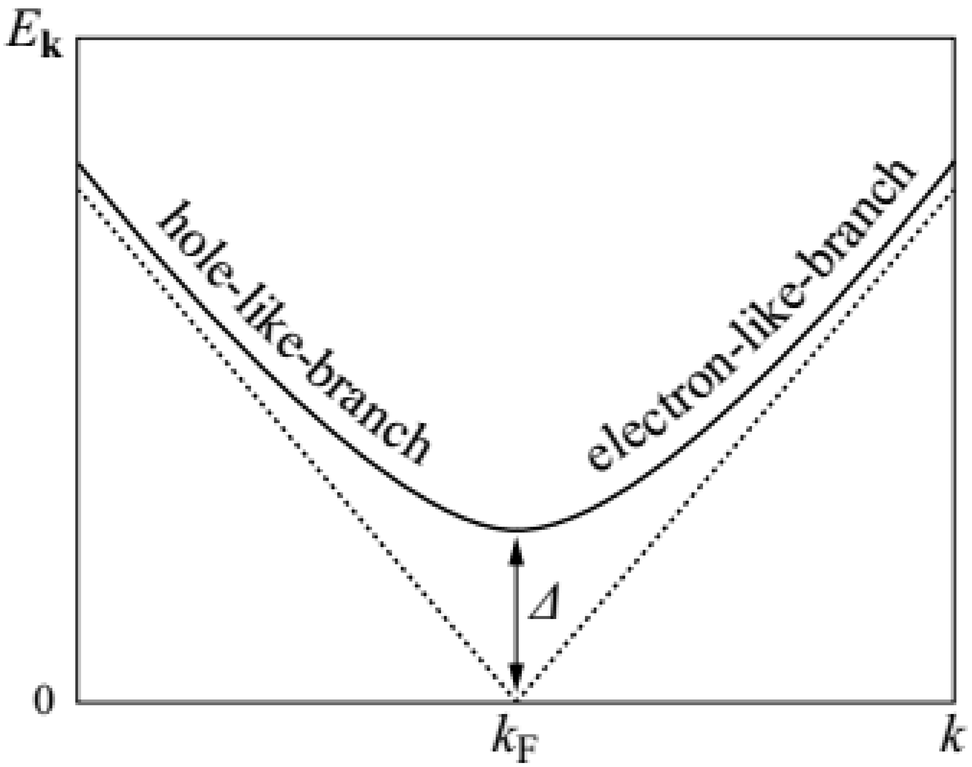
Figure 8.17 - Temperature dependence of the gap

Resultados de BCS : espectro de excitaciones



$$E_{\mathbf{k}}^{\mathrm{n}} = |\xi_{\mathbf{k}}| = \frac{\hbar^2}{2m}\left|k^2 - k_F^2\right| \approx \frac{\hbar^2 k_F}{m}|k - k_F|$$

Metal normal



$$E_{\mathbf{k}} = \sqrt{\Delta^2 + \xi_{\mathbf{k}}^2} = \sqrt{\Delta^2 + \left(\frac{\hbar^2 k^2}{2m} - \varepsilon_F\right)^2}.$$

Superconductor

Tamaño de un par de Cooper
Longitud de coherencia

$$E_{\mathbf{k}} = \sqrt{\Delta^2 + \xi_{\mathbf{k}}^2} = \sqrt{\Delta^2 + \left(\frac{\hbar^2 k^2}{2m} - \varepsilon_{\mathrm{F}}\right)^2}.$$

$k = k_F + \Delta k$

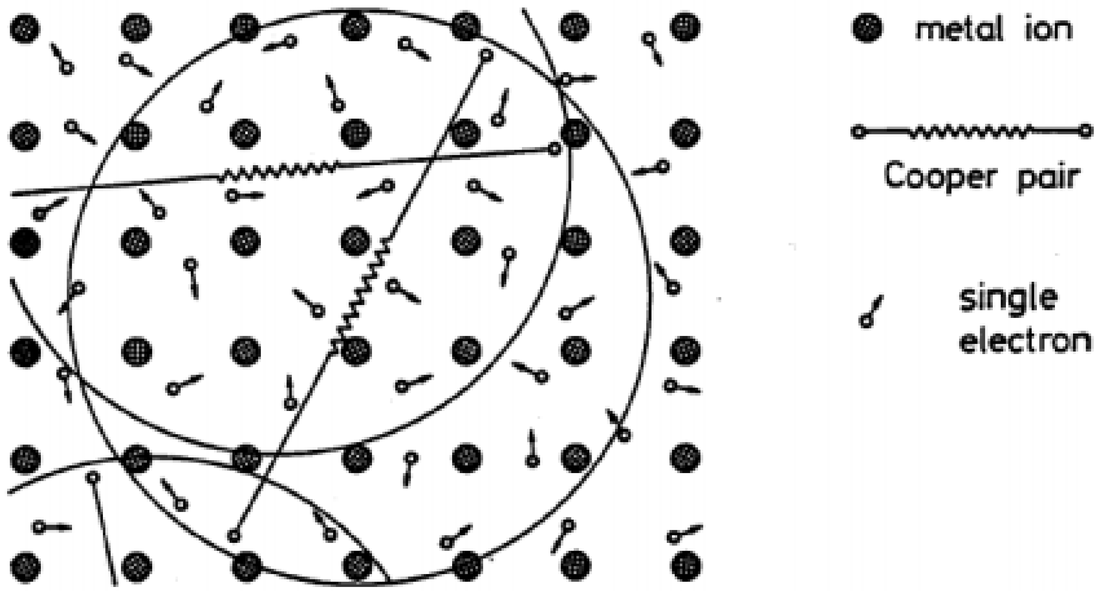
$v_F = \hbar\, k_F / m$

$\Delta E < \hbar\omega_{\mathrm{D}}$

$\Delta x\, \Delta k \sim 1$

$$\Delta x \sim \xi_{\mathrm{BCS}}(0) = \frac{\hbar v_{\mathrm{F}}}{\pi \Delta(0)}$$

$$\Delta x = \xi \approx \frac{\hbar v_F}{k T_c}$$

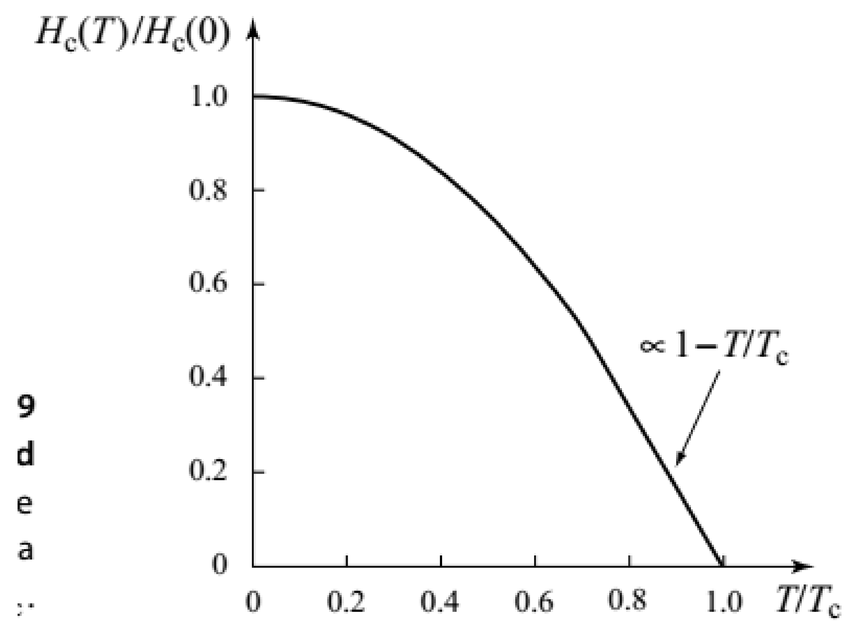


Campo crítico termodinámico

$$(u_s - u_n)(T=0) = -\frac{G(\varepsilon_F)}{2}\Delta^2(0)$$

$$G(\varepsilon_F)\frac{\Delta^2(0)}{2} = \mu_0 \frac{H_c^2(0)}{2}$$

$$H_c(0) = \sqrt{\frac{G(\varepsilon_F)}{\mu_0}}\Delta(0).$$



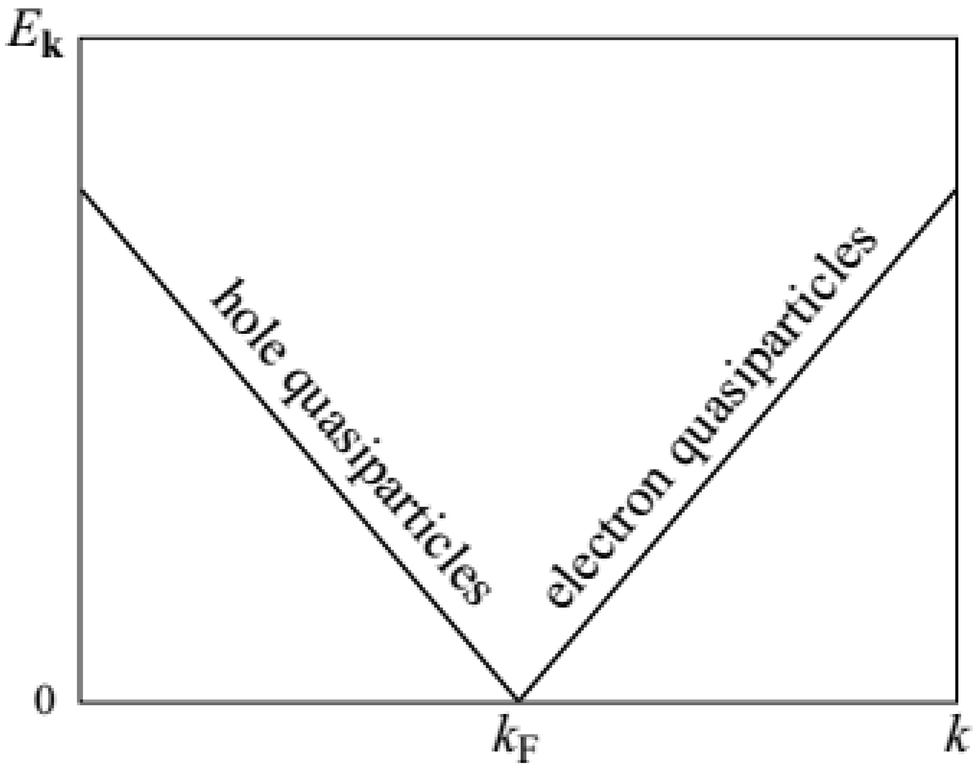
Longitud de penetración

$$G(\varepsilon_F) = \frac{3n}{4\varepsilon_F}.$$

En un gas de Fermi a T = 0

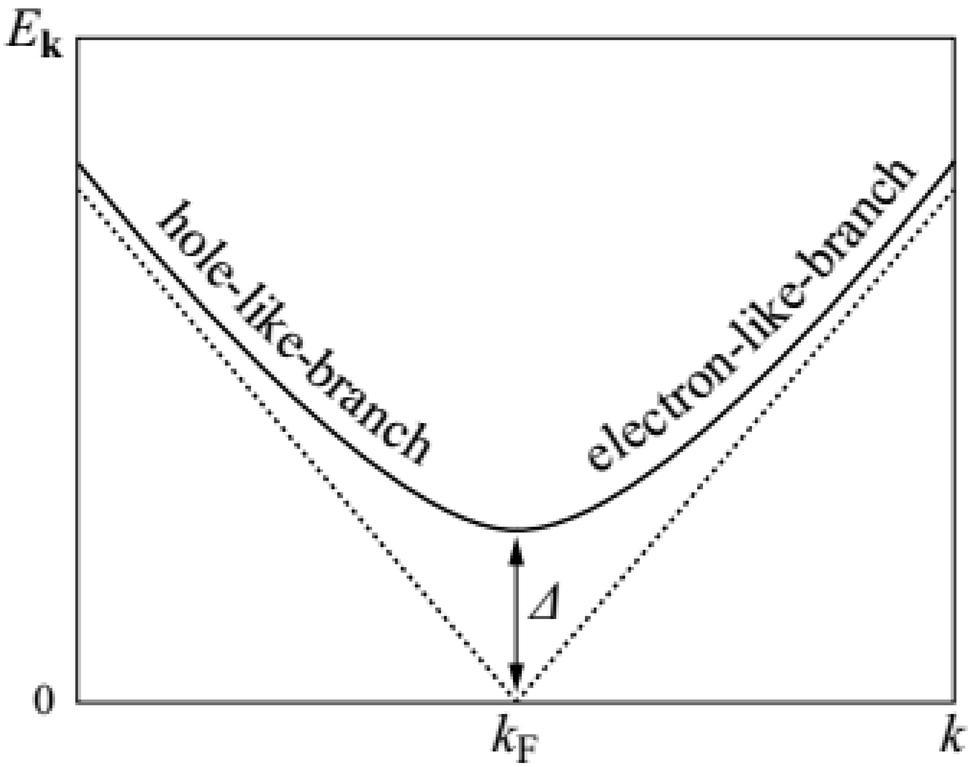
$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

Volvamos al espectro de excitaciones



$$E_{\mathbf{k}}^{\mathrm{n}} = |\xi_{\mathbf{k}}| = \frac{\hbar^2}{2m}\left|k^2 - k_F^2\right| \approx \frac{\hbar^2 k_F}{m}|k - k_F|$$

Metal normal



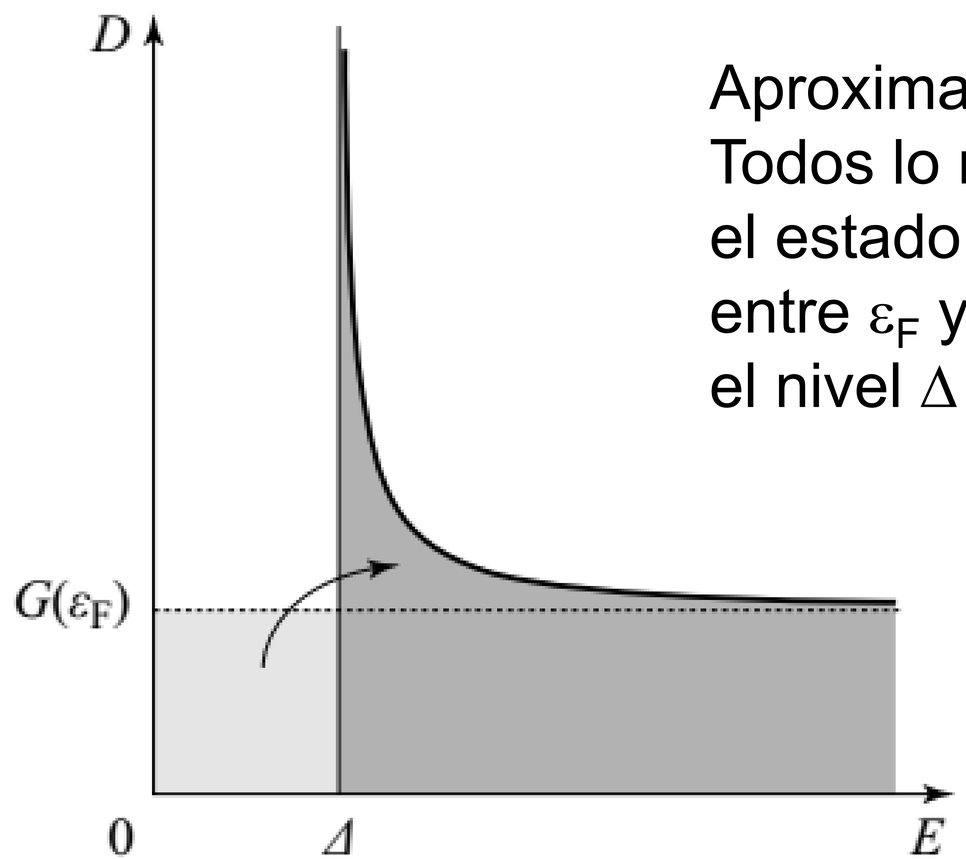
$$E_{\mathbf{k}} = \sqrt{\Delta^2 + \xi_{\mathbf{k}}^2} = \sqrt{\Delta^2 + \left(\frac{\hbar^2 k^2}{2m} - \varepsilon_F\right)^2}.$$

Superconductor

Resultados de BCS : densidad de estados de cuasipartículas

$$D(E)\,\mathrm{d}E = G(k)\,\mathrm{d}k$$

$$D(E) = G(\varepsilon_F)\frac{E}{\sqrt{E^2 - \Delta^2}}.$$



Aproximación a baja T:
Todos lo niveles que en el estado normal estaban entre ε_F y $\varepsilon_F + \Delta$ ocupan el nivel Δ

Calor específico

Aproximación a baja T: Todos lo niveles que en el estado normal estaban entre ε_F y $\varepsilon_F + \Delta$ ocupan el nivel Δ

Podemos pensar en un sistema de dos niveles: 0 y Δ

$$\delta U = \left[2G(\varepsilon_F)\Delta(0)\right]\Delta(0)\, e^{-\frac{\Delta(0)}{k_B T}}$$

$$C_s^{el} = \frac{d(\delta U)}{dT} \approx \frac{2G(\varepsilon_F)\left[\Delta(0)\right]^3}{k_B T^2}\, e^{-\frac{\Delta(0)}{k_B T}}$$

$$C_s^{el}(T) = \frac{2k_B}{(k_B T)^{3/2}} G(\varepsilon_F)\left[\Delta(0)\right]^{5/2} e^{-\frac{\Delta(0)}{k_B T}}.$$

$$C_s^{el} \sim A\, e^{-\frac{\Delta(0)}{k_B T}}$$

Corriente crítica de desapareo

Si, en presencia de una corriente J (en x) , se rompe un par de cooper, y los e adquieren un impulso final p_F + p :

$$E_{\text{final}}^{(c)} - E_{\text{initial}} = 2\Delta - 4\frac{\hbar^2 k_F}{2m}\langle k\rangle.$$

Para que convenga energéticamente:

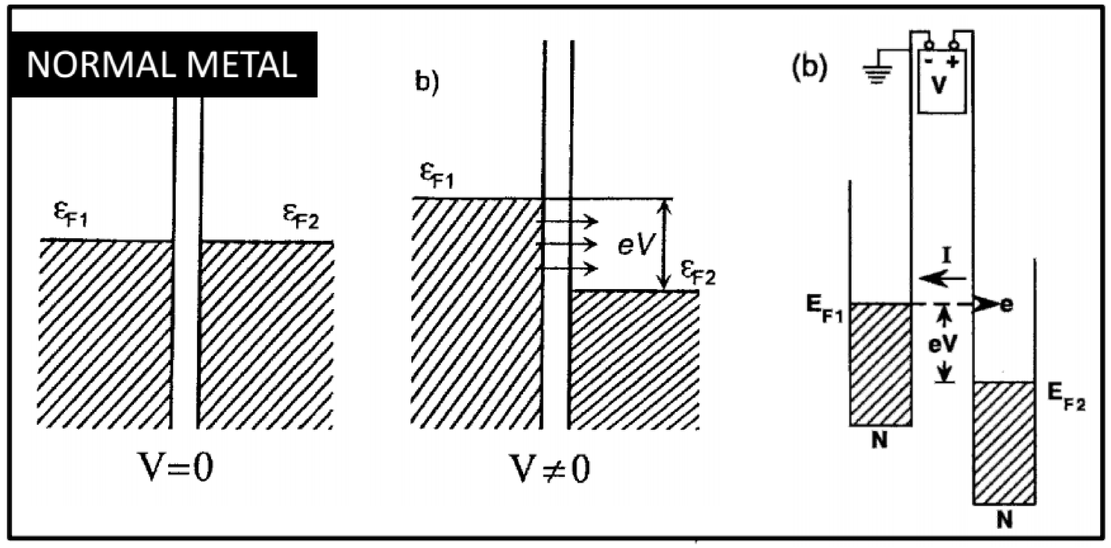
$$2\Delta - 4\frac{\hbar^2 k_F}{2m}\langle k_x\rangle < 0 \qquad \langle k_x\rangle > \frac{m\Delta}{\hbar^2 k_F} \qquad v_c = \frac{\hbar}{m}\frac{m\Delta}{\hbar^2 k_F}.$$

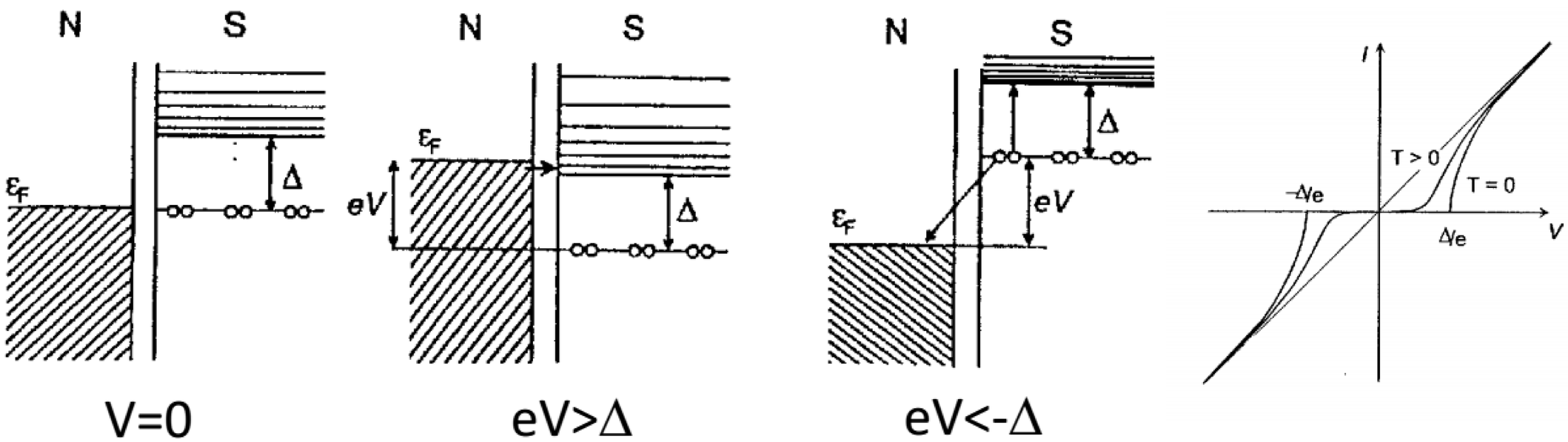
$$j_c = nev_c = \frac{ne\Delta}{\hbar k_F} \qquad j_c = \sqrt{2/3}\sqrt{\frac{\mu_0 ne^2}{m}} = \sqrt{2/3}\,\frac{H_c}{\lambda_L}.$$

Cómo se mide el gap?

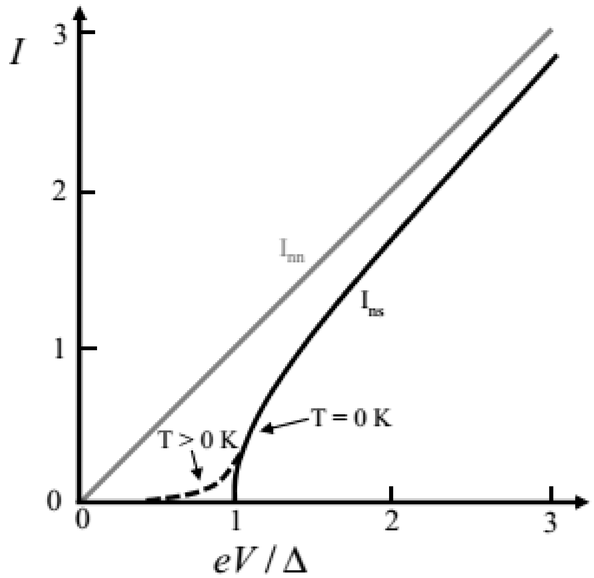
Cómo se mide la densidad de estados?

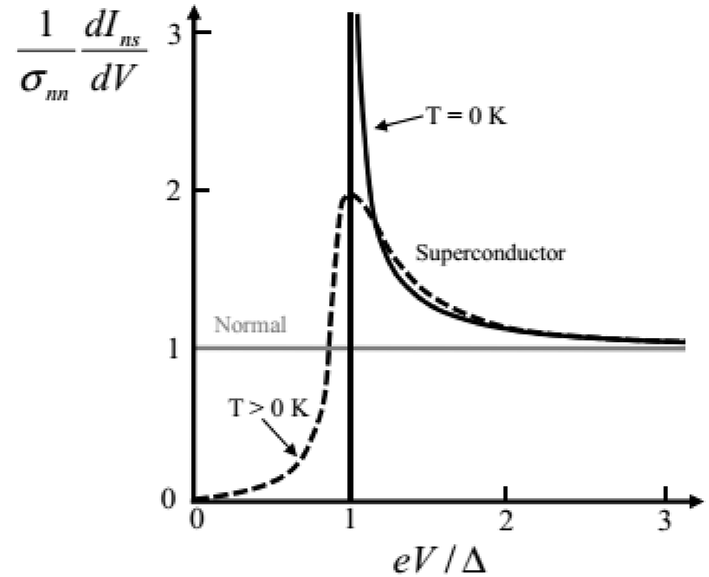
Efecto Tunel

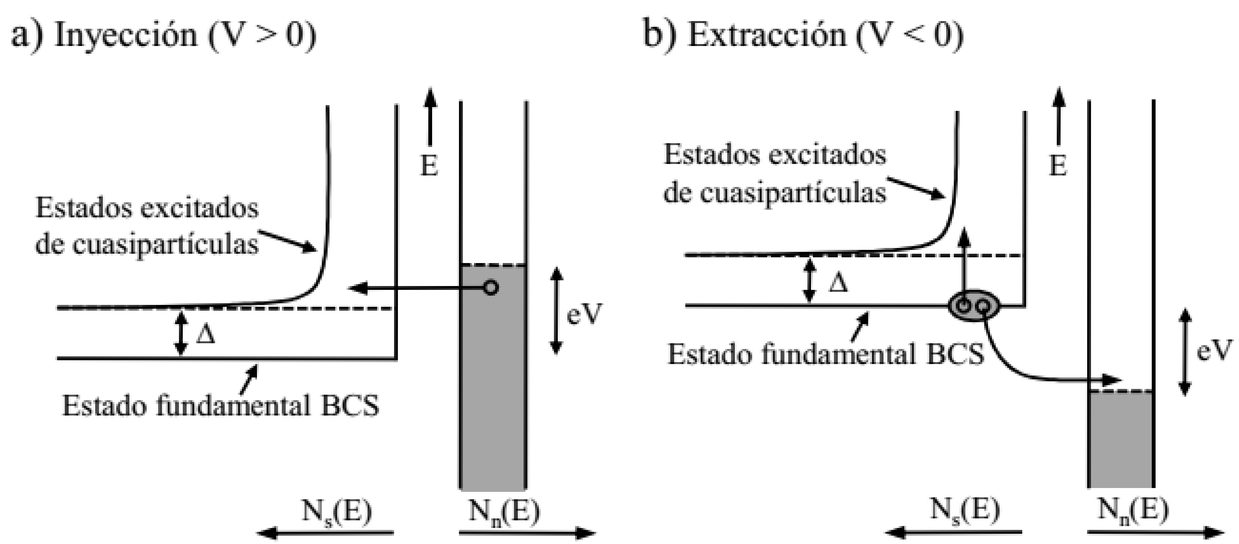




Tunel normal- superconductor







STM

Tunel normal- superconductor entre una punta normal y una muestra superconductora

$$I = \frac{4\pi e}{\hbar} \int_{-\infty}^{\infty} |M|^2 N_m(E - eV) N_p(E) [f(E - eV) - f(E)] dE$$

$$M = \frac{\hbar^2}{2m} \int_{\Sigma} (\Psi_\mu^* \nabla \Psi_\nu - \Psi_\nu \nabla \Psi_\mu^*) d\mathbf{S} \qquad \begin{cases} \Psi_\mu \propto e^{-\kappa z} \\ \Psi_\nu \propto e^{-\kappa(d-z)} \end{cases} \qquad \kappa = \sqrt{\frac{2m\phi}{\hbar^2}} \approx 0.5\sqrt{\phi}\ \text{Å}^{-1}, \phi \text{ en eV}$$

$$I \propto e^{-2\kappa d}$$

STM

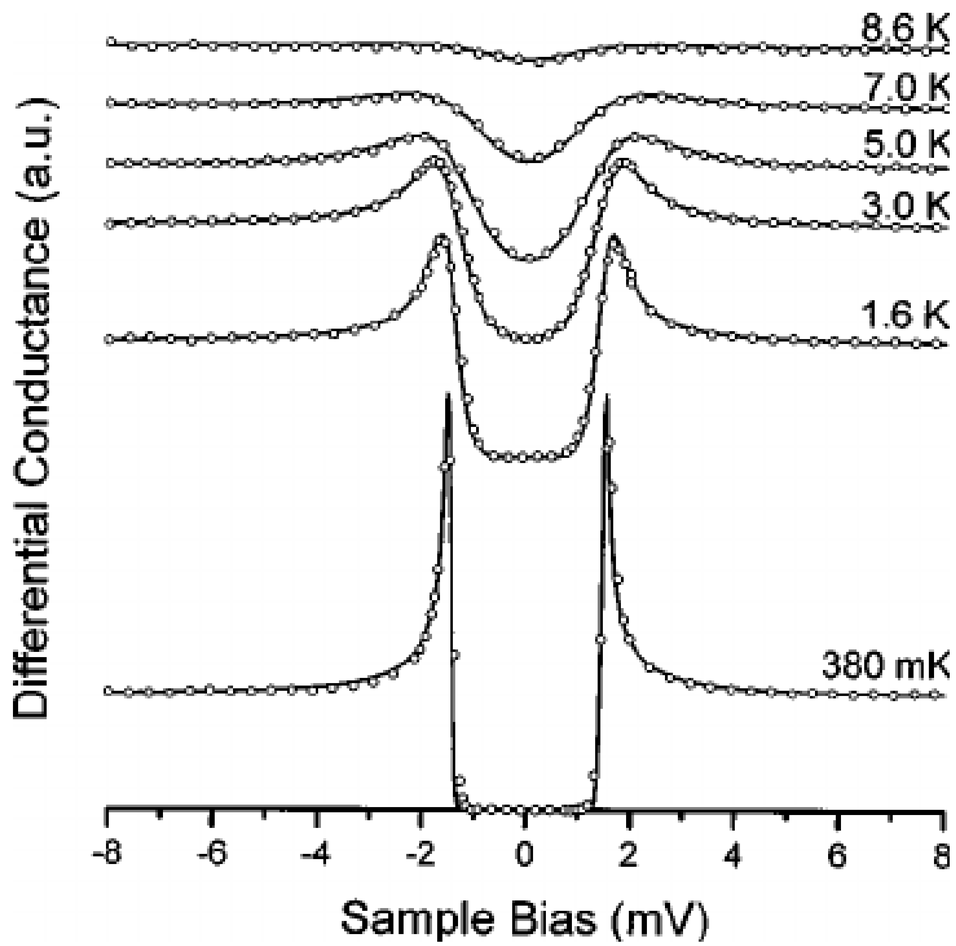
Tunel normal- superconductor entre una punta normal y una muestra superconductora

$$I(V) \propto \int_{-\infty}^{\infty} N_m(E - eV) N_p(E) [f(E - eV) - f(E)] dE$$

$$I(V) \propto \int_{-\infty}^{\infty} N_m(E - eV) [f(E - eV) - f(E)] dE$$

$$\sigma(V) = \frac{dI(V)}{dV} \propto \int_{-\infty}^{\infty} N_S(E) \frac{\partial f(E - eV)}{\partial V} dE$$

Experimentos de espectroscopía por efecto tunel



Measured differential conductance spectra ˋcirclesˆ for tunneling between a Nb tip and a Au surface at several temperatures. The spectra are displaced vertically from each other for clarity. The solid lines are the calculated conductance using a BCS DOS with ⌬ tip (0) ϭ 1.47 meV and the temperatures shown.
J. C. Davis group (1997)

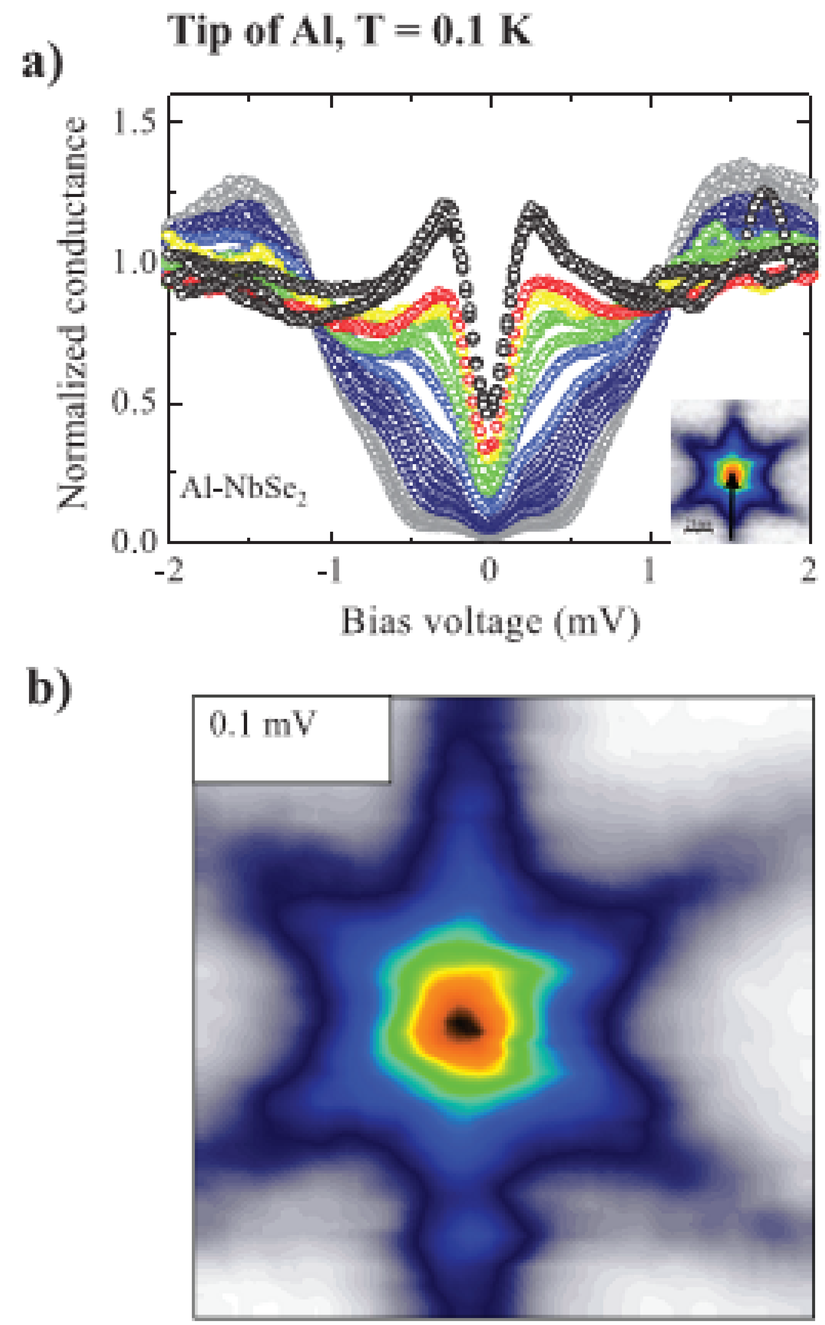
Suponiendo que la densidad de estado de la punta de prueba no depende de V. La conductancia es:

$$G(V) \propto \int_0^{eV} \mathrm{d}E \frac{\mathrm{d}f}{\mathrm{d}V} N_S(E).$$

Gap en superconductores de tipo II imágenes de vórtices

Vortices en $NbSe_2$: no siempre son redondos!!

Grupo e E. Suderow (2009)



Gap en superconductores de tipo II: imágenes de vórtices

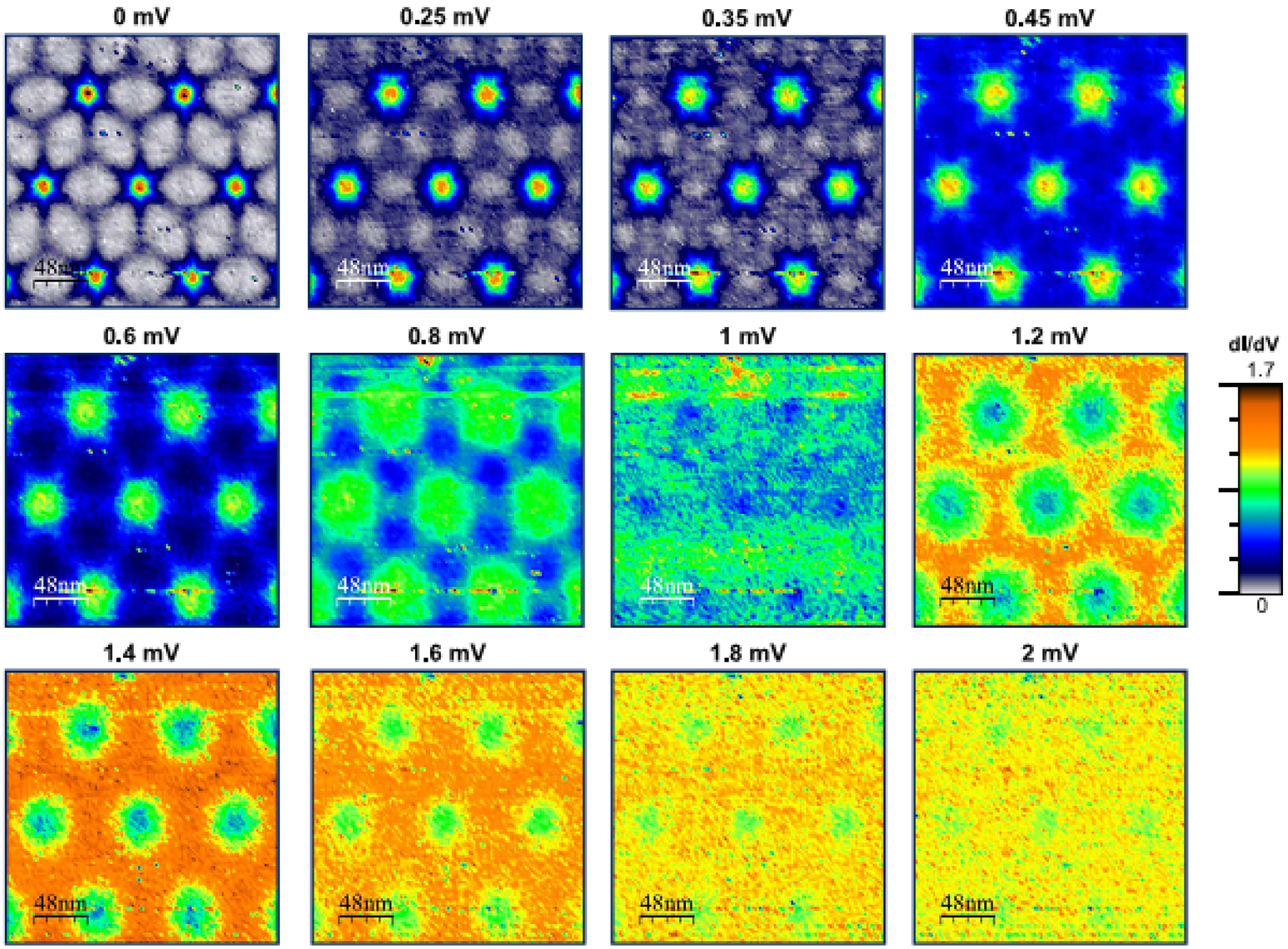


Figure 11. Vortex core structure in 2H-$NbSe_2$ as a function of the bias voltage. Data are from [181]. A normal tip of Au is used, and the applied magnetic field is 0.15 T.