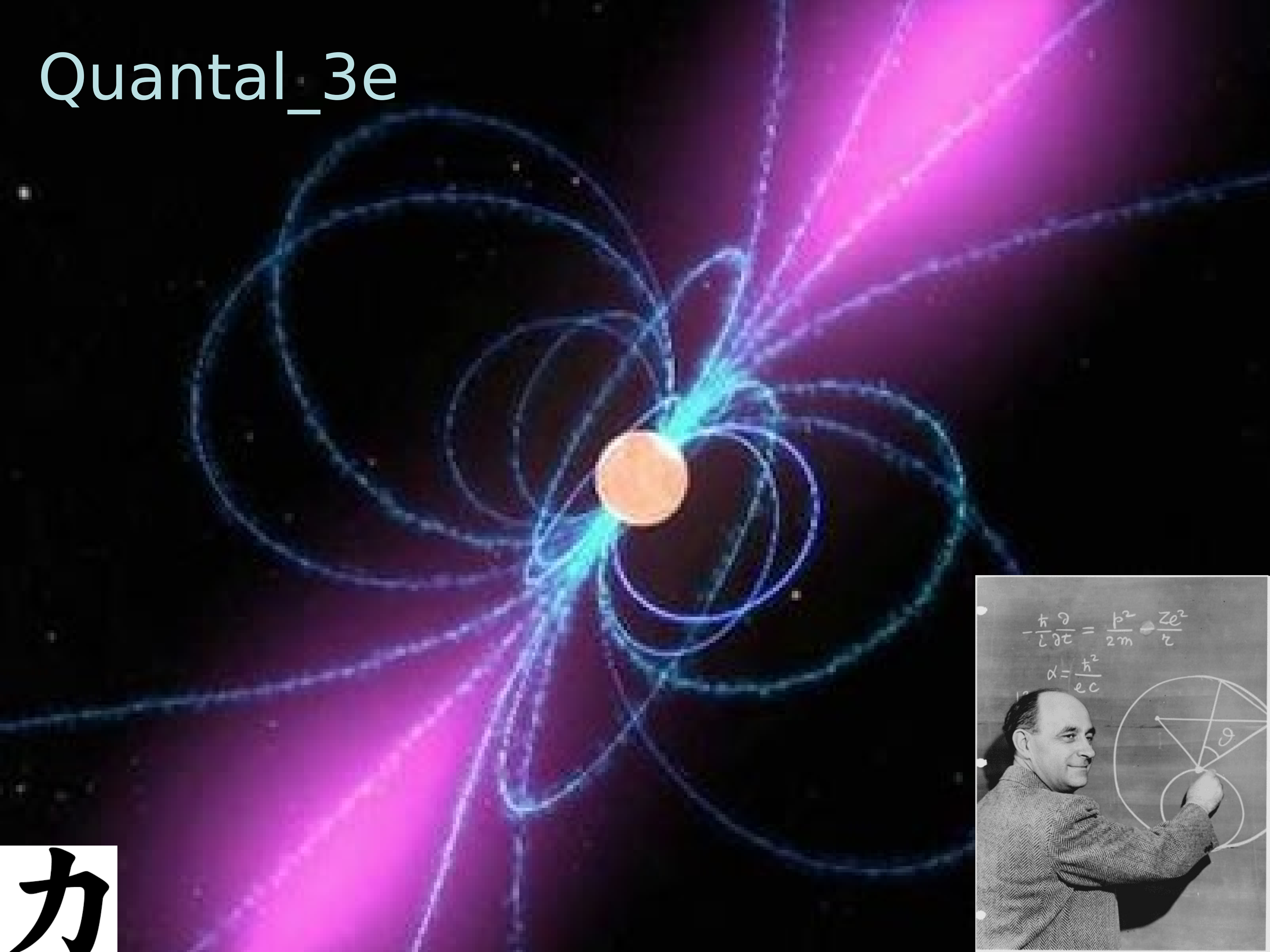
Quantal_3e



$-\frac{\hbar}{i}\frac{\partial}{\partial t} = \frac{p^2}{2m} - \frac{Ze^2}{r}$
$\alpha = \frac{\hbar^2}{ec}$
ϑ

力

Relativistic energy & momentum

Relacion de Einstein para la energia

$$E = mc^2$$

Que incluye a la energia cinetica y la energia asociada a la mas en reposo

La masa relativista es

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0$$

momentum

$$p = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}v = \gamma m_0 v$$

$$p^2c^2 = \frac{m_0^2 v'^2 c^2}{1-\frac{v'^2}{c^2}} = \frac{m_0^2 \frac{v'^2}{c^2} c^4}{1-\frac{v'^2}{c^2}} + \frac{m_0^2 c^4}{1-\frac{v'^2}{c^2}} - \frac{m_0^2 c^4}{1-\frac{v'^2}{c^2}} =$$

$$p^2c^2 = \frac{m_0^2(\frac{v'^2}{c^2}-1)c^4}{1-\frac{v'^2}{c^2}} + \frac{m_0^2 c^4}{1-\frac{v'^2}{c^2}} =$$

Luego

$$p^2c^2 = -m_0^2c^4 + m^2c^4$$

$$p^2c^2 = \underbrace{(mc^2)^2}_{E^2} - m_0^2c^4$$

donde m es la masa relativista y m_0 es la masa en reposo

$$E = \sqrt{p^2c^2 + m_0{}^2c^4}$$

La energia cinetica relativista es

$$E_K = m c^2 - m_0 c^2$$

$$E_K = (\gamma - 1) m_0 c^2$$

$$E_K = m_0 c^4 \left(\frac{1}{\sqrt{1 - \frac{v'^2}{c^2}}} -1 \right)$$

desarrollando

$$E_K = m_0 c^2 \left(\frac{1}{2} \frac{v'^2}{c^2} + \frac{3}{8} \frac{v'^4}{c^4} + \ldots \right) =$$

$$E_K = m_0 \frac{v'^2}{2} + \frac{3}{8} m_0 \frac{v'^4}{c^2} \ldots.$$

Si v' << c…

fotón

Para el foton la expresion $p = \frac{m_0 v'}{\sqrt{1-\frac{v'^2}{c^2}}} = \gamma m_0 v'$ tiene problemas,

entonces usamos

$$E = \sqrt{p^2 c^2 + m_0^2 c^4}$$

Que con $m_0 = 0 \Rightarrow$

fotón

$$E = pc \Rightarrow$$

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

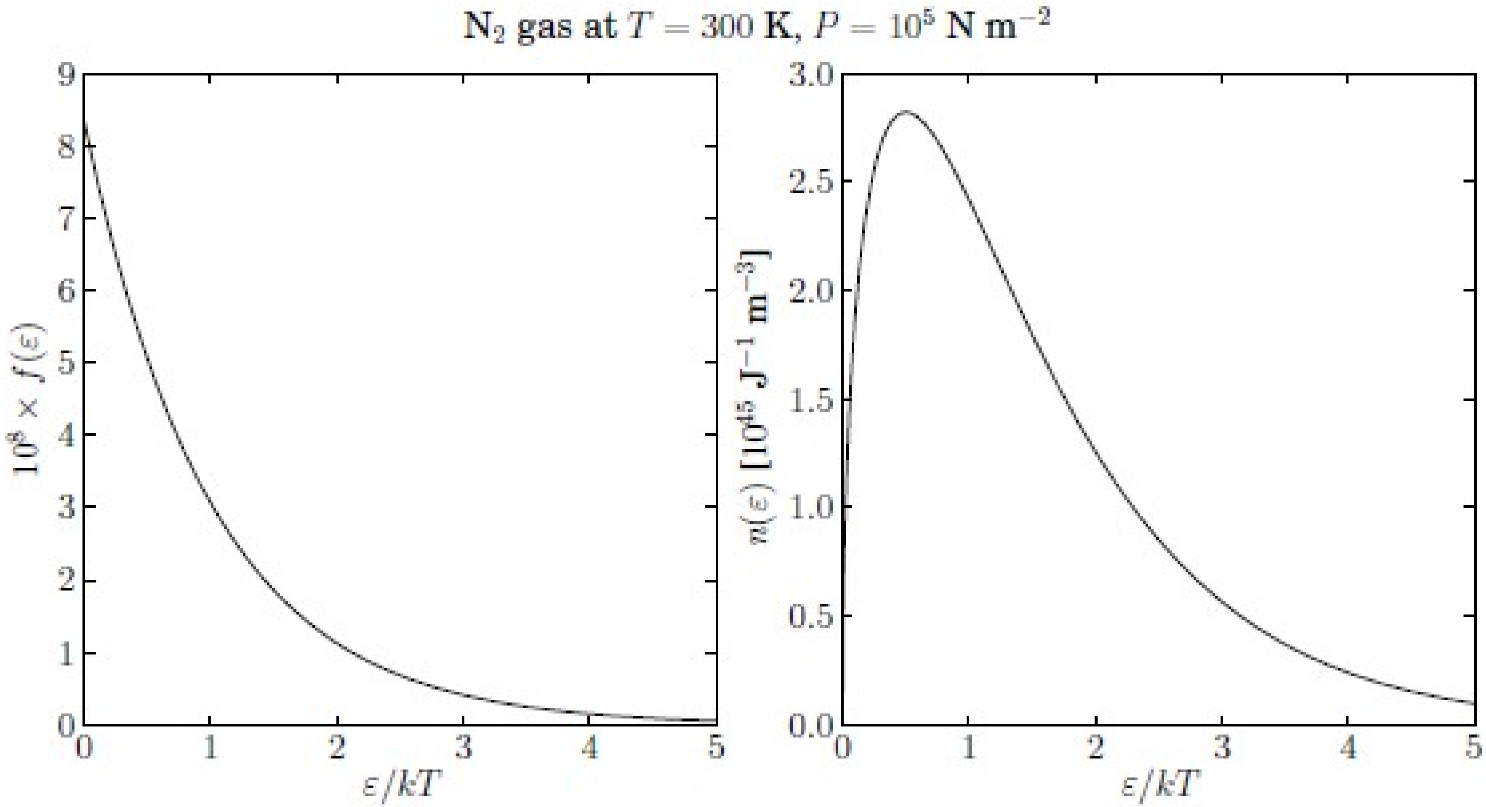


Figure 1: The mean occupation number $f(\varepsilon)$ (left panel) and the corresponding number density of particles $n(\varepsilon)$ (right panel) as a function of energy for a nitrogen (N_2) gas at room temperature and atmospheric pressure. The energy is expressed in units of kT.

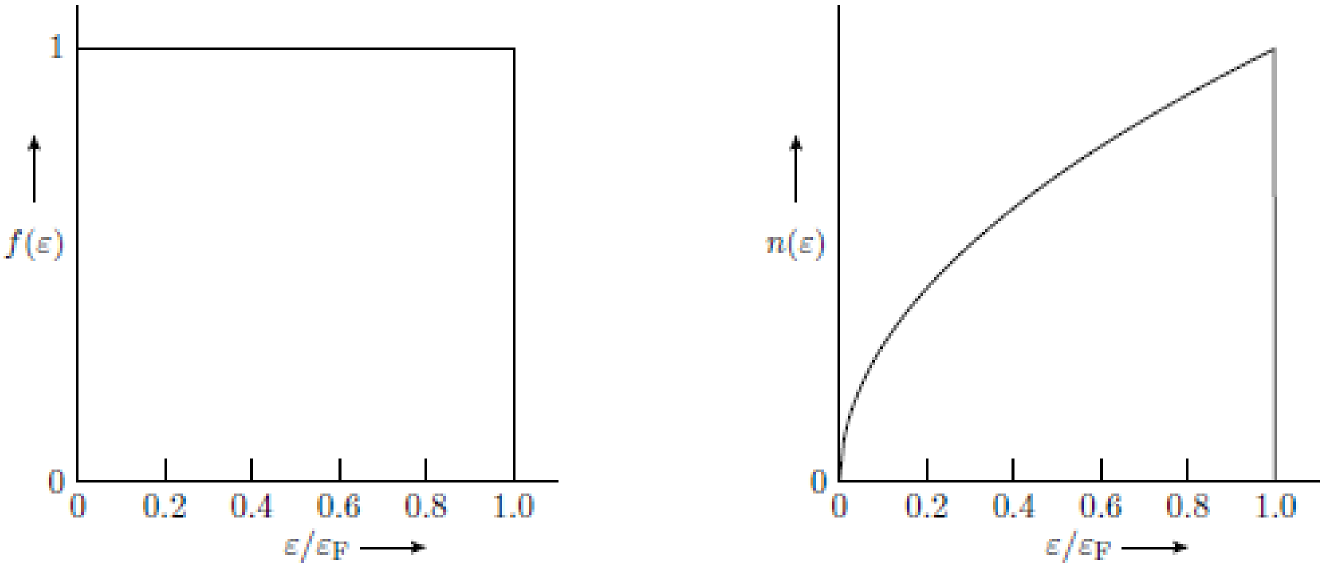


Figure 2: The Fermi-Dirac mean occupation number $f(\varepsilon)$ and energy distribution $n(\varepsilon)$ for a non-relativistic fermion gas at zero temperature: $T = 0$ K. The energy is expressed in units of the Fermi energy ε_F.

$$m \equiv m_0$$

Cosas

$$P = \frac{n}{3} \langle \boldsymbol{p} \cdot \boldsymbol{v} \rangle$$

$$\begin{cases} e_p^2 = p^2 c^2 + m^2 c^4 \\ v = \dfrac{p c^2}{e_p} \end{cases}$$

Si $p << mc$

(no relativista)

$$\begin{cases} v = \dfrac{p}{m} \\ e_p = p^2 / 2m + m c^2 \end{cases}$$

$$P = \frac{2}{3} n \langle \frac{1}{2} m v^2 \rangle$$

$p >> mc$

(relativista)

$$\begin{cases} e_p = p c \\ v = c \end{cases}$$

$$P = \frac{1}{3} n \langle pc \rangle$$

Relativistic Fermi Gas

$E^2 = p^2c^2 + m_0^2c^4$ Sea $m \equiv m_0$

Energía cinética: $\epsilon_p^2 = p^2c^2 + m^2c^4\,,$

Es la energía de una partícula de masa m y momento p

consistente con: $p = \dfrac{mv_p}{\sqrt{1-(v_p/c)^2}}\,.$

velocidad de la partícula $v_p = \dfrac{pc^2}{\epsilon_p}\,,$

Formas útiles de la energía y el momento

$$\frac{\epsilon_p}{mc^2} = \left[\left(\frac{p}{mc}\right)^2 + 1\right]^{1/2}\,,$$

$$\frac{p}{mc} = \left[\left(\frac{\epsilon_p}{mc^2}\right)^2 - 1\right]^{1/2}\,.$$

Relativistic Fermi Gas

$$\epsilon_p^2 = p^2c^2 + m^2c^4 \,,$$

$$\frac{d\epsilon_p}{dp} = \frac{pc^2}{\epsilon_p} = v_p \longrightarrow dp = \frac{\epsilon_p}{pc^2}\, d\epsilon_p = \frac{1}{v_p}\, d\epsilon_p \,,$$

"autoenergias corridas"

$$\epsilon_{\rm kin} = \epsilon_p - mc^2 \,.$$

$$\frac{\epsilon_{\rm kin}}{mc^2} = \left[\left(\frac{p}{mc}\right)^2 + 1\right]^{1/2} - 1 \,,$$

$$\frac{p}{mc} = \left[\left(\frac{\epsilon_{\rm kin}}{mc^2} + 1\right)^2 - 1\right]^{1/2}$$

$$\frac{d\epsilon_{\rm kin}}{dp} = \frac{pc^2}{\epsilon_{\rm kin} + mc^2}$$

$$dp = \frac{\epsilon_{\rm kin} + mc^2}{pc^2}\, d\epsilon_{\rm kin} \,.$$

Relativistic Fermi Gas

$n(\varepsilon_p) =$

$$f(\epsilon_p) = \frac{1}{e^{(\epsilon_p - \mu)/kT} + 1} = \frac{1}{e^{(\epsilon_{\rm kin} - (\mu - mc^2))/kT} + 1},$$

$n(\varepsilon_p)$

$n(\varepsilon_p)$

$n(\varepsilon_p)$

Será Clásico cuando $e^{(mc^2 - \mu)/kT} \gg 1$

recordemos que:

$$g(p)dp = g_s \frac{4\pi V}{h^3} p^2 \, dp,$$

$$n(p) \, dp = f(\epsilon_p) g(p) \, dp, \qquad n = \int_0^\infty f(\epsilon_p) g(p) \, dp.$$

Entonces el gas puede ser :

ultrarelativistico → un numero significativo de particulas tiene

$$p \gg mc$$

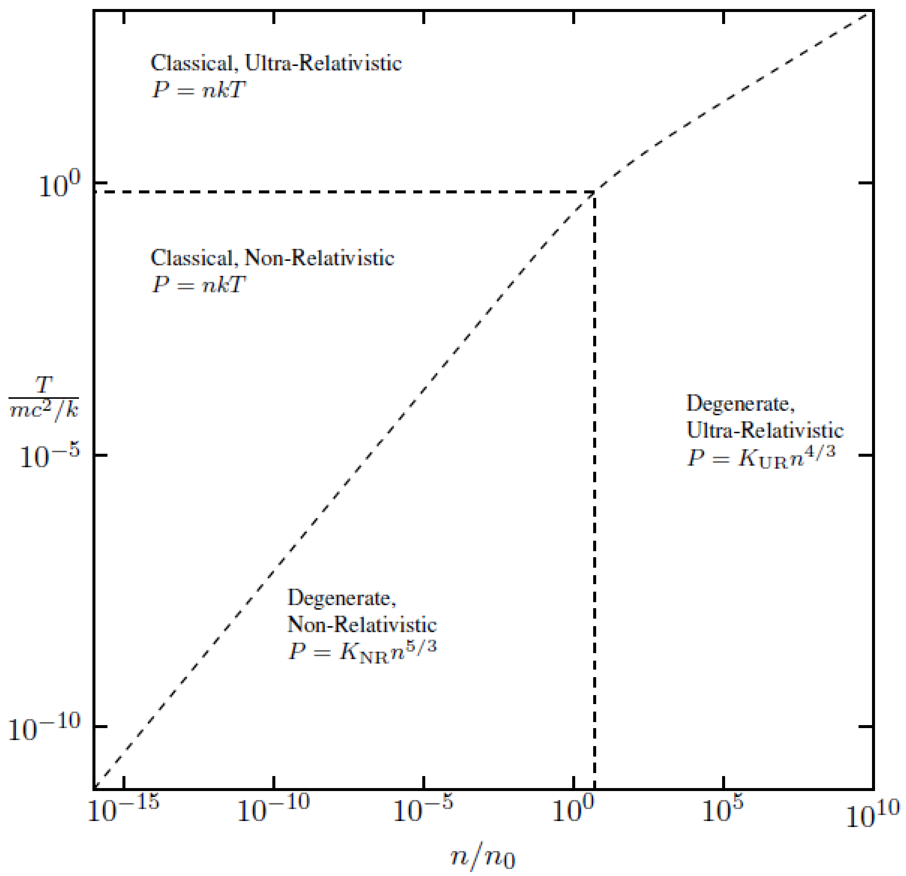
no relativistico → un numero significativo de particulas tiene

$$p \ll mc.$$

Relativistic Fermi Gas

Ademas el gas puede ser degenerado o no.

Entonces se verá de esta forma (n en unidades de n_0 y T en mc^2/k)



Relativistic Fermi Gas

sea entonces

$$\bar{\mu} = \mu - mc^2 \, .$$

$n(\varepsilon_{kin})$

$n(\varepsilon_{kin})$

$n(\varepsilon_p)$

luego

$$f(\epsilon_{\rm kin}) = \frac{1}{e^{(\epsilon_{\rm kin} - \bar{\mu})/kT} + 1} \, ,$$

y resulta

$$n(p)\, dp = f(\epsilon_{\rm kin}) g(p)\, dp \, .$$

$$n(p)\, dp = n(\epsilon_{\rm kin})\, d\epsilon_{\rm kin}$$

$$n(\epsilon_{\rm kin}) = n(p) \frac{dp}{d\epsilon_{\rm kin}} \, .$$

Relativistic Fermi Gas

a partir de

$$n(p)\,dp = f(\epsilon_{\rm kin}) g_s \frac{4\pi}{h^3} p^2\,dp\,. \qquad n(\epsilon_{\rm kin}) = n(p)\frac{dp}{d\epsilon_{\rm kin}}\,.$$

$$\begin{aligned} n(\epsilon_{\rm kin})\,d\epsilon_{\rm kin} &= f(\epsilon_{\rm kin}) g_s \frac{4\pi}{h^3} p^2 \frac{dp}{d\epsilon_{\rm kin}}\,d\epsilon_{\rm kin} \\ &= f(\epsilon_{\rm kin}) g_s \frac{4\pi}{h^3} p \frac{\epsilon_{\rm kin} + mc^2}{c^2}\,d\epsilon_{\rm kin} \\ &= f(\epsilon_{\rm kin}) g_s \frac{4\pi}{h^3} \frac{p}{mc} \frac{m}{c} mc^2 (\epsilon_{\rm kin}/mc^2 + 1)\,d\epsilon_{\rm kin}\,. \end{aligned}$$

usando

$$\frac{p}{mc} = \left[\left(\frac{\epsilon_{\rm kin}}{mc^2} + 1\right)^2 - 1\right]^{1/2}\,,$$

y expresando en términos de mc

$$n\left(\frac{\epsilon_{\rm kin}}{mc^2}\right)\,d\left(\frac{\epsilon_{\rm kin}}{mc^2}\right) =$$

$$= f(\epsilon_{\rm kin}) g_s \frac{4\pi}{h^3} \frac{m}{c} \left[\left(\frac{\epsilon_{\rm kin}}{mc^2} + 1\right)^2 - 1\right]^{1/2} mc^2 \left(\frac{\epsilon_{\rm kin} + mc^2}{mc^2}\right) mc^2\,d\left(\frac{\epsilon_{\rm kin}}{mc^2}\right)$$

Relativistic Fermi Gas

con $\epsilon'_{\rm kin} = \epsilon_{\rm kin}/mc^2.$

$$n(\epsilon'_{\rm kin}) = f(\epsilon'_{\rm kin}) g_s 4\pi \left(\frac{mc}{h}\right)^3 (\epsilon'_{\rm kin}+1)\left((\epsilon'_{\rm kin}+1)^2-1\right)^{1/2}$$

calculamos la energía de Fermi del modo usual

$$n = g_s 4\pi \left(\frac{mc}{h}\right)^3 \int_0^{\epsilon_{\rm F}} (\epsilon'_{\rm kin}+1)\left((\epsilon'_{\rm kin}+1)^2-1\right)^{1/2} d\epsilon'_{\rm kin}$$

$$= g_s 4\pi \left(\frac{mc}{h}\right)^3 \frac{1}{3}\left((\epsilon'_{\rm F}+1)^2-1\right)^{3/2} \qquad [*]$$

En el limite $\epsilon_{\rm F}/mc^2 \ll 1$ se recupera:

$$n = g_s \frac{2\pi}{h^3}(2m)^{3/2}\int_0^{\varepsilon_{\rm F}} \varepsilon^{1/2}\, d\varepsilon = g_s \frac{2\pi}{h^3}(2m)^{3/2}\frac{2}{3}\varepsilon_{\rm F}^{3/2},$$

Relativistic Fermi Gas

de [*] →
$$\frac{\epsilon_{\mathrm{F}}}{mc^2} = \left(\left(\frac{3}{4\pi} \frac{n}{g_s} \right)^{2/3} \left(\frac{h}{mc} \right)^2 + 1 \right)^{1/2} - 1$$

$$\frac{\epsilon_{\mathrm{F}}}{mc^2} = \left(\left(\frac{3n}{g_s 4\pi (mc/h)^3} \right)^{2/3} + 1 \right)^{1/2} - 1$$

$$\frac{p_{\mathrm{F}}}{mc} = \left(\frac{3n}{g_s 4\pi (mc/h)^3} \right)^{1/3}$$

Relativistic Fermi Gas , fases

Tenemos las expresiones :

No relativista (NR)

$$\varepsilon_{\mathrm{F}} = \frac{h^2}{2m}\left(\frac{3}{g_s 4\pi}\right)^{2/3} n^{2/3} = \frac{\hbar}{2m}\left(\frac{6\pi^2}{g_s}\right)^{2/3} n^{2/3} .$$

Relativista (UR)

$$\frac{\epsilon_{\mathrm{F}}}{mc^2} = \left(\left(\frac{3}{4\pi}\frac{n}{g_s}\right)^{2/3}\left(\frac{h}{mc}\right)^2 + 1\right)^{1/2} - 1 ,$$

Relativistic Fermi Gas , fases

$$\frac{\epsilon_{\mathrm{F,NR}}}{mc^2} = \frac{1}{2}\left(\frac{3n}{g_s 4\pi (mc/h)^3}\right)^{2/3} = \frac{1}{2}\left(\frac{n}{n_0}\right)^{2/3}$$

con

$$n_0 = g_s \frac{4\pi}{3}\left(\frac{mc}{h}\right)^3 .$$

$$\frac{\epsilon_{\mathrm{F}}}{mc^2} = \left(\left(\frac{n}{n_0}\right)^{2/3} + 1\right)^{1/2} - 1 ,$$

$$\frac{p_{\mathrm{F}}}{mc} = \left(\frac{n}{n_0}\right)^{1/3}$$

Relativistic Fermi Gas , fases

Hay 4 regiones

D degenerado

↓

ND no degenerado

$$\frac{3}{2}kT \approx \epsilon_F \,,$$

NR no relativista

↓

UR ultra relativista

$$(3/2)kT \gg mc^2$$

$$\epsilon_F \gg mc^2.$$

Relativistic Fermi Gas

Se pasa de NR a UR si

o

$$\epsilon_F \gg mc^2.$$

Luego la línea que separa estos regimenes es:

$$\frac{3}{2}kT \approx mc^2 \quad \vee \quad \frac{\epsilon_F}{mc^2} \approx 1\,,$$

recordando

$$\frac{\epsilon_F}{mc^2} = \left(\left(\frac{n}{n_0} \right)^{2/3} + 1 \right)^{1/2} - 1\,,$$

$$T = \frac{2}{3}\frac{mc^2}{k} \quad \vee \quad n = 3^{3/2} n_0\,.$$

si lo hacemos 1

Relativistic Fermi Gas

La línea de separación entre degenerado y no-degenerado

$$\frac{3}{2}kT \approx \epsilon_F ,$$

o sea

$$T = \frac{2}{3}\frac{mc^2}{k}\left[\left(\left(\frac{n}{n_0}\right)^{2/3} + 1\right)^{1/2} - 1\right]$$

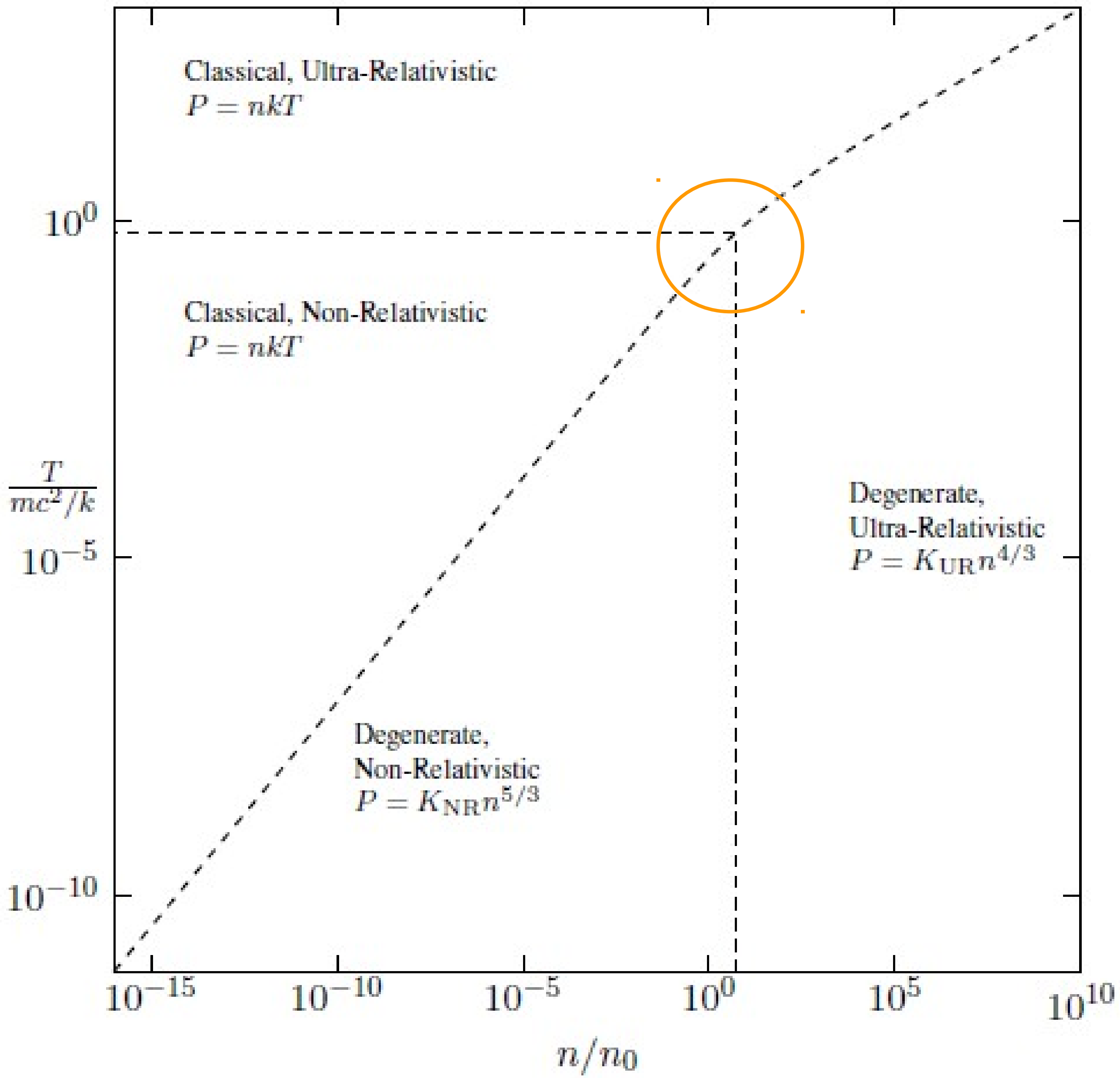
no relativista → $(n \ll n_0)$

$$T_{NR} = \frac{1}{3}\frac{mc^2}{k}\left(\frac{n}{n_0}\right)^{2/3}$$

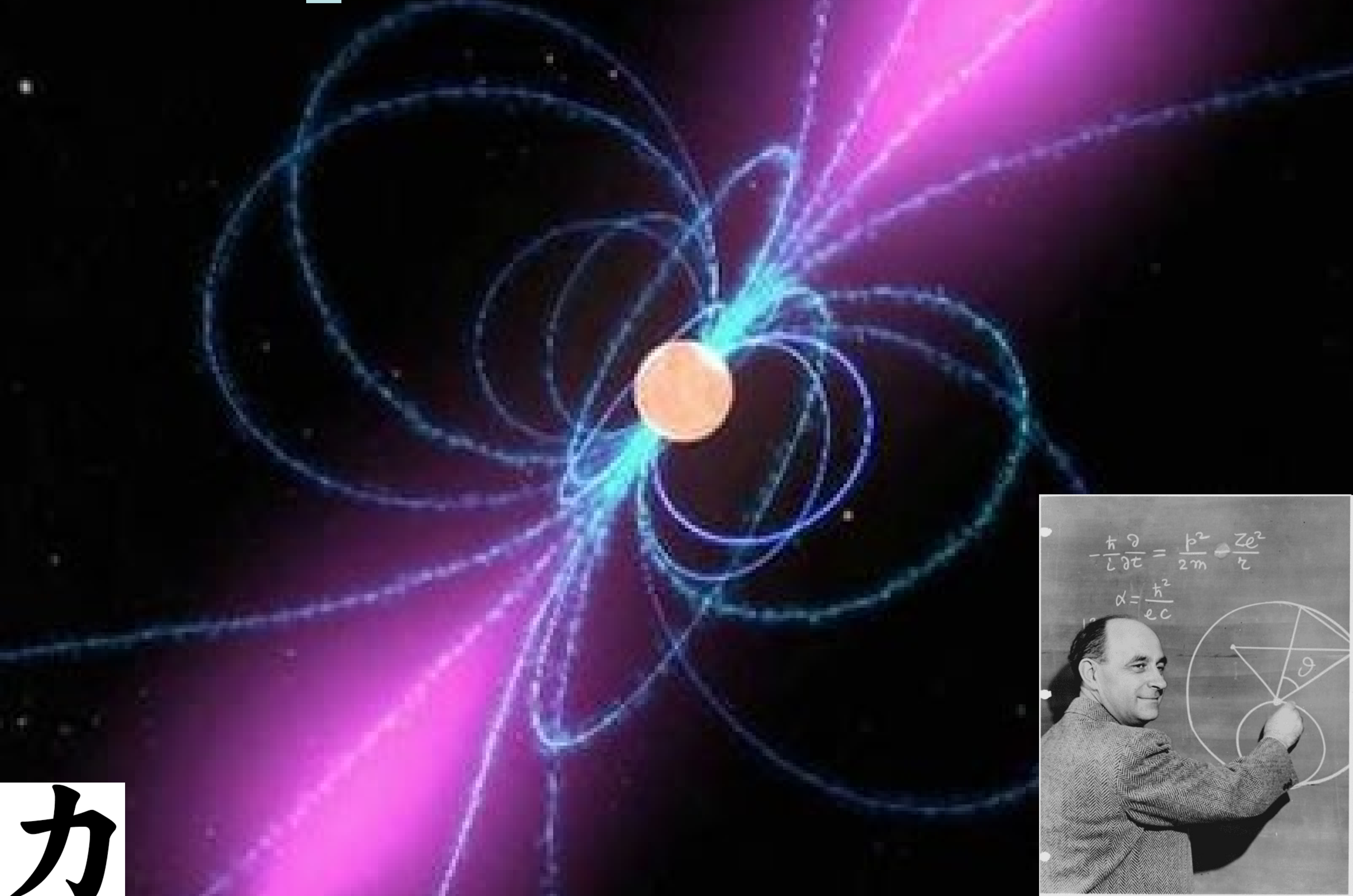
ultra relativista → $(n \gg n_0)$

$$T_{UR} = \frac{2}{3}\frac{mc^2}{k}\left(\frac{n}{n_0}\right)^{1/3} .$$

Fermi Gas, fases

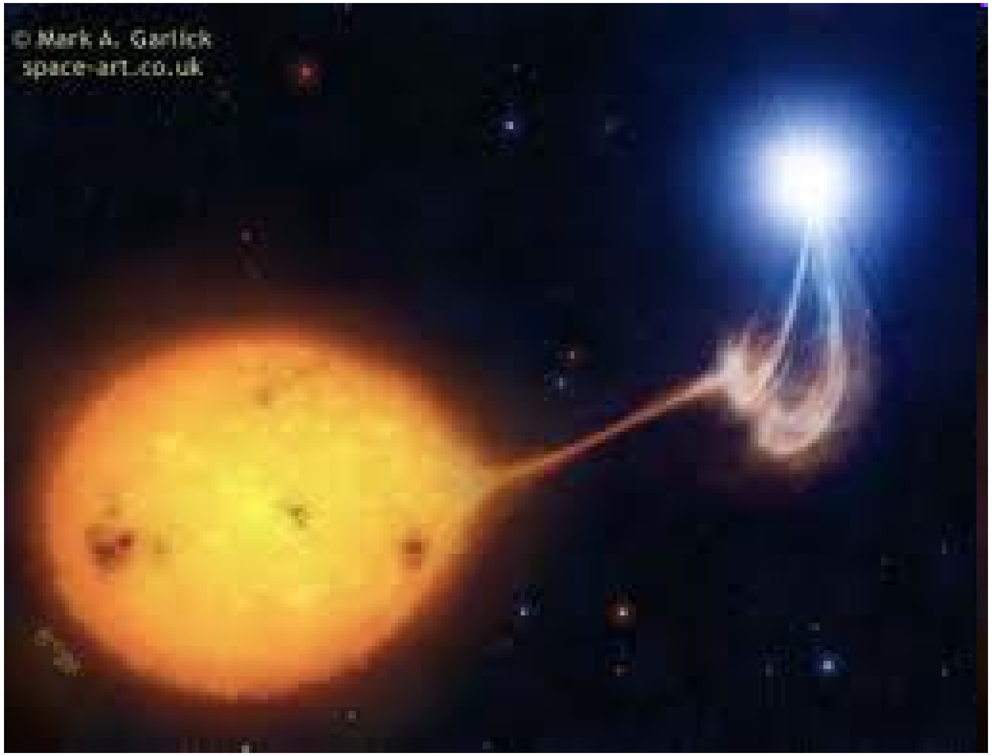


Quantal_3e

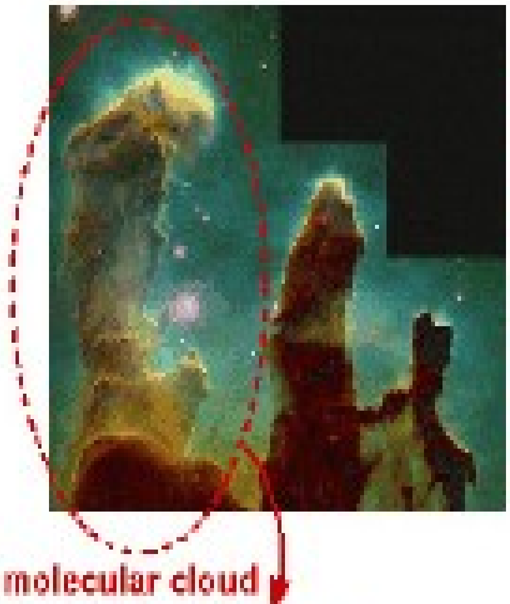

$-\frac{\hbar}{i}\frac{\partial}{\partial t} = \frac{p^2}{2m} - \frac{Ze^2}{r}$
$\alpha = \frac{\hbar^2}{ec}$
ϑ

力

De las estrellas



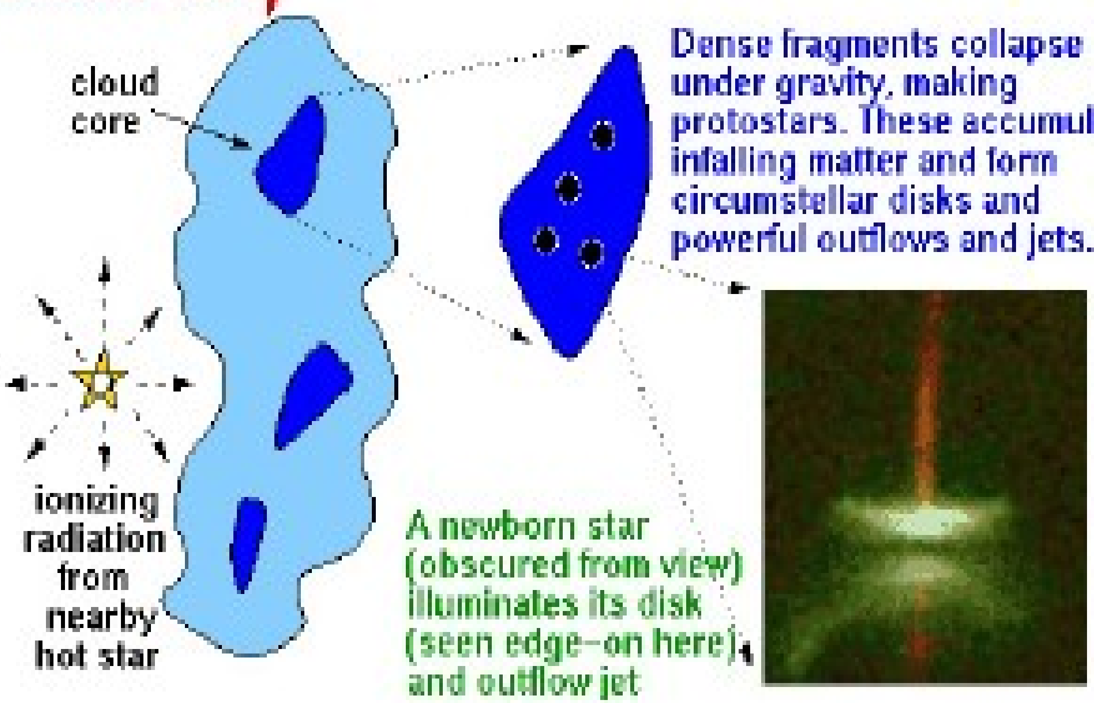
All stars, as far as we know, are born from the gravitational collapse of the core of a molecular cloud.



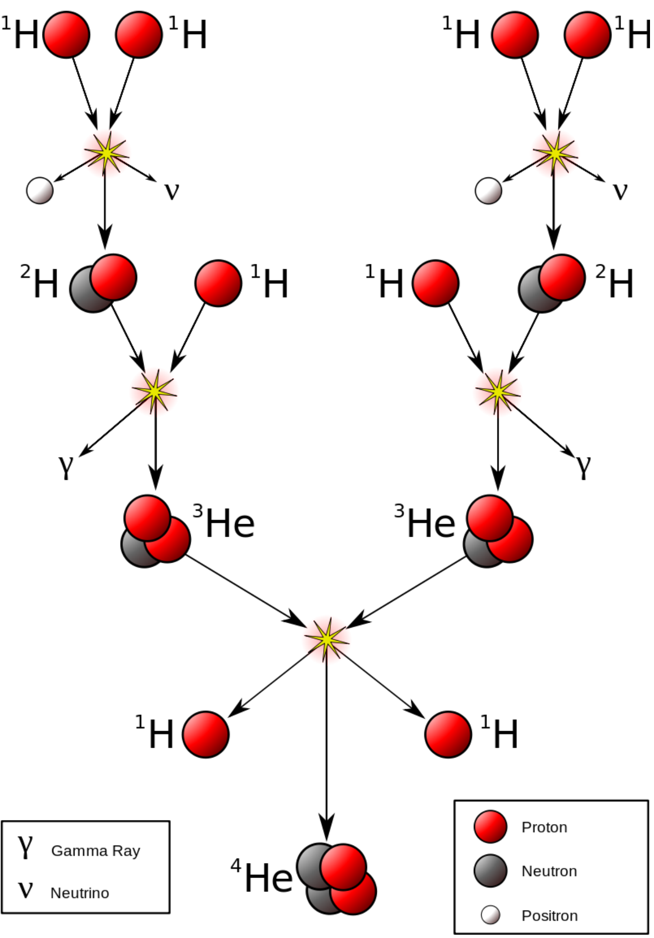
Molecular clouds are cold, dark, giant condensations of dust and molecular gas which serve as "stellar nurseries".

All stars are born in molecular clouds, including our Sun. Molecular clouds are the "stuff" we're made of!

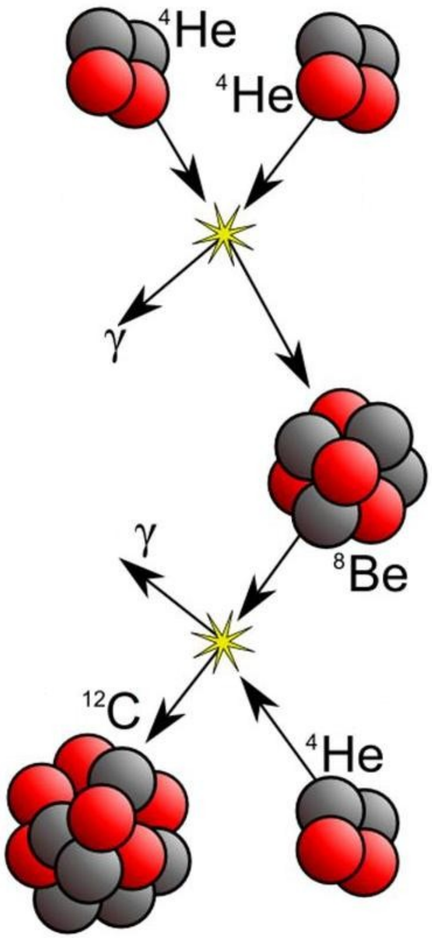
Because of their dusty content, visible light cannot penetrate into a molecular cloud. Thus, infrared and submillimeter observations are needed to "see" the star-forming process.



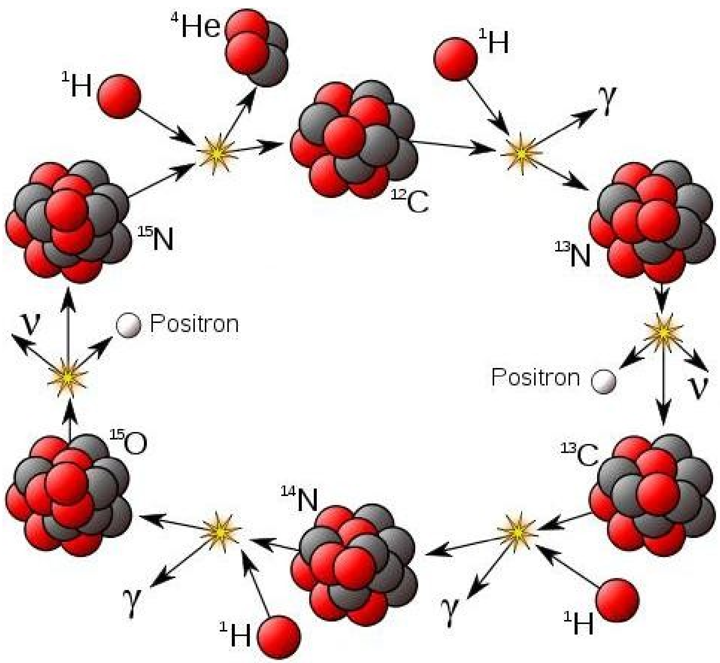
A Closer Look at Nucleosynthesis



H to He



He to C



CNO cycle

White dwarfs (enanas blancas)

Algunas unidades

proton mass $938.3 MeV/c^2 (1.6726 \times 10^{-27} kg)$ (1836 veces la masa del electron)

densidad nuclear $2.7 \times 10^{14} g\ cm^{-3}$ = $0.16 fm^{-3}$

El Sol

Mean diameter $1.392 \times 10^6 km$ (109 Earth diameters)

Circumference $4.373 \times 10^6 km$(342 Earth diameters)

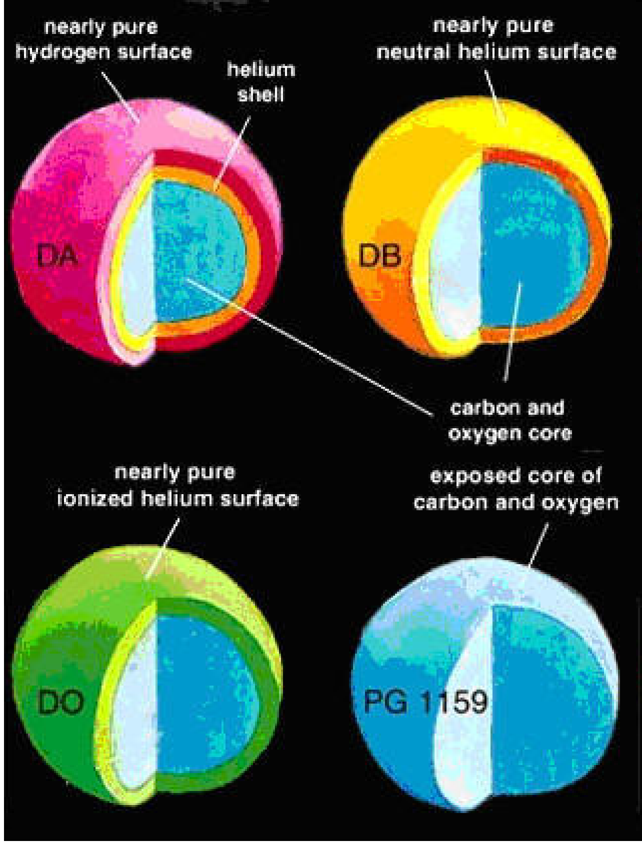
Surface area $6.09 \times 10^{12} km^2$(11,900 Earths)

Volume $1.41 \times 10^{18} km^3$ (1,300,000 Earths)

Mass $1.988\,435 \times 10^{30} kg$ (332,946 Earths)

Density $1.408 g/cm^3$

Surface gravity $273.95 ms^{-2}$ (27.9 g)



Las Enanas Blancas

Estrellas de luminosidad muy baja

Estan compuestas basicamente por Helio

Con una densidad de $10^{7} g/cm^{3}$ (o sea $10^{7}\rho_{\odot}$)

Masa $10^{33} g\ (\approx M_{\odot})$

Temperatura en el centro $10^{7} K\ (\approx T_{\odot})$

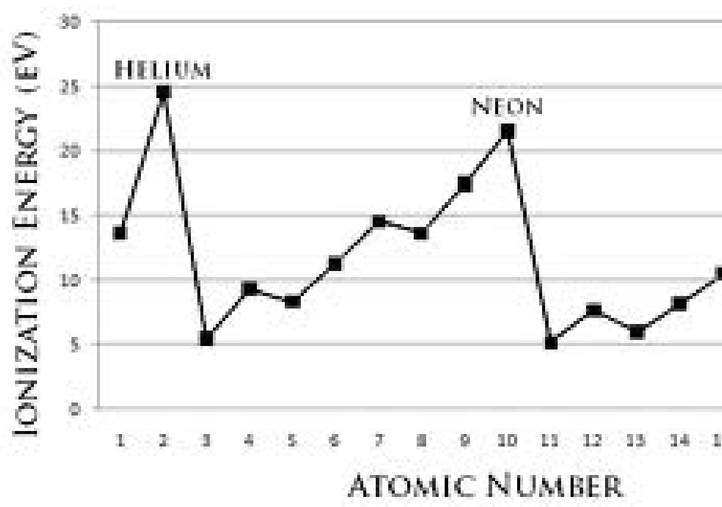
$(\approx 1000eV)(eV \approx 11602K)$

$0.008R_{\odot} \leq$ Radio de una Enana $\leq 0.02R_{\odot}$

M ≈ 1.0 M sol
R ≈ 5800 km
$V_{esc} \approx 0.02c$
Comparación de una enana blanca con la Tierra

Material	Density in kg/m^3	Notes
Water (fresh)	1,000	At STP
Osmium	22,610	Near room temperature
The core of the Sun	~150,000	
White dwarf star	1×10^{9}[1]	
Atomic nuclei	2.3×10^{17}[31]	Does not depend strongly on size of nucleus
Neutron star core	$8.4 \times 10^{16} - 1 \times 10^{18}$	
Black hole	2×10^{30}[32]	Critical density of an Earth-mass black hole

A esta energia los atomos de Helio estan completamente ionizados y por lo tanto tenemos un sopa de nucleos de Helio y electrones libres.



Calculamos las propiedades del “gas de electrones”

Para estos electrones

$$\epsilon_{F_e} = \frac{\hbar^2}{2m_e}\frac{1}{v^{2/3}}$$

$\hbar = 1.05457 \text{x} 10^{-34} Js$

$m_e = 9.10938 \text{ x} 10^{-31} kg$

$6.24150636309 \times 10^{12}\ MeV$

$$\epsilon_F = \frac{(1.05457 \text{x} 10^{-34})^2 kg^2 m^4 s^2}{2 \cdot 9.10938 \text{x} 10^{-31} kg\ s^4}\left(\frac{10^7 g/cm^3}{9.10938 \text{x} 10^{-28} g}\right)^{2/3}$$

$$= \frac{kg\ m^4}{m^2 s^2}\frac{(1.05457)^2}{2(9.10938)^{5/3}}\frac{10^4 10^{-68}(10^{35})^{2/3}}{10^{-31}}$$

$$\frac{10^{-68}(10^{7})^{2/3}(10^{28})^{2/3}10^{4}}{10^{-31}}\frac{(1.05457)^{2}}{2(9.10938)^{5/3}} = 3.0151 \times 10^{-12} Js =$$

$$(6.24150636309 \times 10^{12})(0.30151 \times 10^{-11}) MeV =$$

$$\varepsilon_F = 18.819 MeV \sim 2 \times 10^{7} eV \sim 10^{12} K.$$

Entonces si la correspondinete Temperatura de Fermi es del orden de $T_F \approx 10^{12} K$, la Temperatura de la estrella es $10^{7} K$ es mucho menor y por lo tanto los electrones estan en un estado muy degenerado.→ gas ideal a temperatura ≈ 0.

Mas propiedades del gas de electrones

$$m_e = 9.10938x10^{-31}kg$$

$$c = 2.9979x10^{8}m/s$$

de donde

$$m_e c \approx 2.7x10^{-23}kgm/s$$

mientras que

$$p_F = \left(\frac{3N}{4\pi g V}\right)^{1/3} h \approx 1.05457x10^{-34}Js\left(\frac{3}{8\pi}\right)^{1/3}\left(\frac{N}{V}\right)^{1/3}$$

$\longrightarrow$

con

$$\left(\frac{N}{V}\right)^{1/3} = \left(\frac{10^7 g/cm^3}{9.10938x10^{-28}g}\right)^{1/3} \approx (10^{34}/cm^3)^{1/3} \approx 10^{34/3}\frac{1}{cm} = 10^{40/3}\frac{1}{m}$$

$$\left(\frac{3}{8\pi}\right)^{1/3} = 0.49237$$

resulta

$$p_F \approx 10^{-22}kgm/s$$

$$m_e c \approx 2.7x10^{-23} kgm/s$$

$$p_F \approx 10^{-22} kgm/s$$

como son comparables resulta que el gas lo podemos considerar relativistico

De esta forma el sistema es : un gas de N electrones en el estado fundamental, a muy alta densidad (los tratamos relativisticamente) moviendose en un fondo neutralizador de $N/2$ nucleos de Helio. Son estos los que "proveen" el campo gravitatorio que balancea la presion del gas de Fermi.

No es solo electrones! sistema neutro!
Coulomb no explota.

Un par de aproximaciones

Sea la densidad de electrones

electrones por nucleon

densidad central

$$n_e = Y_e \frac{\rho_c}{m_H}$$

Sea un gas de electrones no relativista degenerado

$$P = K_{NR} n^{5/3} = \frac{h^2}{5m} \left[\frac{3}{8\pi} \right]^{2/3} n^{5/3}$$

Cual es la presion necesaria para mantener a la Estrella ?

Contraccion gravitatoria

Sea una masa en el centro:

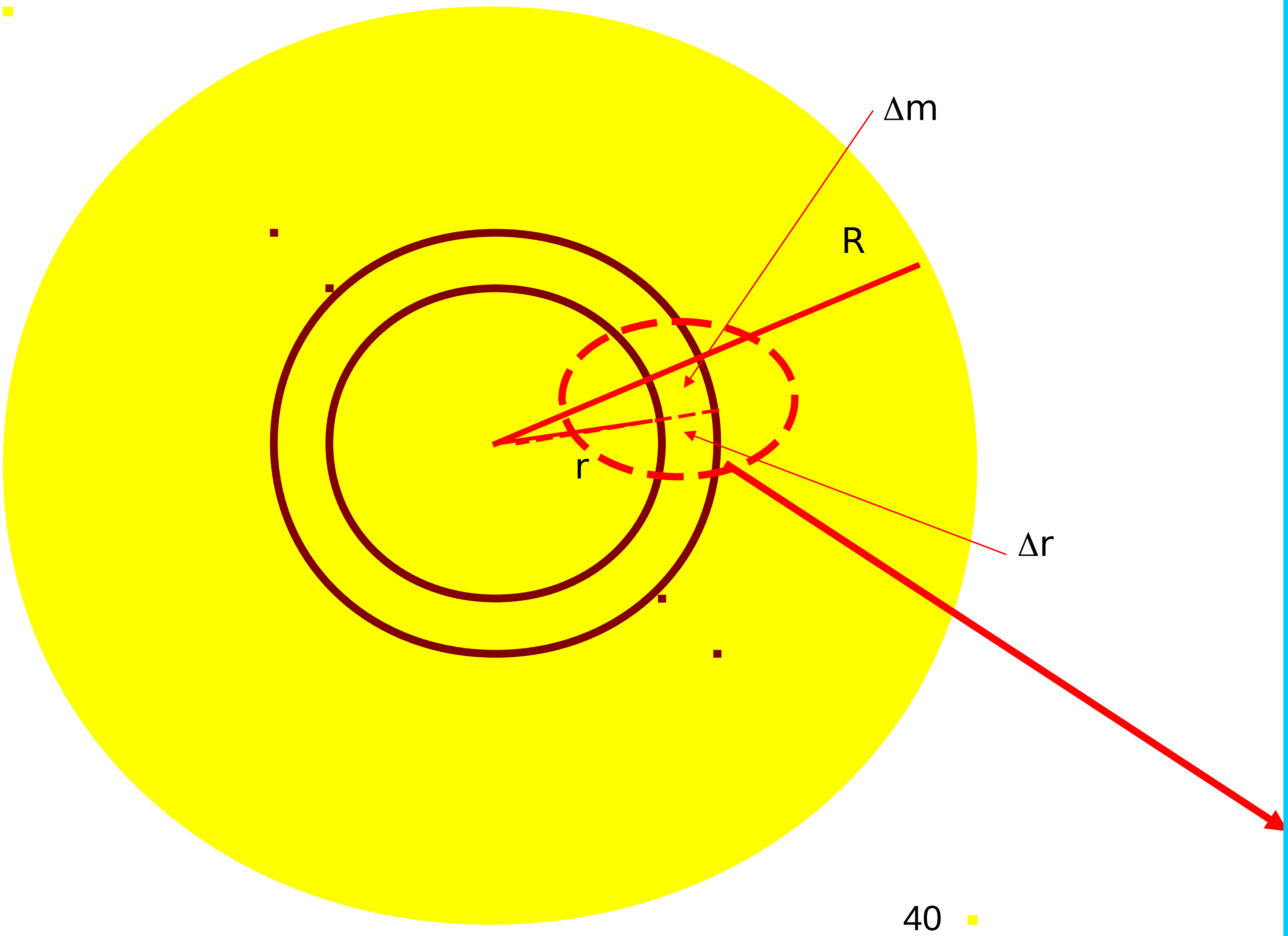
$$m(r) = \int_0^r \rho(r')4\pi r'^2 dr'$$

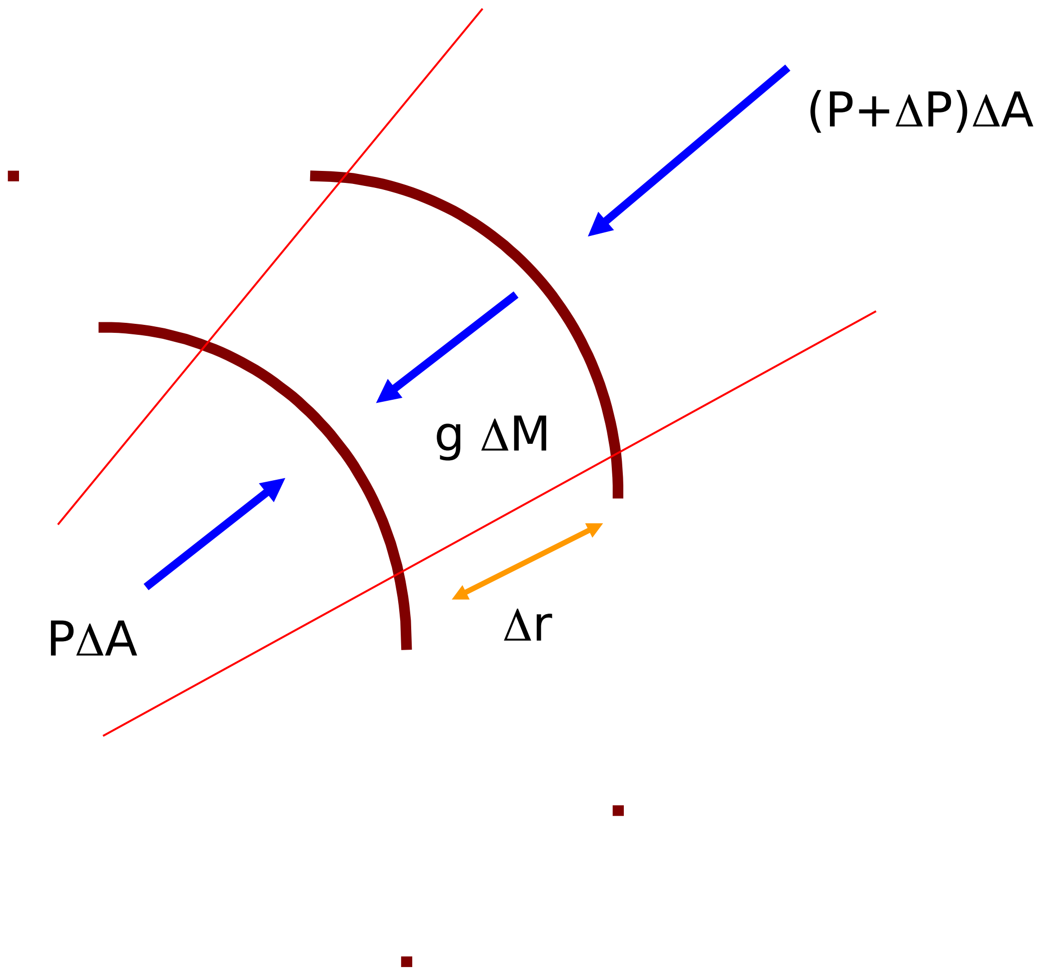
que genera una aceleración gravitatoria en r

$$g(r) = \frac{Gm(r)}{r^2}$$

Para un elemento de volumen

Simple sistema y el equilibrio hidrostático




(P+ΔP)ΔA
g ΔM
PΔA
Δr

Para un elemento de volumen el balance de presiones es:

$$\left[P(r) + \frac{dP(r)}{dr}\Delta r - P(r)\right]\Delta A = \frac{dP(r)}{dr}\Delta r \Delta A$$

la aceleracion del elemento de volumen

$$\Delta M = \rho(r)\Delta r \Delta A$$

$$\Rightarrow \Delta r \Delta A = \frac{\Delta M}{\rho(r)}$$

es:

$$-\frac{d^2 r}{dt^2} = g(r) + \frac{1}{\rho(r)}\frac{dP}{dr}$$

En equilibrio hidrostatico $-\frac{d^2r}{dt^2} = 0$, entonces

$$\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

Para un equilibrio global (relación entre presión interna y energía gravitatoria)

$$\int_0^R \frac{4\pi r^3}{3} \frac{dP}{dr} dr = -\int_0^R \frac{4\pi r^3}{3} \frac{Gm(r)\rho(r)}{r^2} dr$$

$$\int_0^R 4\pi r^3 \frac{dP}{dr} dr = -\int_0^R 4\pi r^2 \frac{Gm(r)\rho(r)}{r} dr$$

El termino de la derecha es la energia potencial gravitatoria del sistema

$$-\int_{m=0}^{M=M} \frac{Gm(r)}{r} dm = E_{GR}$$

El termino de la iquierda es (integrando por partes)

$$\int_0^R 4\pi r^3 \frac{dP}{dr} dr = \left[P(r) 4\pi r^3\right]_0^R - 3\int_0^R P(r) 4\pi r^2 dr = 0 - 3\langle P\rangle V$$

De donde

$$\langle P\rangle = -\frac{1}{3}\frac{E_{GR}}{V}$$

- Si estamos a baja densidad, los electrones e iones forman un gas clasico y

$$P = nKT$$

- A altas densidades los electrones forman un gas cuantico degenerado

$$\varepsilon_p = mc^2 + p^2/2m$$

con una presion dada por

$$P = \frac{h^2}{5m}\left[\frac{3}{8\pi}\right]^{2/3} n_e^{5/3} = n\frac{p_F^3}{5m}$$

En este caso

$$\left[\frac{3n}{8\pi}\right]^{1/3} h = p_F << mc$$

Como $p_F = \left[\frac{3n}{8\pi}\right]^{1/3} h$. lo anterior es equivalente a

$$n << \left(\frac{mc}{h}\right)^3$$

-Si las densidades son muy altas

$$n >> \left(\frac{mc}{h}\right)^3$$

el gas se hace ultrarelativista entonces

$$\varepsilon_p = pc$$

de donde

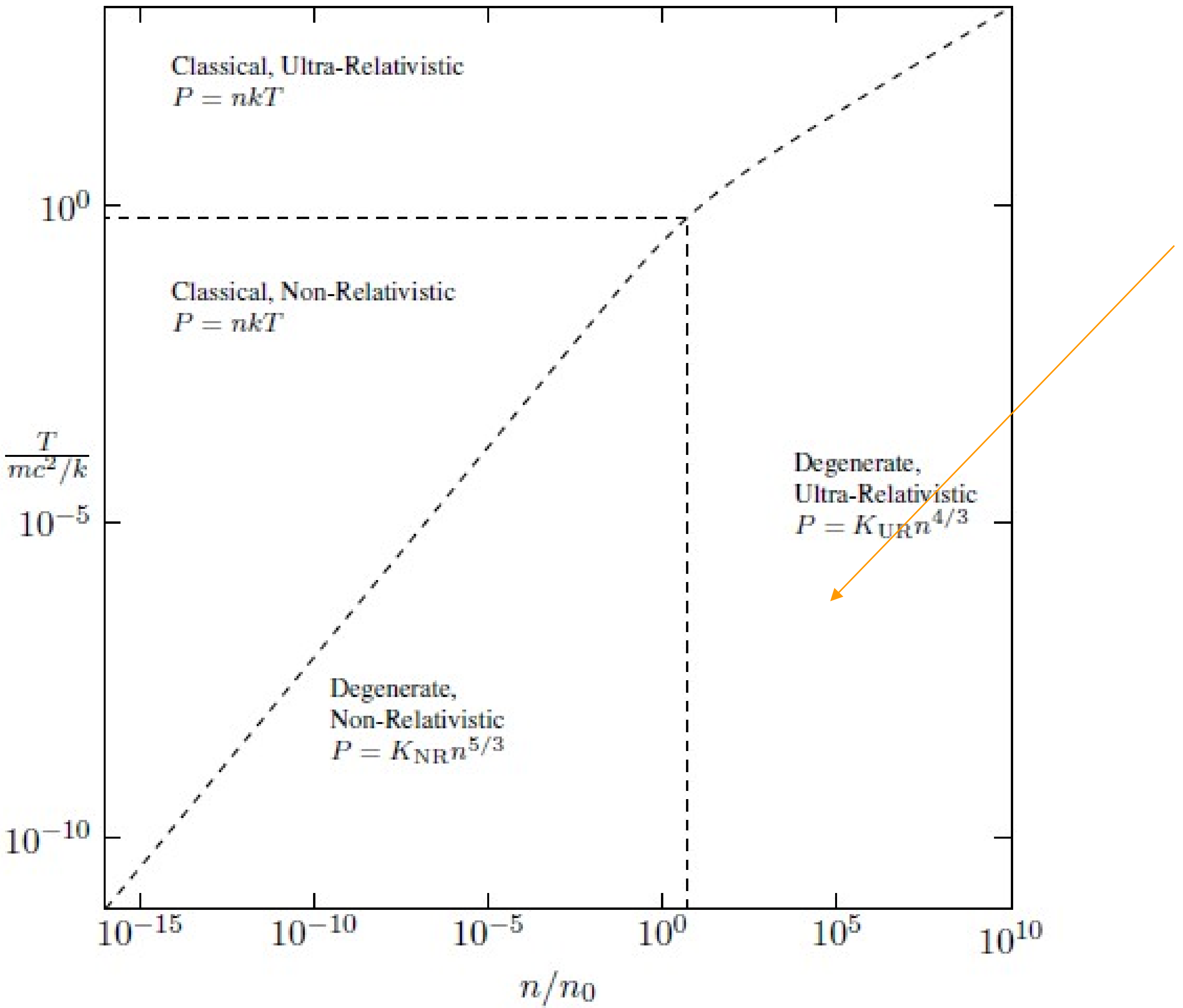
$$E = \int_0^{p_F} pc\, g_s \frac{V}{h^3} 4\pi p^2 dp = N \frac{3}{4} p_F\, c$$

$$N = \frac{8\pi V}{h^3} p_F^3$$

y la presion se aproxima a

$$P = \frac{hc}{4} \left[\frac{3}{8\pi} \right]^{1/3} n_e^{4/3}$$

$$\frac{P_{\rm UR}}{n_0 mc^2} = \frac{1}{4} \left(\frac{p_{\rm F}}{mc} \right)^{4/3} = \frac{1}{4} \left(\frac{n}{n_0} \right)^{4/3} .$$



Classical, Ultra-Relativistic
$P = nkT$
Classical, Non-Relativistic
$P = nkT$
Degenerate,
Ultra-Relativistic
$P = K_{\mathrm{UR}} n^{4/3}$
Degenerate,
Non-Relativistic
$P = K_{\mathrm{NR}} n^{5/3}$
$\frac{T}{mc^2/k}$
10^0
10^{-5}
10^{-10}
10^{-15}
10^{-10}
10^{-5}
10^0
10^5
10^{10}
n/n_0

Presion ejercida por el gas de electrones

Los estados del gas son (tomar en cuenta que $s = \pm\frac{1}{2}$)

$$\epsilon_{ps} = \left[(pc)^2 + (m_e c^2)^2\right]^{1/2}$$

El momento de Fermi es

$$\frac{2V}{h^3}\left(\frac{4}{3}\pi p_F^3\right) = N = \frac{8\pi V}{h^3}\int_0^{p_f} p^2 dp$$

de donde

$$p_F = \hbar\left(\frac{3\pi^2}{v}\right)^{1/3} = \left(\frac{3}{8\pi v}\right)^{1/3} h$$

La energia del estado fundamental es

$$E_0 = 2 \sum_{|\mathbf{p}| \leq p_F} \left[(pc)^2 + (m_e c^2)^2 \right]^{1/2}$$

$$= \frac{2V}{h^3} \int_0^{p_F} dp 4\pi p^2 \left[(pc)^2 + (m_e c^2)^2 \right]^{1/2}$$

Si calculamos la presion

$$P = \frac{1}{3} \frac{N}{V} \left\langle p \frac{d\epsilon}{dp} \right\rangle$$

$$P = \frac{1}{3}\frac{N}{V}\left\langle p\frac{d\epsilon}{dp}\right\rangle$$

Para un gas relativista en el estado fundamental (huang)

con $\epsilon = \left[(pc)^2 + (m_e c^2)^2\right]^{1/2} = m_e c^2\left\{1 + [\frac{p}{m_e c}]^2\right\}^{1/2} \Rightarrow$

$$v = \frac{d\epsilon}{dp} = \frac{1}{2}\frac{2m_e c^2 p/m_e^2 c^2}{\left\{1+[\frac{p}{m_e c}]^2\right\}^{1/2}} = \frac{p/m_e}{\left\{1+[\frac{p}{m_e c}]^2\right\}^{1/2}}$$

De esta forma

$$P_0 = \frac{8\pi}{3h^3}\int_0^{p_F}\frac{p/m_e}{\left\{1 + [\frac{p}{m_e c}]^2\right\}^{1/2}}p\,p^2 dp$$

Se define $p = mc(\sinh\theta)$

con $\sinh\theta = (e^{x} - e^{-x})/2$

con $\cosh\theta = (e^{x} + e^{-x})/2$

$dp = mc(\cosh\theta)\, d\theta$

que cumplen $\cosh^2\theta - \sinh^2\theta = 1$

Entonces la integral es

$$P_0 = \frac{8\pi}{3h^3}\int_0^{p_F} \frac{m^4c^5\sinh^4\theta}{\{\cosh^2\theta\}^{1/2}}\cosh\theta\, d\theta = \frac{8\pi}{3h^3}m^4c^5\int_0^{p_F}\sinh^4\theta d\theta$$

Que resulta dar

$$P_0 = \frac{\pi m^4 c^5}{3h^3} A(x) = \frac{\pi m^4 c^5}{3h^3}[x(x^2+1)^{1/2}(2x^2-3) + 3\sinh^{-1} x] =$$

$$= ((\pi m^4 c^5)/(3h^3)) \cdot A(x)$$

Donde

$$x = \sinh \theta_F = \frac{p_F}{mc}$$

Se puede considerar el siguiente proceso :

Cuando se equilibra este sistema

Como sabemos,el gas de fermi necesita "paredes" para estar confinado

En este caso tenemos el potencial gravitatorio!

Sea un contenedor esferico que realiza un cambio de volumen V (si $dV > 0 \Rightarrow \Delta E < 0$) $\rightarrow$

$$dE_0 = -P_0(\frac{1}{V})dV = -P_0(R)4\pi R^2 dR$$

Que pasa si esto ocurre para un sistema que se sostiene confinado por accion de la gravedad?

Haciendo varias suposiciones (el sistema no es homogeneo, etc) calculamos el cambio de energia potencial gravitatoria debido a un cambio de volumen como

$$dE_g = \frac{dE_g}{dR} dR = \alpha \frac{GM^2}{R^2} dR$$

α aparece para tener en cuenta efectos de no homogeneidad.

Para un sistema en equilibrio la energia total se mantiene constante entonces $dE_0 = -dE_g$ dando

$$P_0(R) 4\pi R^2 = \alpha \frac{GM^2}{R^2} \Rightarrow P_0(R) = \alpha \frac{GM^2}{4\pi R^4}$$

Como la masa la concentran los H_e (núcleos)

$$M = N(m_e + 2m_p) \approx 2Nm_p$$

Utilizando la relacion para P_0 con $A(x) = A\left(\left[\frac{9\pi M}{8m_p}\right]^{1/3}\frac{\hbar/mc}{R}\right)$

se obtiene

$$A\left(\left[\frac{9\pi M}{8m_p}\right]^{1/3}\frac{\hbar/mc}{R}\right) = 6\pi\alpha\left(\frac{\hbar/mc}{R}\right)^3\frac{GM^2/R}{mc^2}$$

x

Queda una relacion implicita (ver forma de A)

Como hay formas asintoticas para $x \gg 1$ y $x \ll 1$, o sea:

$$P_0 = \frac{\pi m^4 c^5}{3h^3}A(x) = \frac{\pi m^4 c^5}{3h^3}\left[x(x^2+1)^{1/2}(2x^2-3)+3\sinh^{-1}x\right]$$

$$[p_F/(mc)] \gg 1 \longrightarrow R \ll 10^8\,cm$$

o

$$[p_F/(mc)] \ll 1 \longrightarrow R \gg 10^8\,cm$$

Se puede demostrar que:

$x_F=[p_F/(mc)]>>1$ corresponde al limite ultrarelativista

$x_F<<1$ corresponde al limite no relativista

radio solar $\approx 6.96\ 10^8$ metros

$[p_F/(mc)] \gg 1 \longrightarrow R \ll 10^8 cm$

o

$[p_F/(mc)] \ll 1 \longrightarrow R \gg 10^8 cm$

para $R \gg 10^8 cm$ y $R \ll 10^8 cm$ para las cuales se obtiene

si $R \gg 10^8 cm \Rightarrow R \propto M^{-1/3}$

para $R \ll 10^8 cm \Rightarrow R \approx \frac{\hbar}{mc}\left[\frac{9\pi}{8}\frac{M}{m_p}\right]^{1/3}\left[1-\left(\frac{M}{M_0}\right)^{2/3}\right]^{1/2}$

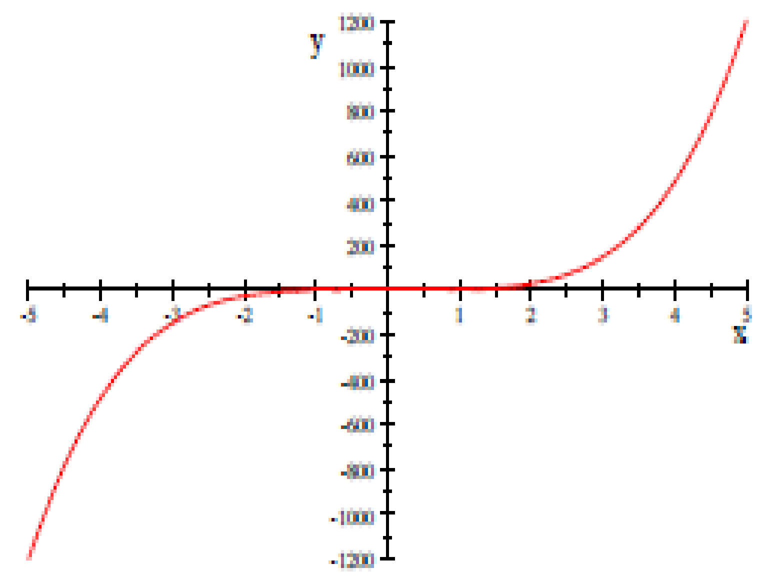
con

$M_0 = \frac{9}{64}(3\pi/\alpha^3)^{1/2}\frac{(\hbar c/G)^{3/2}}{m_p^2}$

debe ser [<1] para solucion

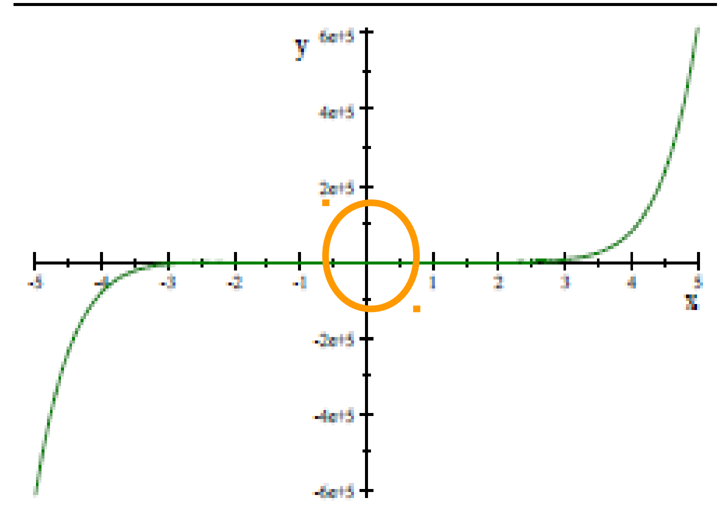
A(x)

$x(x^2+1)^{1/2}(2x^2-3)+3\ln(x+\sqrt{1+x^2})$



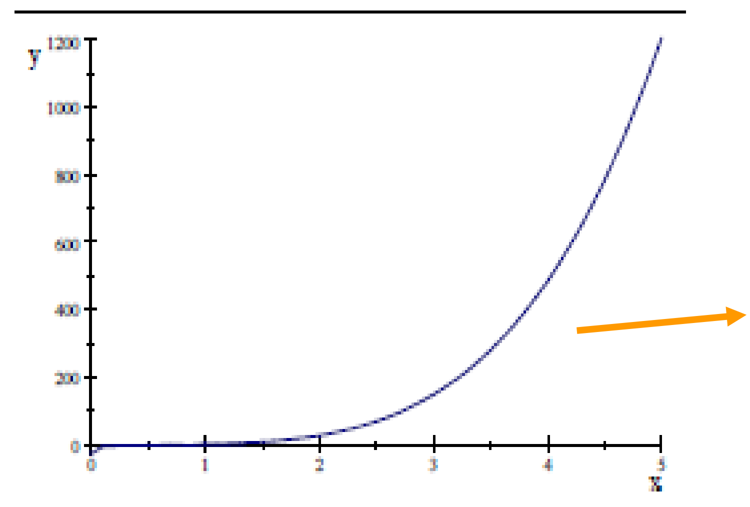
X<<1

$\frac{8}{5}x^5-\frac{4}{7}x^7+\frac{1}{3}x^9$

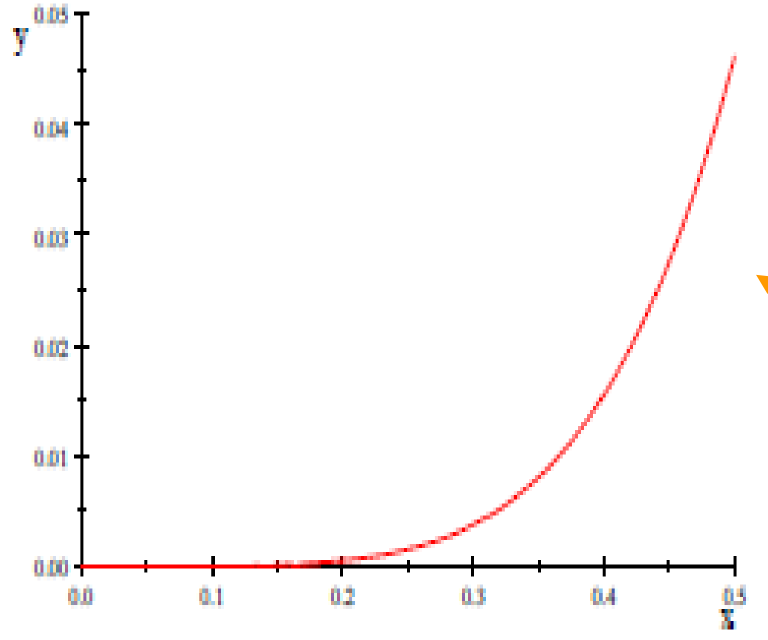


X>>1

$2x^4-2x^2+3(\ln(2x)-\frac{7}{12})$

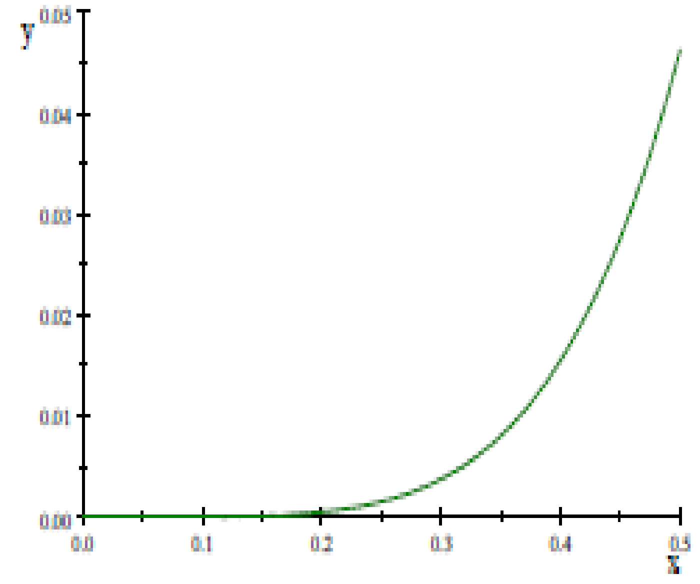


$x(x^2+1)^{1/2}(2x^2-3)+3\ln(x+\sqrt{1+x^2})$



X<<1

$\frac{8}{5}x^5 - \frac{4}{7}x^7 + \frac{1}{3}x^9$



De donde

a) al crecer la masa disminuye el radio

b) para $M > M_0$ no hay solucion real

$\Rightarrow$una enana blanca en equilibrio debe tener

$$M < M_0$$

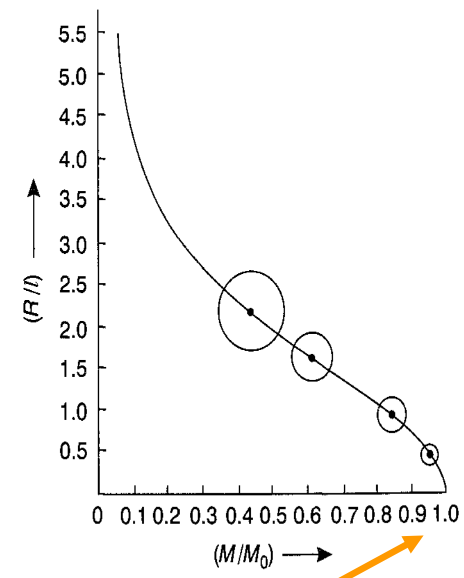


FIG. 8.9. The mass–radius relationship for white dwarfs (after Chandrasekhar, 1939). The masses are expressed in terms of the limiting mass M_0 and the radii in terms of a characteristic length l, which is given by $7.71\mu_e^{-1} \times 10^8$ cm $\simeq 3.86 \times 10^8$ cm.

Chandrasekhar calculo:

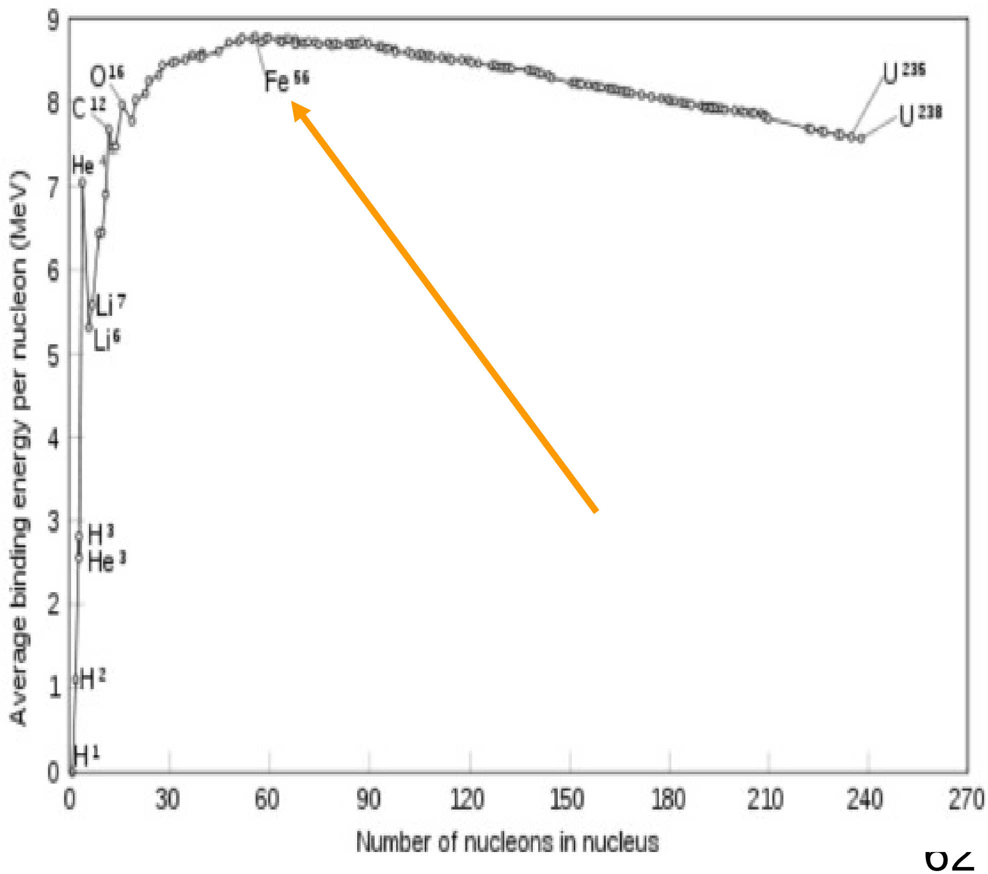
$$M_0 = \frac{5.75}{\mu_e^2} M_{\odot}$$

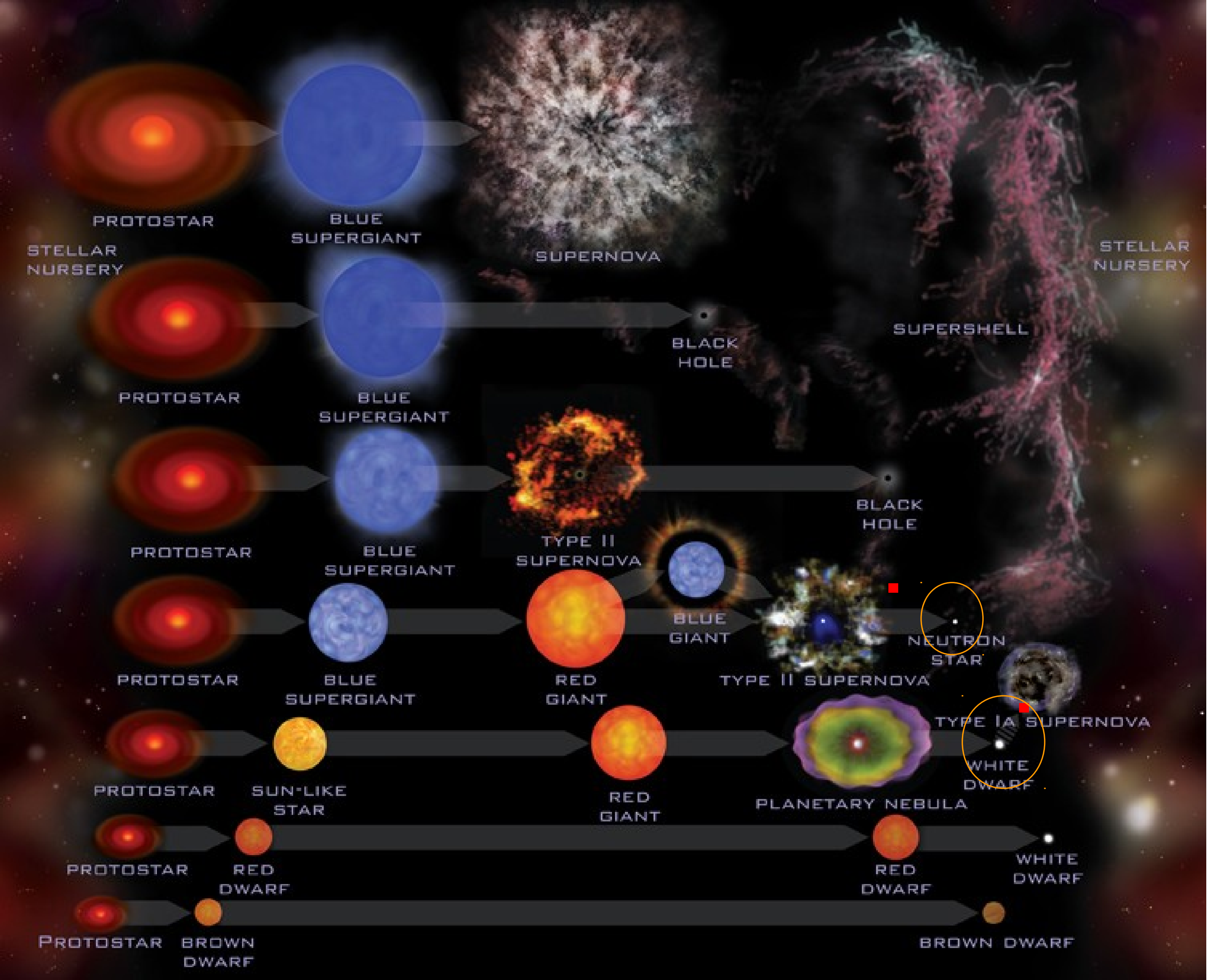
con $\mu_e = M/Nm_H$ grado de ionizacion (usualmente del orden de 2)$\Rightarrow$

$$M_0 \approx 1.44 \cdot M_{\odot}$$

Que es lo que ocurre? porque este limite?

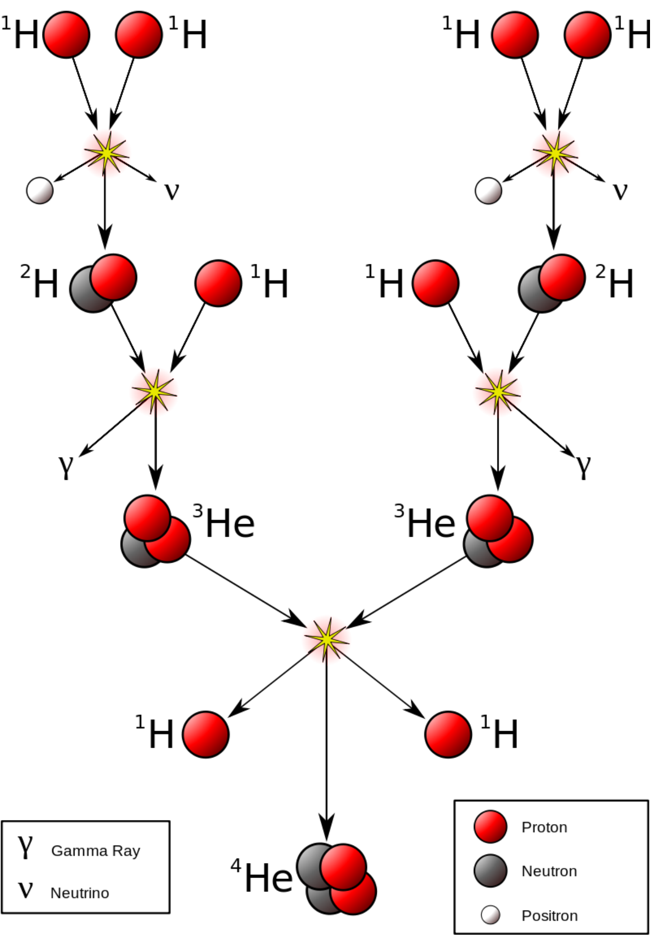
Si la masa supera el valor de Chandrasekhar la presion electronica no es suficiente para soportar la compresion gravitatoria y la estrella sigue colapsando.

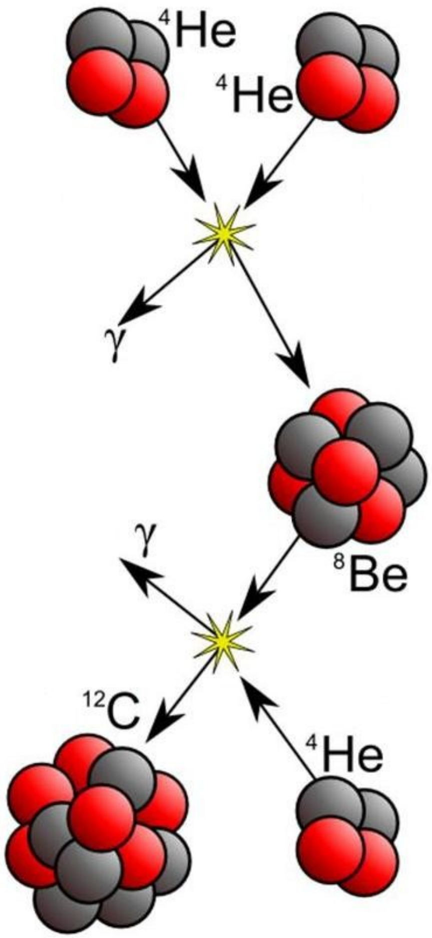


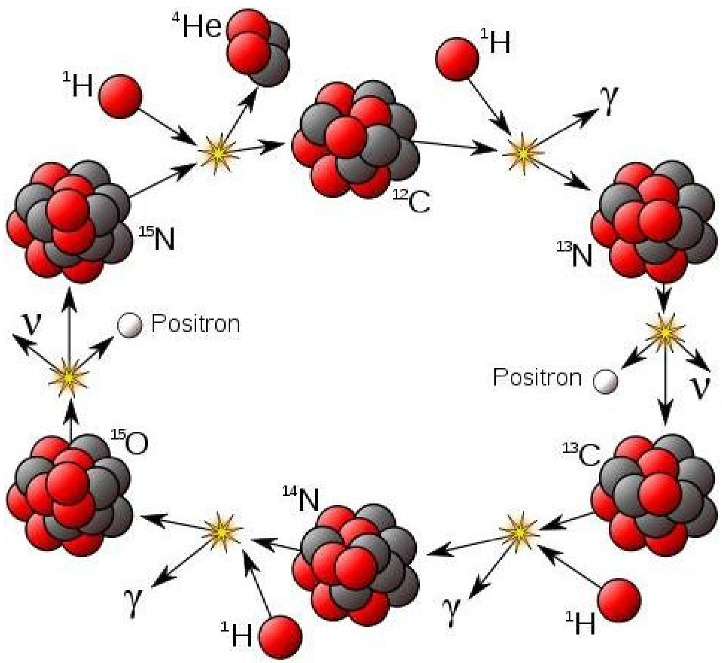


PROTOSTAR
BLUE SUPERGIANT
SUPERNOVA
STELLAR NURSERY
STELLAR NURSERY
PROTOSTAR
BLUE SUPERGIANT
BLACK HOLE
SUPERSHELL
PROTOSTAR
BLUE SUPERGIANT
TYPE II SUPERNOVA
BLACK HOLE
PROTOSTAR
BLUE SUPERGIANT
RED GIANT
BLUE GIANT
TYPE II SUPERNOVA
NEUTRON STAR
TYPE IA SUPERNOVA
PROTOSTAR
SUN-LIKE STAR
RED GIANT
PLANETARY NEBULA
WHITE DWARF
PROTOSTAR
RED DWARF
RED DWARF
WHITE DWARF
PROTOSTAR
BROWN DWARF
BROWN DWARF

A Closer Look at Nucleosynthesis



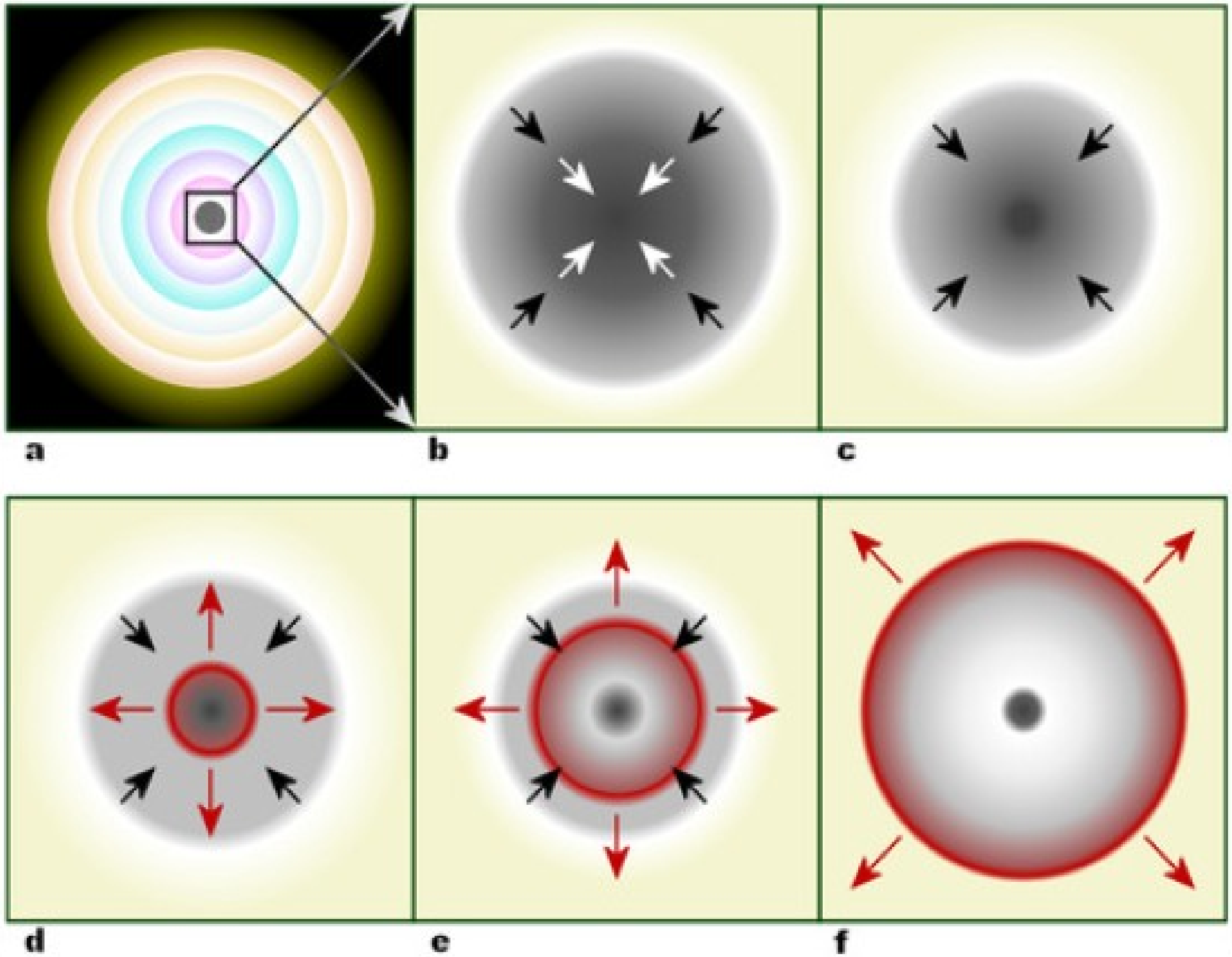




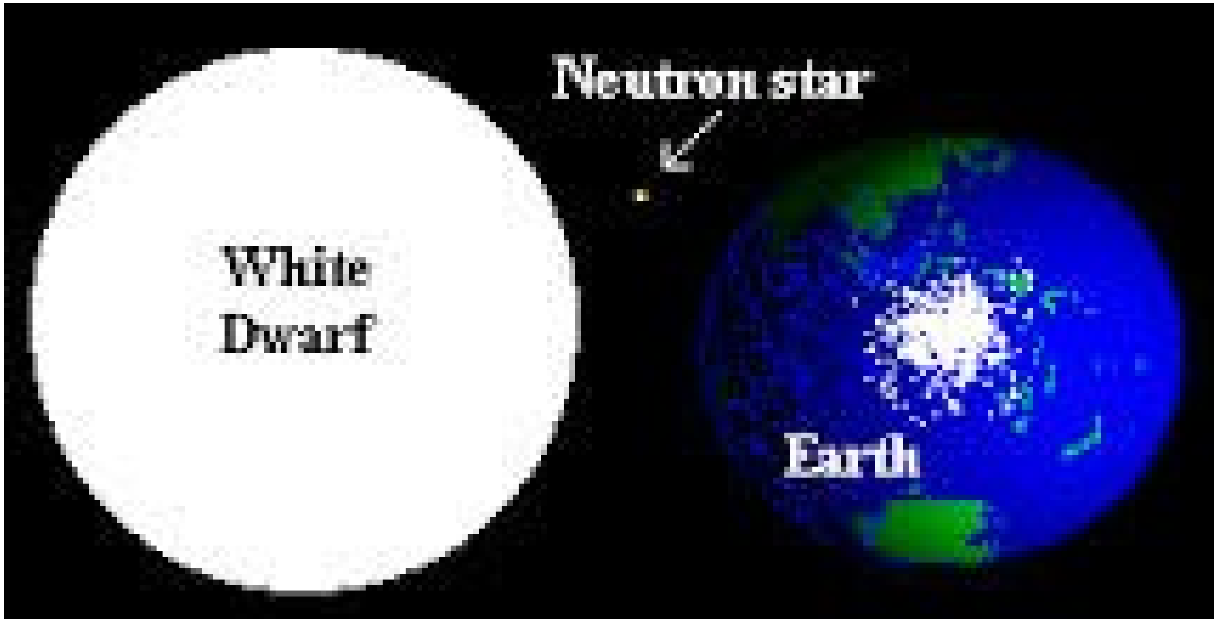
H to He

He to C

CNO cycle



Within a massive, evolved star (a) the onion-layered shells of elements undergo fusion, forming a nickel-iron core (b) that reaches Chandrasekhar-mass and starts to collapse. The inner part of the core is compressed into neutrons (c), causing infalling material to bounce (d) and form an outward-propagating shock front (red). The shock starts to stall (e), but it is re-invigorated by neutrino interaction. The surrounding material is blasted away (f), leaving only a degenerate remnant.


Neutron star
White
Dwarf
Earth

La materia dentro de la estrella de neutrones

Un neutrón decae en unos 15 minutos en

$$n \rightarrow p + e^{-} + \overline{\nu}_e$$

En el seno de la estrella de neutrones, a altas densidades, hay bloqueo debido al Ppo. De Pauli

Consideremos un sistema compuesto por gases ideales (n.p,e)

Consideremos al neutrón mas energético presente.
Los neutrones no decaen si

$$\epsilon_F(n) < \epsilon_F(p) + \epsilon_F(e)$$

Y no decaerán si

$$\epsilon_F(n) > \epsilon_F(p) + \epsilon_F(e)$$

La condición de equilibrio

$$\epsilon_F(n) = \epsilon_F(p) + \epsilon_F(e)$$

En equilibrio hay un balance

$$n \rightarrow p + e^{-} + \overline{\nu_e} \qquad\qquad p + e^{-} \rightarrow n + \overline{\nu_e}$$

Tomando en cuenta que

$$p_F = \left[\frac{3n}{8\pi}\right]^{\frac{1}{3}} h \qquad \text{n es la densidad}$$

Para densidades del orden de ρ_{nuc}

Consideramos los neutrones y protones no relativistas

$$\epsilon_F(n) \approx m_n c^2 + \frac{p_F(n)^2}{2m_n} \qquad\qquad \epsilon_F(p) \approx m_p c^2 + \frac{p_F(p)^2}{2m_p}$$

Los electrones son mas livianos y entonces $\epsilon_F(e) \approx p_F(e)c$

co
n

$$p_F = \left[\frac{3n}{8\pi}\right]^{\frac{1}{3}} h \qquad \epsilon_F(n) \approx m_n c^2 + \frac{p_F(n)^2}{2m_n} \qquad \epsilon_F(p) \approx m_p c^2 + \frac{p_F(p)^2}{2m_p}$$

$$\epsilon_F(e) \approx p_F(e) c$$

El balance se escribe

$$m_n c^2 + \frac{p_F(n)^2}{2m_n} \approx m_p c^2 + \frac{p_F(p)^2}{2m_p} + p_F(e) c$$

$$m_n c^2 - m_p c^2 \approx \left[\frac{3n_p}{8\pi}\right]^{\frac{2}{3}} \frac{h^2}{2m_p} + \left[\frac{3n_p}{8\pi}\right]^{\frac{1}{3}} hc - \left[\frac{3n_n}{8\pi}\right]^{\frac{2}{3}} \frac{h^2}{2m_n}$$

Como

$$m_n - m_p \approx 1.3\, MeV/c^2$$

Densidad típica $\rho = 2 \, x \, 10^{17} \, kg \, m^{-3} \quad \Rightarrow$

$$\begin{cases} n_n \approx 1 \, x \, 10^{44} \, m^{-3} \\ n_e = n_p \approx n_n / 200 \end{cases}$$

Estrella de Neutrones

Neutron Star

Neutron stars are the collapsed cores of some massive stars (8 → 20-30 solar masses)

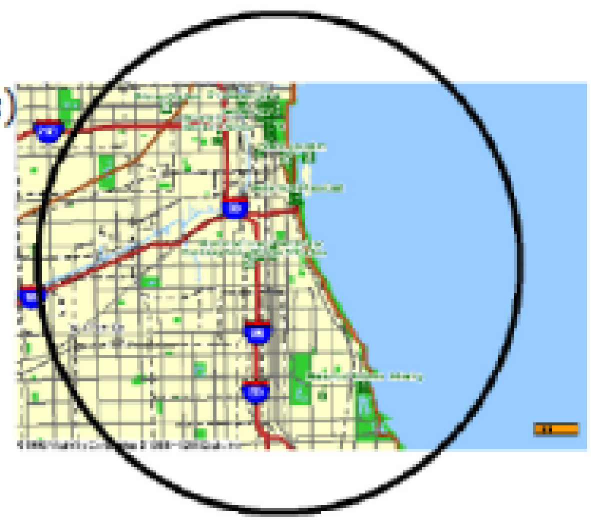
1 solar mass < NS Mass < 3 solar masses

NS radius ≅ 10^{-5} Solar Radius ≅ 10 Km

NS Density ≅ 10^{15} g/cm³ (water ≅ 1.0 gr/cm³)

Electrically Neutral

Neutron star vs. Chicago

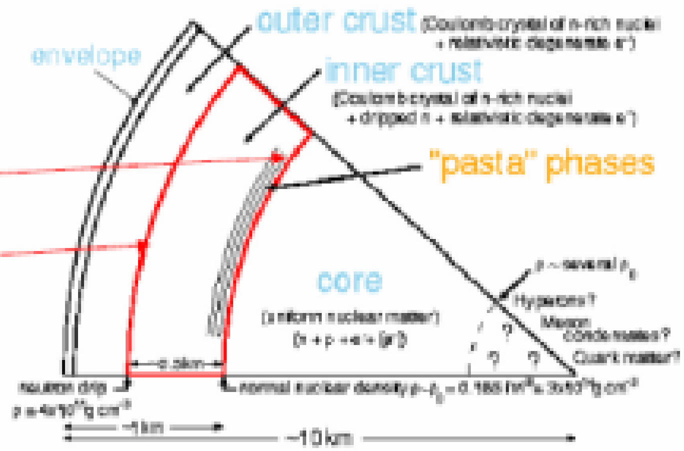


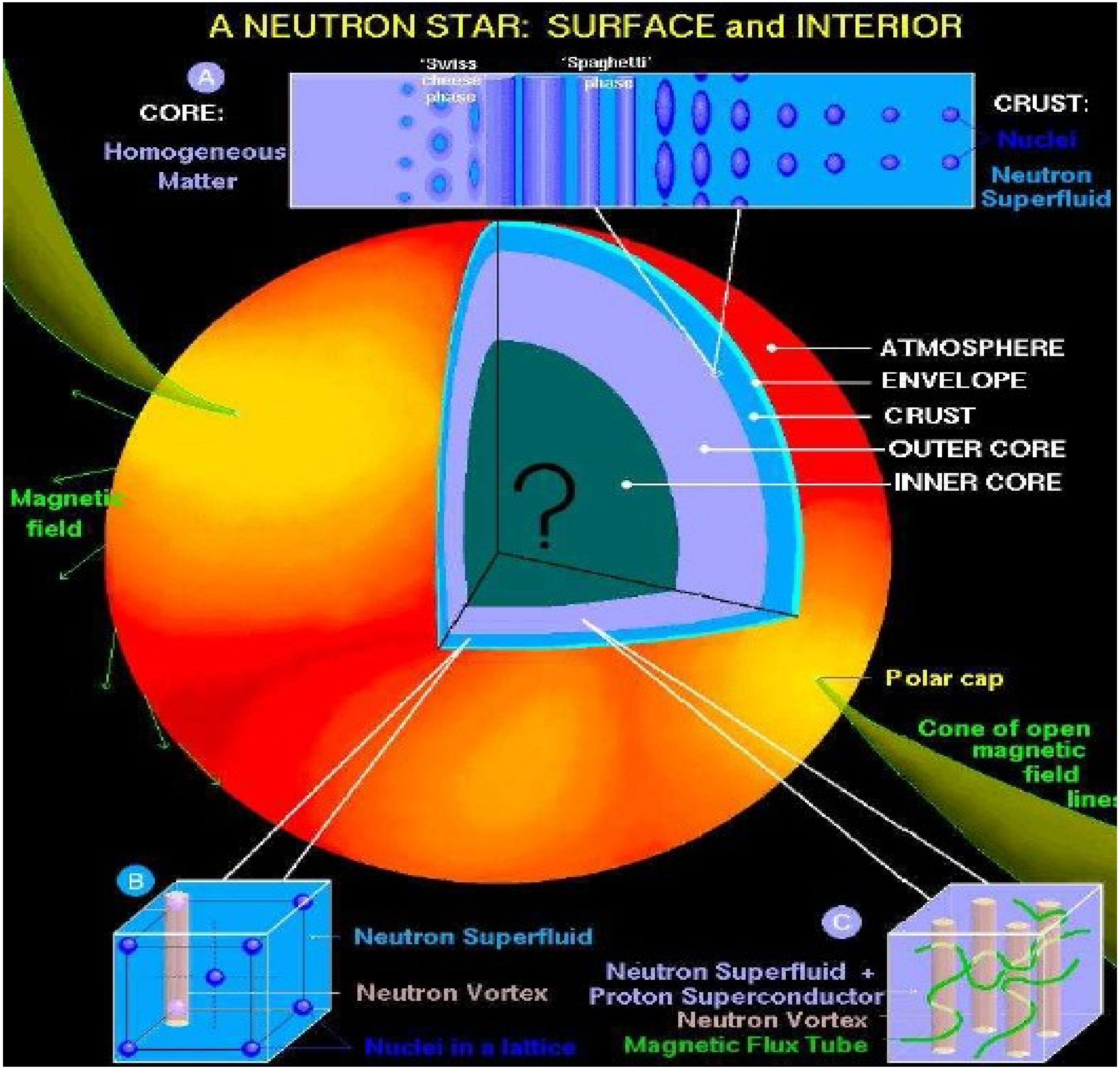
Mass=1.4 M_{sun}, Radius=10 km
Spin rate up to 38,000 rpm
Density~10^{14} g/cc, Magnetic field~10^{12} Gauss

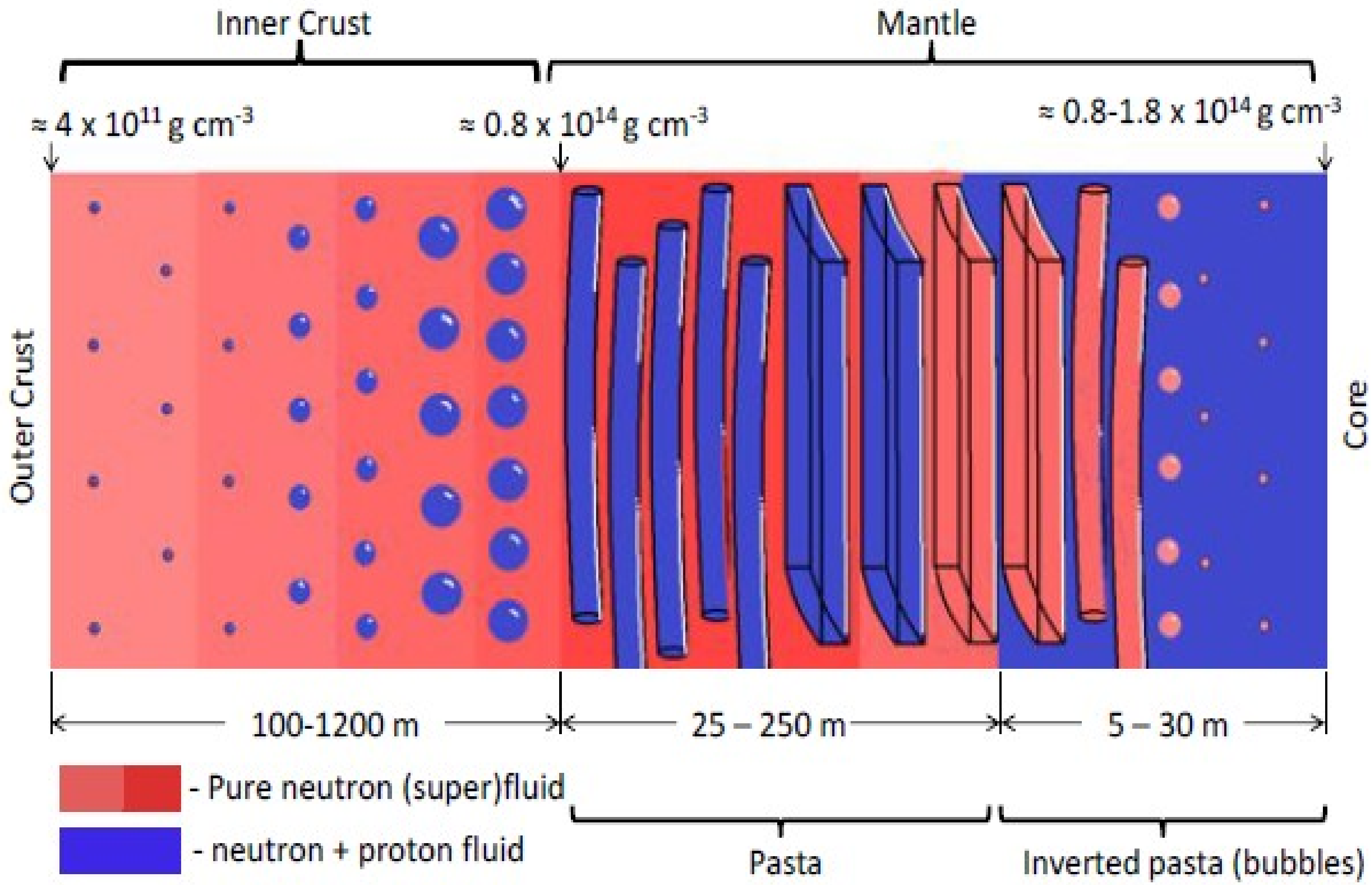
This magnitudes are depth dependent

$\rho_0 \sim \rho \cong 3\times10^{14}$ gr/cm³

$\rho \cong 4\times10^{11}$ gr/cm³




A NEUTRON STAR: SURFACE and INTERIOR
A
'Swiss cheese' phase
'Spaghetti' phase
CORE:
Homogeneous Matter
CRUST:
Nuclei
Neutron Superfluid
ATMOSPHERE
ENVELOPE
CRUST
OUTER CORE
INNER CORE
?
Magnetic field
Polar cap
Cone of open magnetic field lines
B
Neutron Superfluid
Neutron Vortex
Nuclei in a lattice
C
Neutron Superfluid + Proton Superconductor
Neutron Vortex
Magnetic Flux Tube



Inner Crust
Mantle
≈ 4 x 10¹¹ g cm⁻³
≈ 0.8 x 10¹⁴ g cm⁻³
≈ 0.8-1.8 x 10¹⁴ g cm⁻³
Outer Crust
Core
100-1200 m
25 – 250 m
5 – 30 m
- Pure neutron (super)fluid
- neutron + proton fluid
Pasta
Inverted pasta (bubbles)

CMD Model

$$V_{np}(r) = V_r\left[exp(-\mu_r r)/r - exp(-\mu_r r_c)/r_c\right] - V_a\left[exp(-\mu_a r)/r - exp(-\mu_a r_a)/r_a\right]$$

$$V_{nn}(r) = V_{pp}(r) = V_0\left[exp(-\mu_0 r)/r - exp(-\mu_0 r_c)/r_c\right]$$

Potential

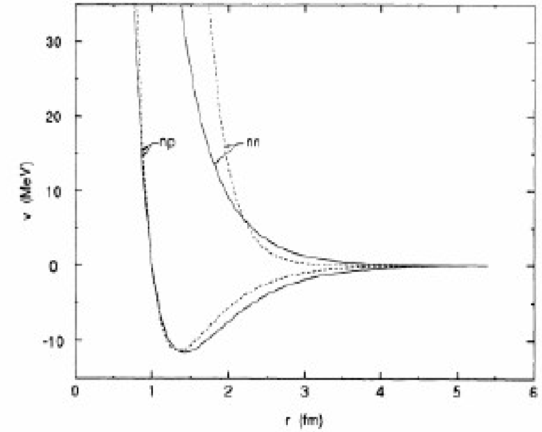
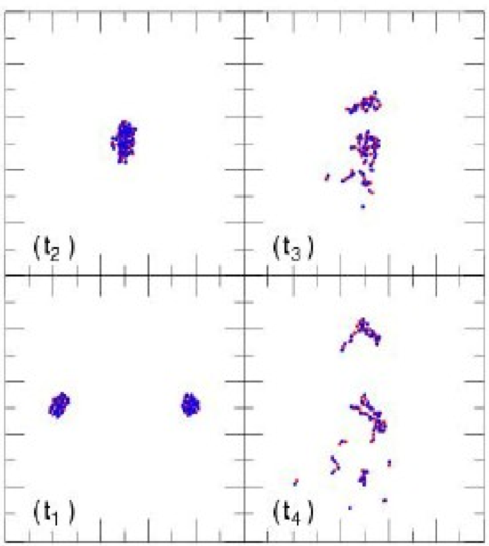
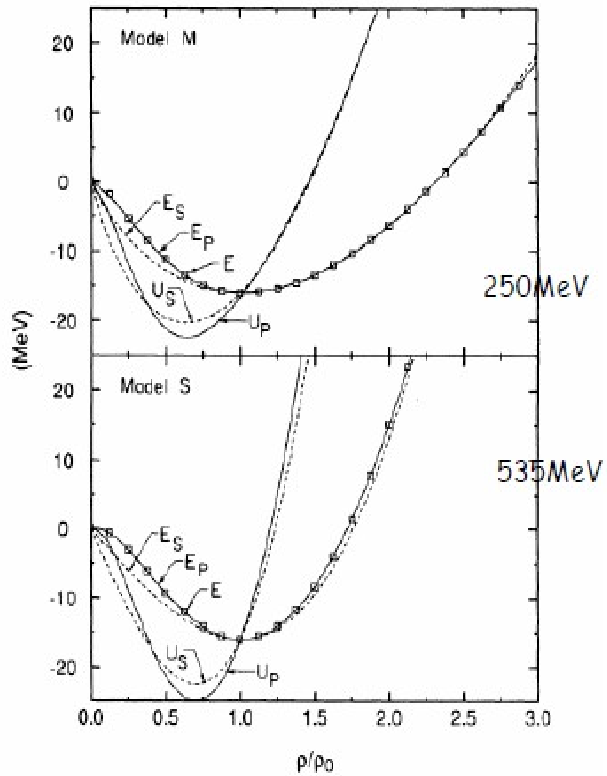


FIG. 1. The interparticle potentials v_{nn} and v_{np} for the M (solid) and S (dashed) models.



Dynamical evolution :

CMD Model



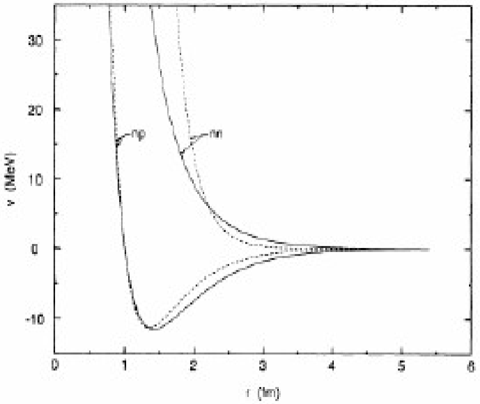


FIG. 1. The interparticle potentials v_{nn} and v_{np} for the *M* (solid) and *S* (dashed) models.

TABLE II. Energies of nuclei (MeV/nucleon) for the *M* and *S* models compared to experimental binding energies.

A	*Z*	E_M	E_S	E_{exp}
2	1	−5.76	−5.67	−1.11
3	1	−7.14	−7.26	−2.83
4	2	−7.91	−7.05	−7.07
5	2	−7.47	−6.22	−5.47
16	8	−10.39	−10.49	−7.98
40	20	−10.44	−10.60	−8.55
90	40	−9.93	−10.25	−8.71
139	57	−9.12	−9.58	−8.38
197	79	−8.38	−8.84	−7.92

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Accuracy of the Vlasov-Nordheim approximation in the classical limit

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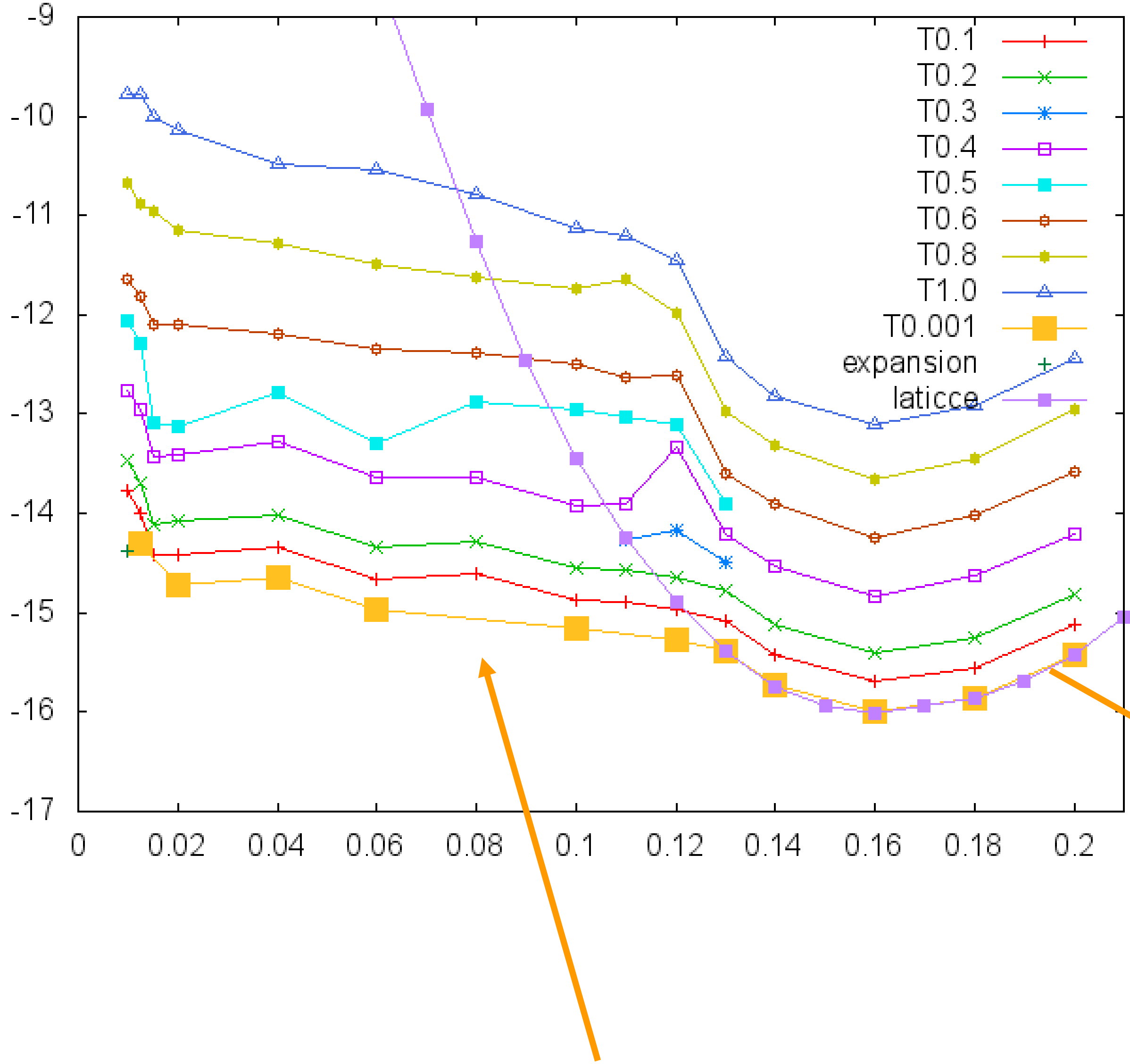
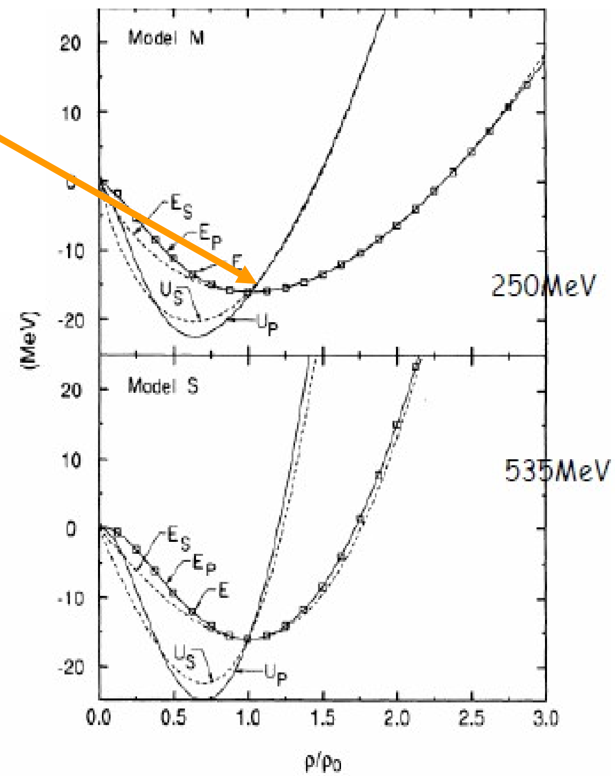
(Received 1 March 1990)



T0.1
T0.2
T0.3
T0.4
T0.5
T0.6
T0.8
T1.0
T0.001
expansion
laticce



Model M
Model S
250MeV
535MeV
(MeV)
ρ/ρ0

CMD Model

Table 1. Comparison of coefficients obtained for different models.

Coefficient	Stiff	Medium	Experimental
C_v	16.1	17.37	15.75
C_s	−11.73	−14.38	−17.8
C_c	−0.197	−0.226	−0.177
C_{sym}	−34.07	−25.08	−23.7

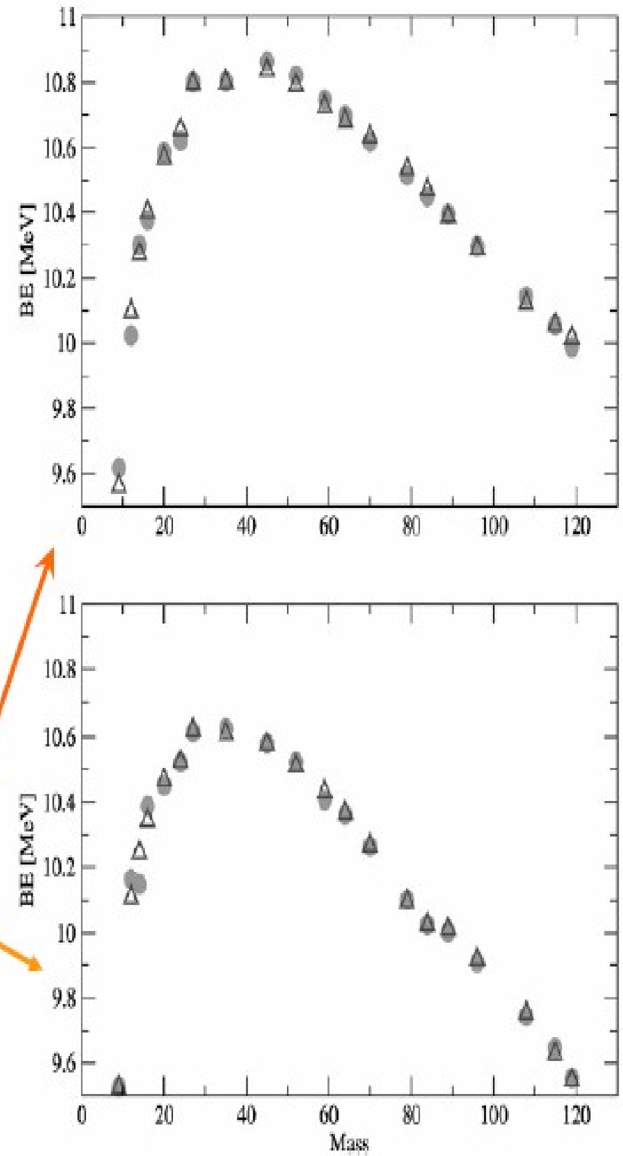
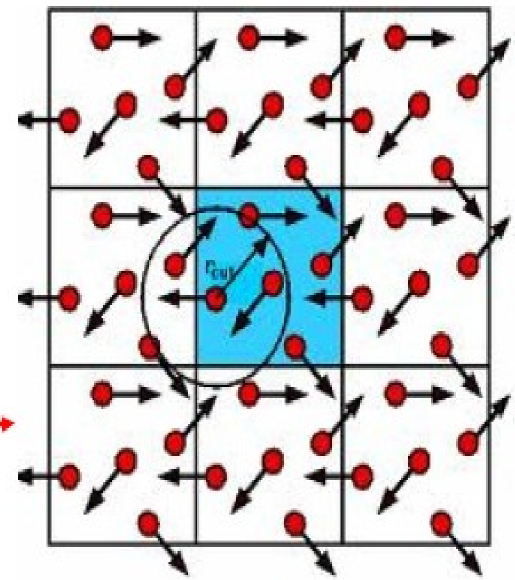


Figure 1. Energies obtained with the mass formula fit (triangles) for the stiff and medium models (top and bottom panels, respectively) together with the corresponding ground states calculated using frictional molecular dynamics (circles).

Our work: use molecular dynamics to study neutron stars



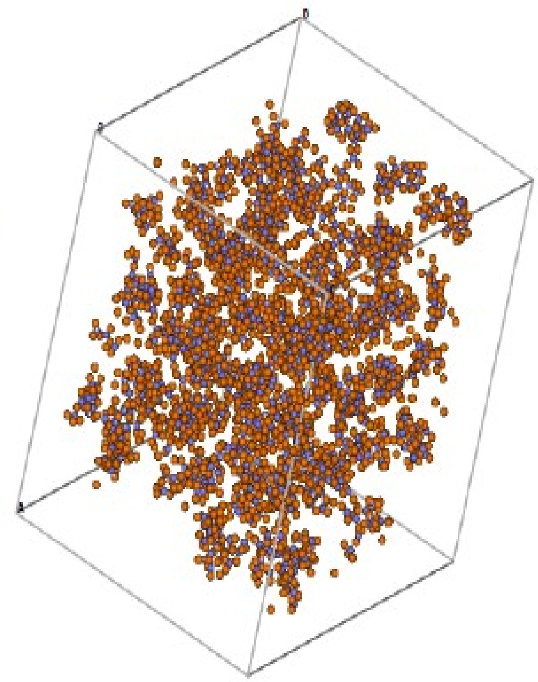
Solve the dynamics as before but...

Simulate the infinte systems using Periodic boundary conditions

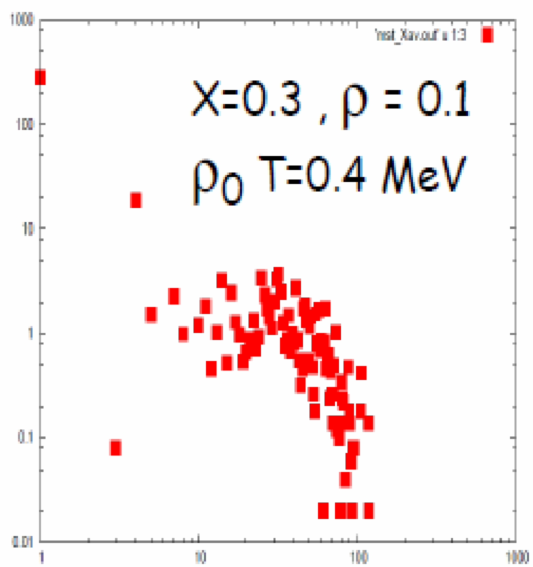
For Coulomb use Thomas Fermi approximation

$$V_C(r) \approx \frac{e^2}{r}$$

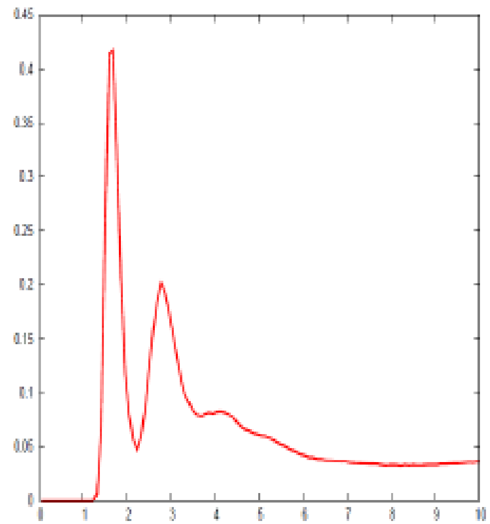
$$V_C^{TF}(r) \approx \frac{e^2}{r}\exp(-r/\lambda)$$



Study:

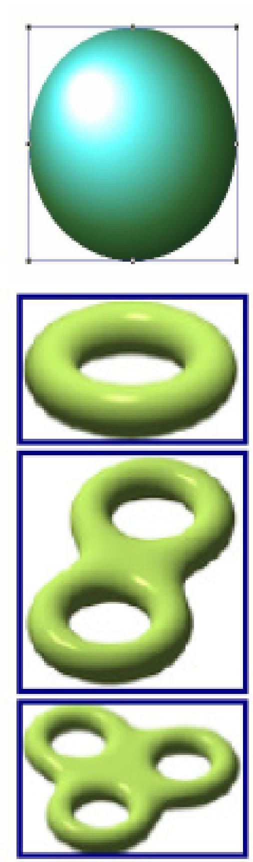


Fragments and their mass distribution



Pair correlation function

Topology (why?)



Euler

1

0

-1

-2

Minkowski functionals

Results: Nuclear Pasta!

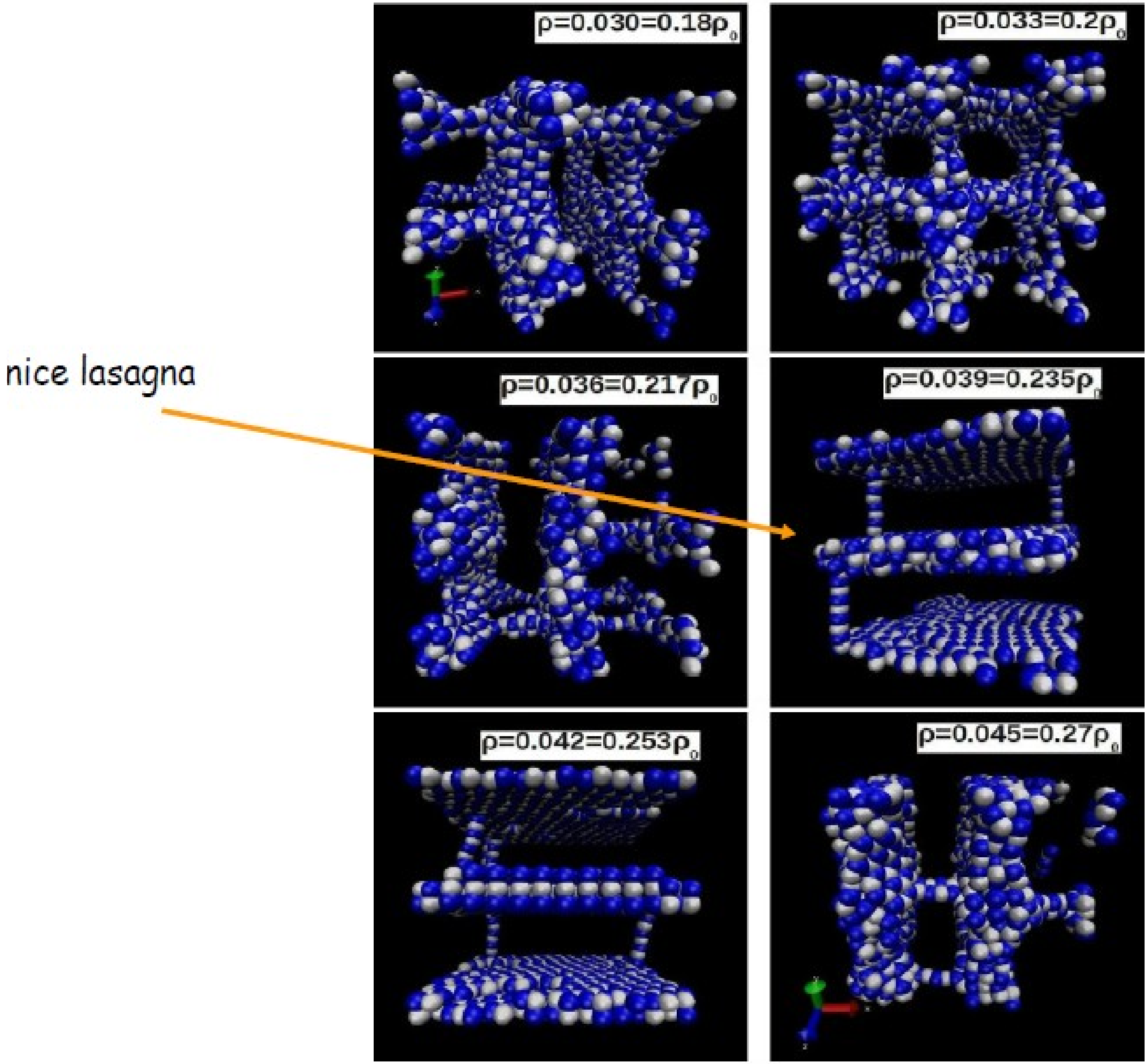
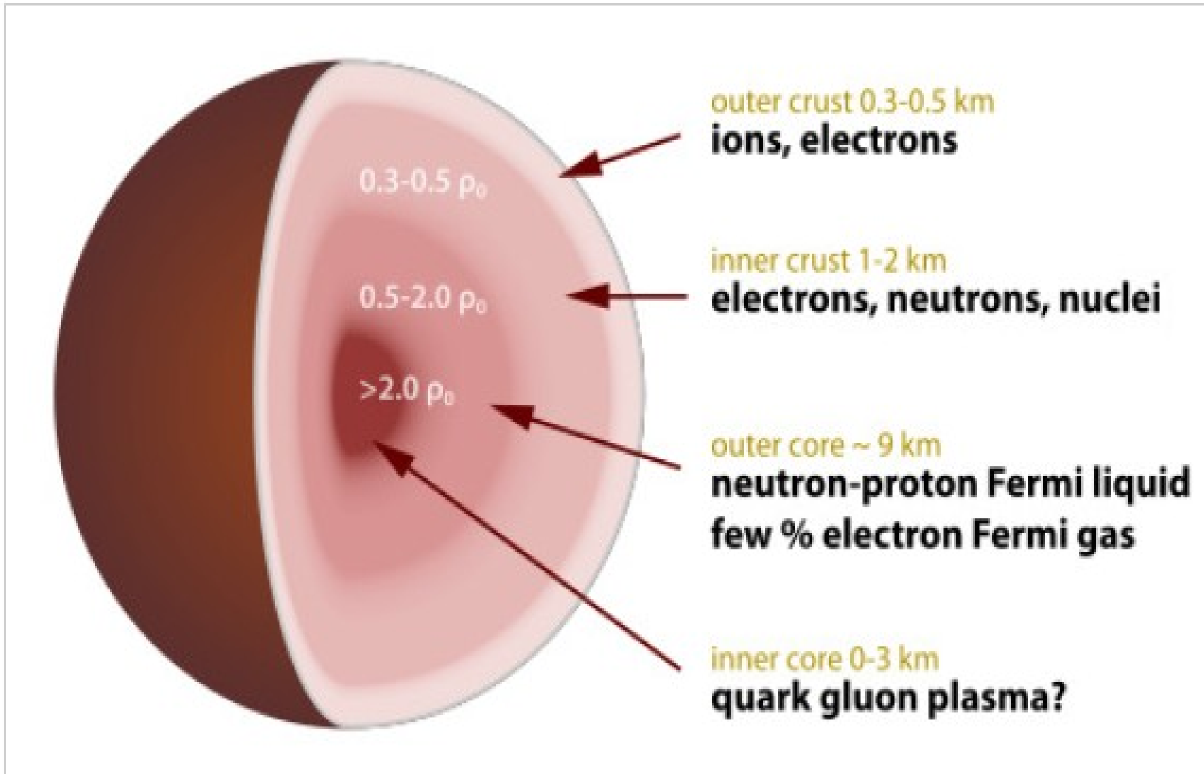


Figure 8. Configuration obtained with the screened Coulomb potential with $x = 0.5$, $T = 0.1\ MeV$ and densities between $\rho = 0.03\ fm^{-3}$ to $0.045\ fm^{-3}$

A **neutron star** is a type of stellar remnant that can result from the gravitational collapse of a massive star during a Type II, Type Ib or Type Ic supernova event. Such stars are composed almost entirely of neutrons, which are subatomic particles without electrical charge and with slightly larger mass than protons. Neutron stars are very hot and are supported against further collapse by quantum degeneracy pressure due to the Pauli exclusion principle. This principle states that no two neutrons (or any other fermionic particles) can occupy the same place and quantum state simultaneously.



Cross-section of neutron star. Densities are in terms of ρ_0 the saturation nuclear matter density, where nucleons begin to touch.

Core-burning nuclear fusion stages for a 25-solar mass star

Process	Main fuel	Main products	25 M☉ star[6]		
			Temperature (Kelvin)	Density (g/cm^3)	Duration
hydrogen burning	hydrogen	helium	7×10^7	10	10^7 years
triple-alpha process	helium	carbon, oxygen	2×10^8	2000	10^6 years
carbon burning process	carbon	Ne, Na, Mg, Al	8×10^8	10^6	10^3 years
neon burning process	neon	O, Mg	1.6×10^9	10^7	3 years
oxygen burning process	oxygen	Si, S, Ar, Ca	1.8×10^9	10^7	0.3 years
silicon burning process	silicon	nickel (decays into iron)	2.5×10^9	10^8	5 days

When the core's mass exceeds the Chandrasekhar limit of about 1.4 solar masses, degeneracy pressure can no longer support it, and catastrophic collapse ensues.[10] The outer part of the core reaches velocities of up to 70,000 km/s (23% of the speed of light) as it collapses toward the center of the star.[11] The rapidly shrinking core heats up, producing high-energy gamma rays that decompose iron nuclei into helium nuclei and free neutrons via photodisintegration. As the core's density increases, it becomes energetically favorable for electrons and protons to merge via inverse beta decay,

producing neutrons and elementary particles called neutrinos. Because neutrinos rarely interact with normal matter they can escape from the core, carrying away energy and further accelerating the collapse, which proceeds over a timescale of milliseconds. As the core detaches from the outer layers of the star, some of these neutrinos are absorbed by the star's outer layers, beginning the supernova explosion.[12]

For Type II supernovae, the collapse is eventually halted by short-range repulsive neutron-neutron interactions, mediated by the strong force, as well as by degeneracy pressure of neutrons, at a density comparable to that of an atomic nucleus. Once collapse stops, the infalling matter rebounds, producing a shock wave that propagates outward. The energy from this shock dissociates heavy elements within the core. This reduces the energy of the shock, which can stall the explosion within the outer core.[13]

$$\sigma_{\text{total}} = \sigma_{\text{free neutron}} \times S(q) \tag{4}$$

The neutrino scattering cross section of a free neutron is given by:

$$\sigma_{\text{free neutron}} = \frac{G_F^2 E_\nu^2}{6\pi} \tag{5}$$

with G_F the Fermi coupling and E_ν the energy of the neutrino. With this in mind, the cross section is:

$$\sigma_{\text{total}} = \frac{G_F^2 E_\nu^2}{6\pi} S(q) \tag{6}$$

The structure factor $S(q)$ is the Fourier transform of the pair distribution function:

$$S_{nn}(q) = 1 + \rho \int_V \mathrm{d}r\, \mathrm{e}^{-i\,q\,r} \left[g_{nn}(r) - 1\right] \tag{8}$$

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The neutrino opacity of neutron rich matter

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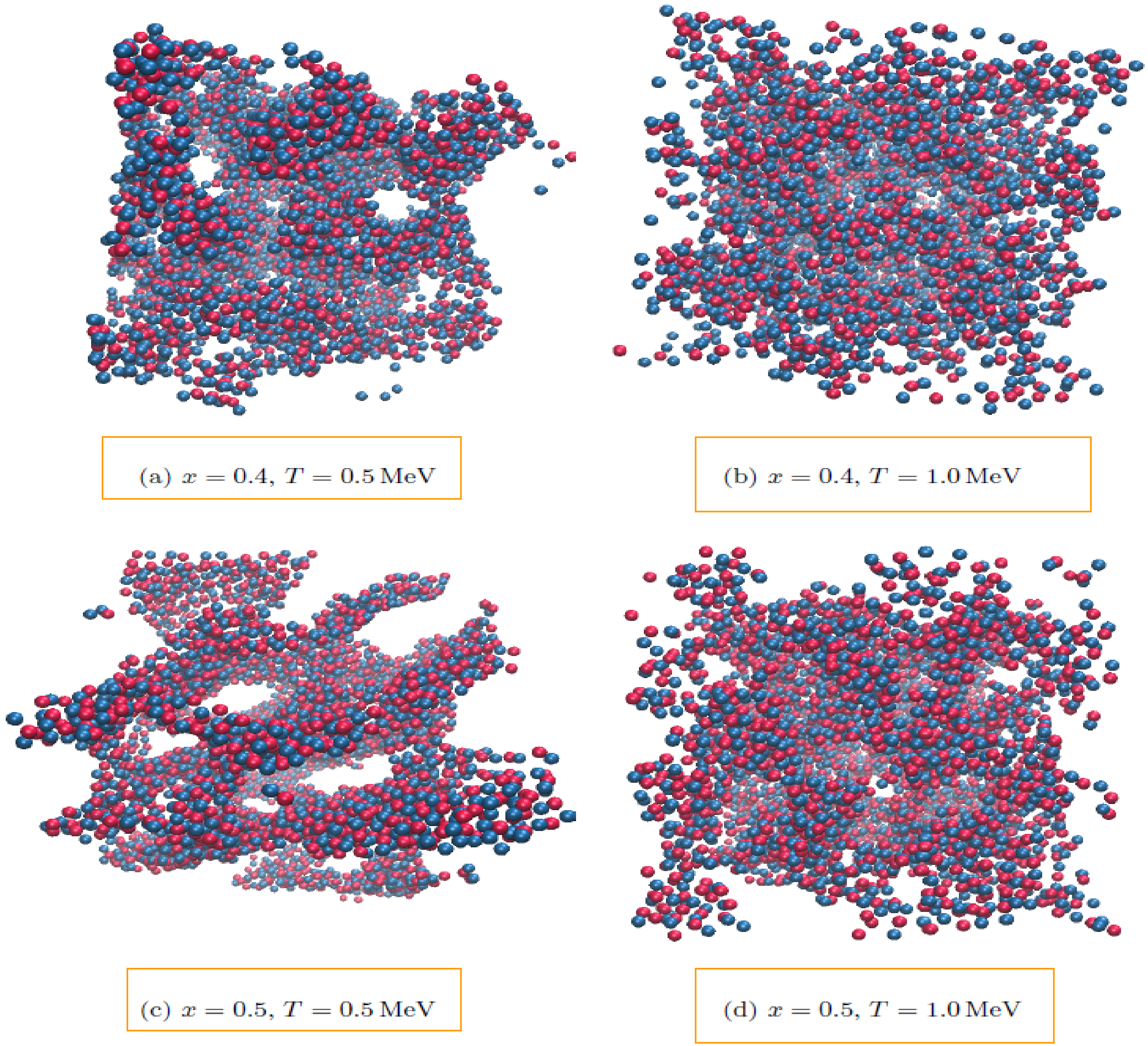


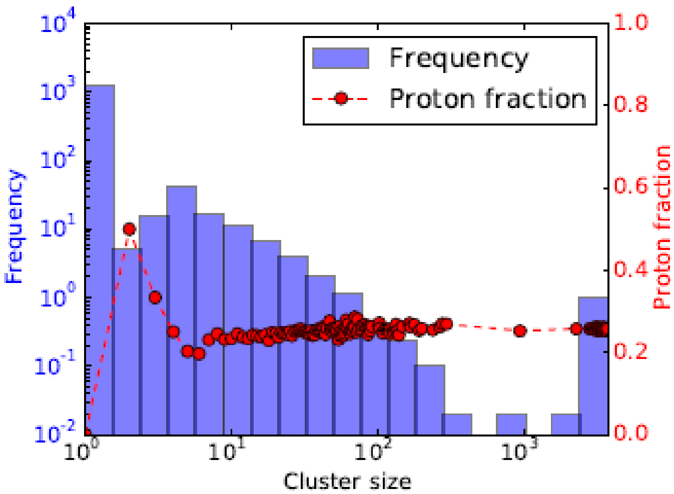
(a) $x = 0.4$, $T = 0.5\,\mathrm{MeV}$

(b) $x = 0.4$, $T = 1.0\,\mathrm{MeV}$

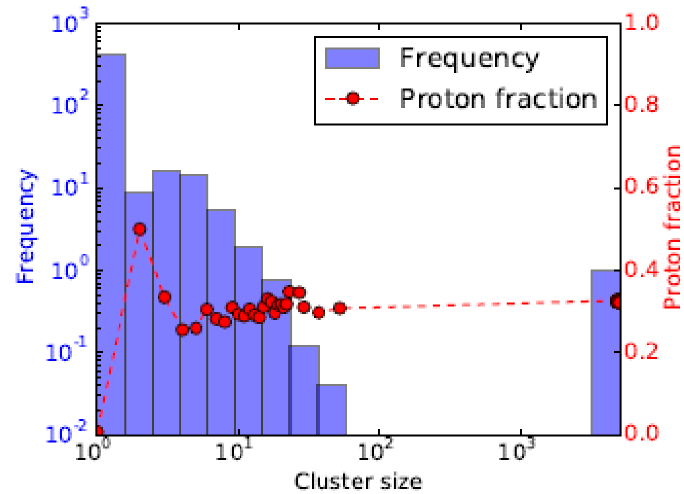
(c) $x = 0.5$, $T = 0.5\,\mathrm{MeV}$

(d) $x = 0.5$, $T = 1.0\,\mathrm{MeV}$

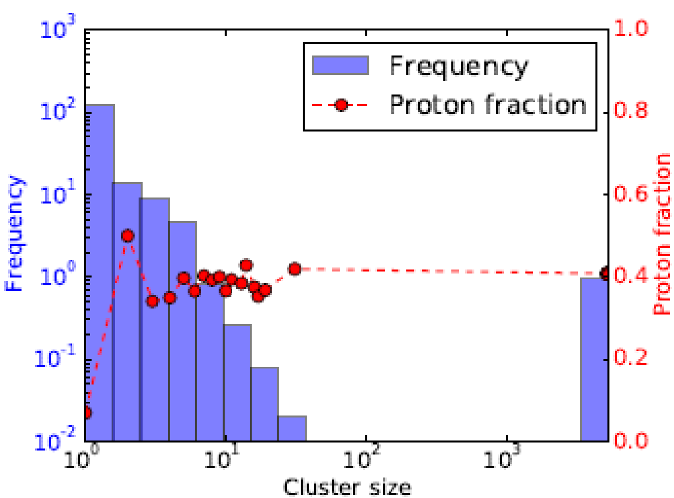
Figure 3: (Color online) Snapshots of a system with density $\rho = 0.04\,\mathrm{fm}^{-3}$ for different values of proton fraction and temperature, generated with VMD [34]. Structures obtained at $T = 0.5\,\mathrm{MeV}$ differ substantially. Nevertheless both show inhomogeneities.



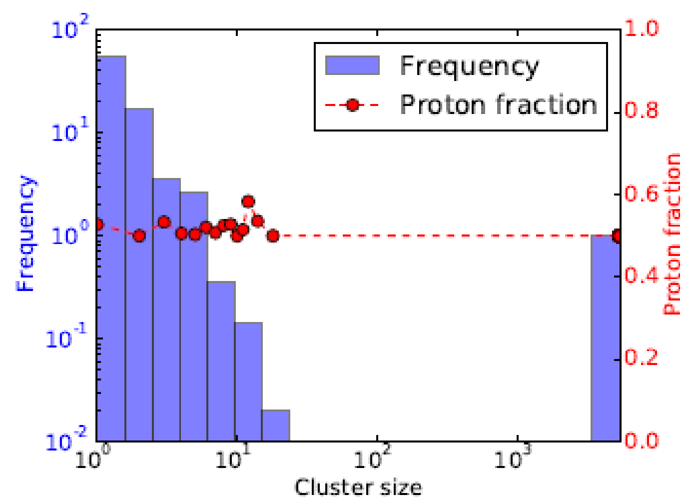
(a) $x = 0.2$



(b) $x = 0.3$



(c) $x = 0.4$



(d) $x = 0.5$

Figure 4: (Color online) Cluster distribution with MSTE algorithm for temperature $T = 2.0\,\mathrm{MeV}$, density $\rho = 0.04\,\mathrm{fm}^{-3}$ and different proton fractions. For the lowest of the studied proton fractions, $x = 0.2$, the large cluster has a larger proton fraction (about 30% higher) and there are many isolated neutrons.

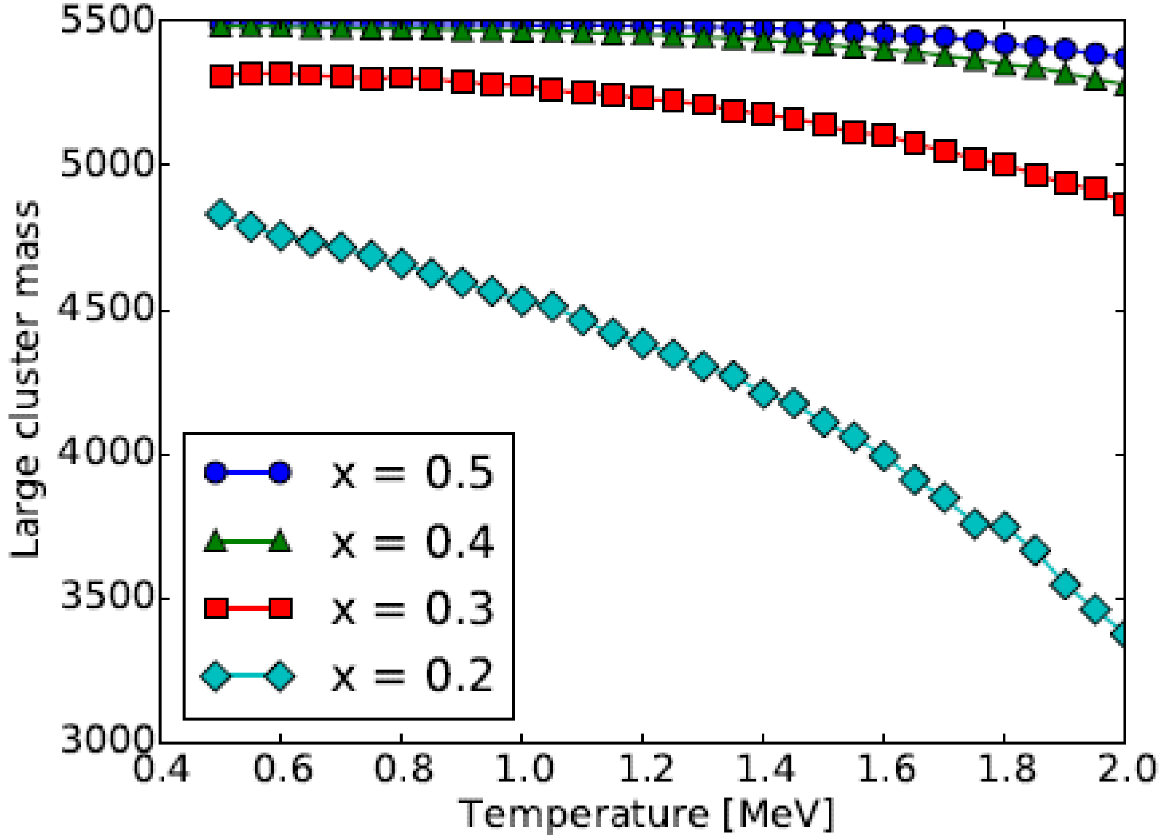
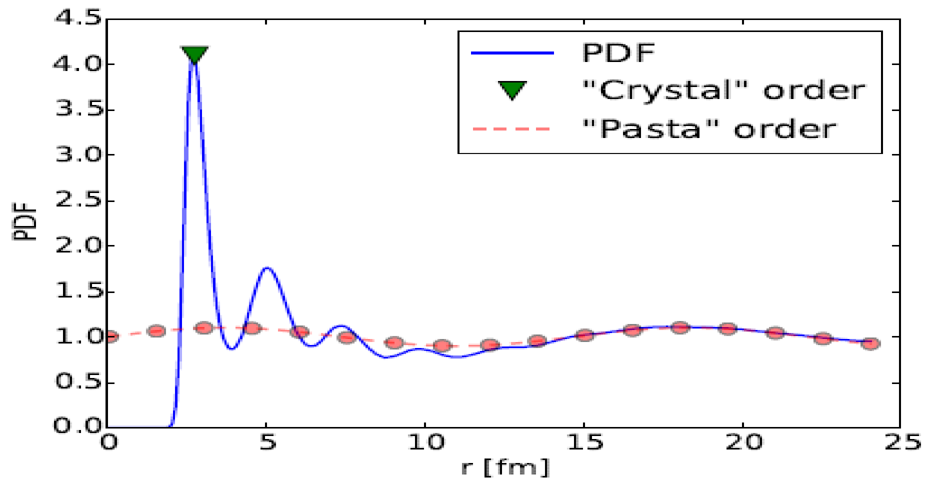
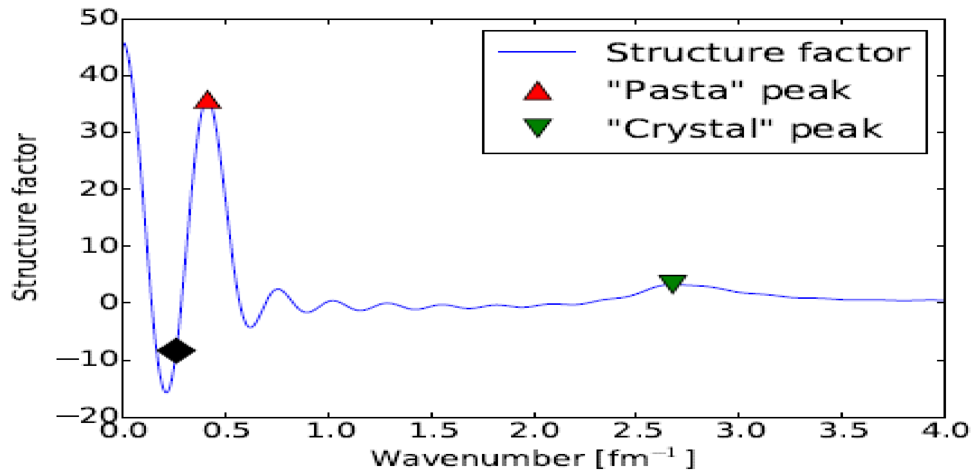


Figure 5: (Color online) Mass of the largest cluster for $\rho = 0.04\,\mathrm{fm}^{-3}$ for different values of x.

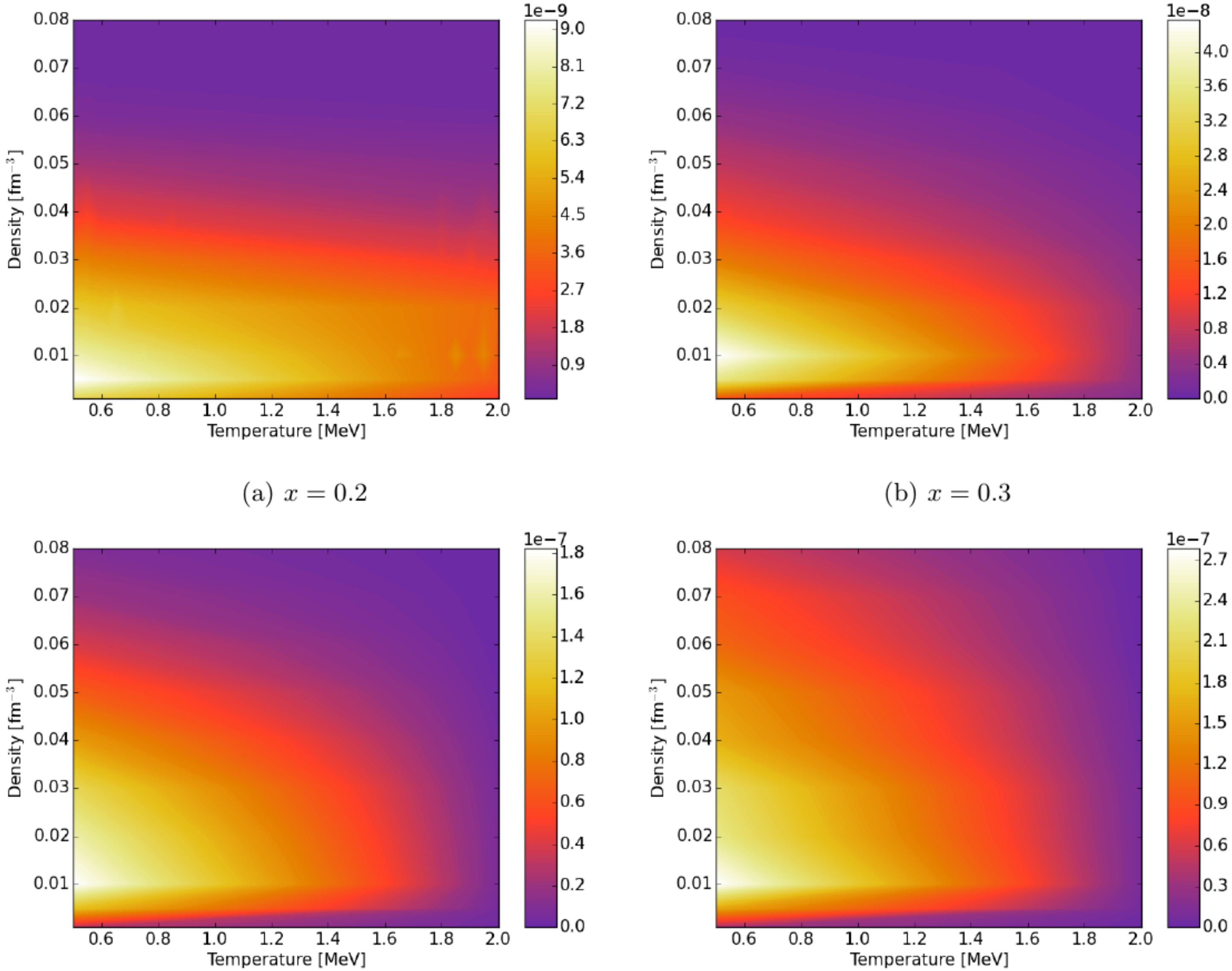


(a) Pair distribution function for neutrons. The first peak due to crystalline structures are marked with ▼, while the very long range order is marked with a dashed line — —.



(b) Structure factor for neutrons. Peak due to the crystalline structure is marked with ▼, and the very long range order is marked with ▲. The minimum momentum transfer that can be measured (due to finite size effects) is marked with ◆.

Figure 6: (Color online) Pair distribution function 6a and structure factor 6b for a system with proton fraction $x = 0.5$, density $\rho = 0.05\,\mathrm{fm}^{-3}$ and temperature $T = 0.5\,\mathrm{MeV}$.



(a) $x = 0.2$

(b) $x = 0.3$

(c) $x = 0.4$

(d) $x = 0.5$

Figure 9: (Color online) Opacity peak height in the very long wavelength for different proton fractions as a function of temperature and density. It can be seen that the opacity decreases drastically for $T \gtrsim 0.8\,\text{MeV}$. We also show here that the opacity is affected by the proton fraction, as it can be noted from the scales on the color bar. Also note that in the opacity for $x = 0.2$ and $x = 0.3$, the results are governed by noise.

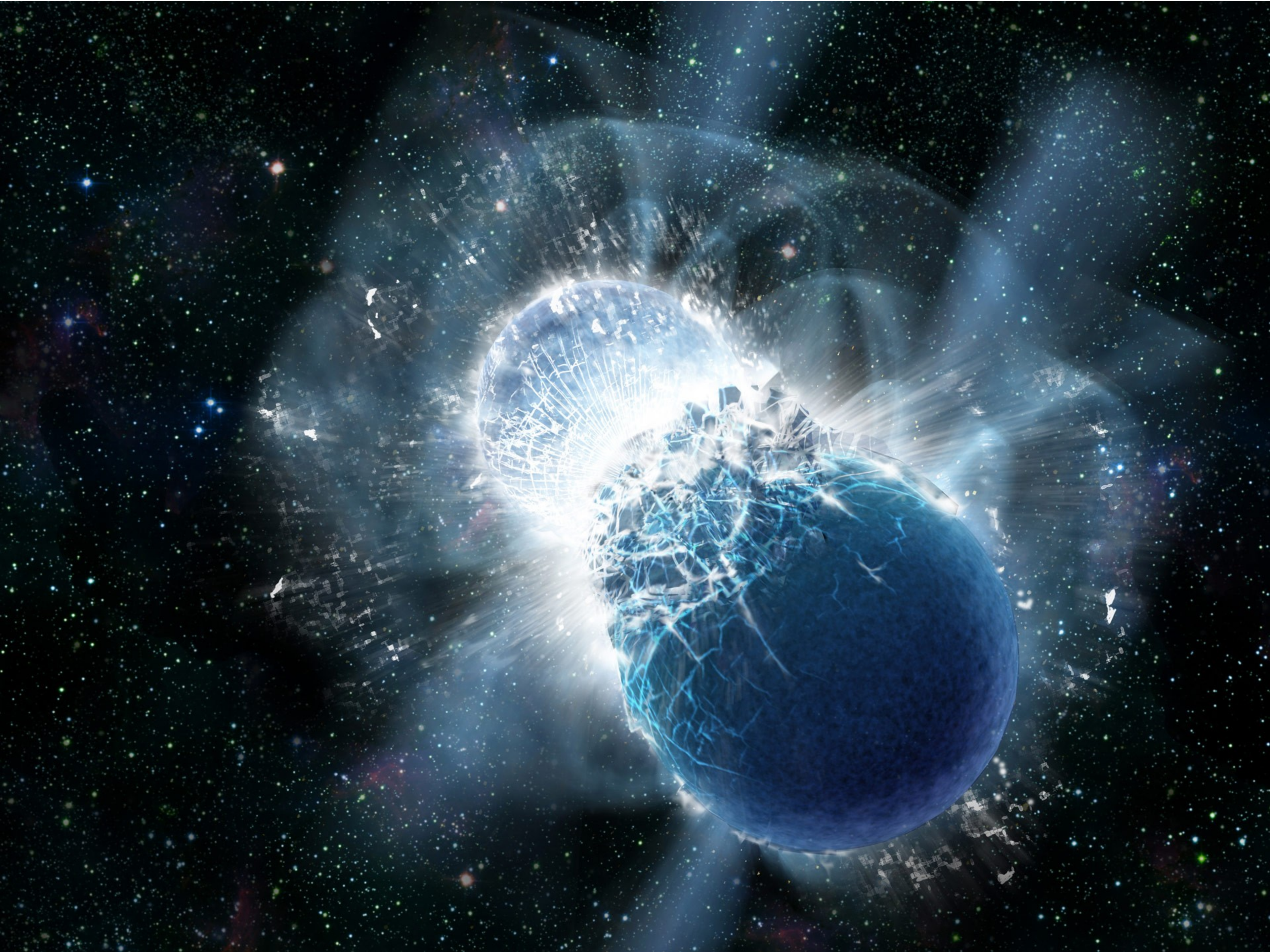


Fig. 1: Where did gold form? For a long time, the cosmic production site of this rare metal and of other very heavy chemical elements has been unknown. New theoretical models now confirm that it could be forged in the merger events of two neutron stars.
Image: Natural gold nuggets from California and Australia; Natural History Museum, London

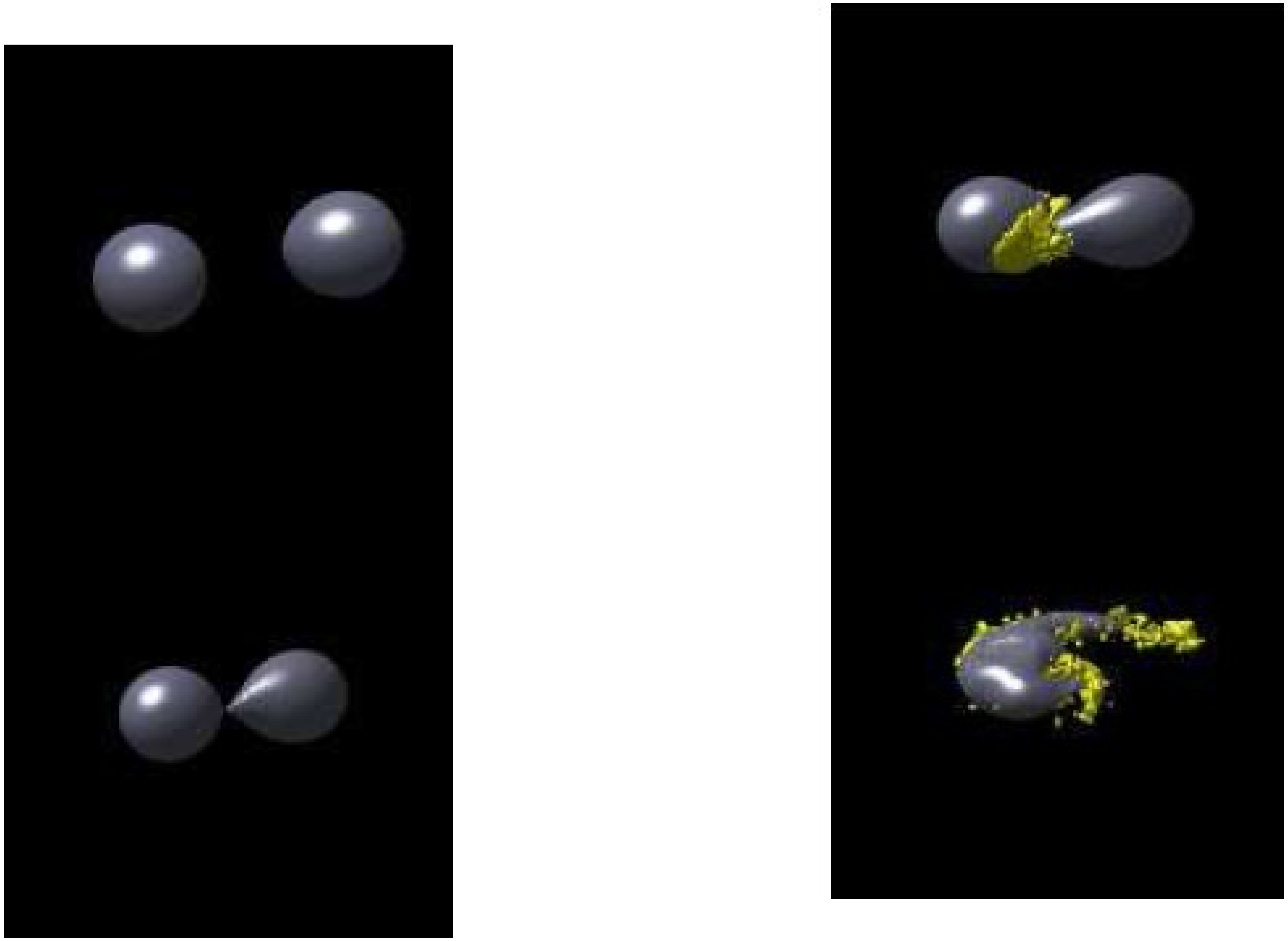


Fig. 2: Various stages of the merger of two neutron stars; the sequence of images covers a period of about one hundredths of a second. Once the stars collide material is squeezed out between the stars and gets stripped off from tidal tails. In the material ejected a multitude of nuclear reactions take place producing heavy elements.

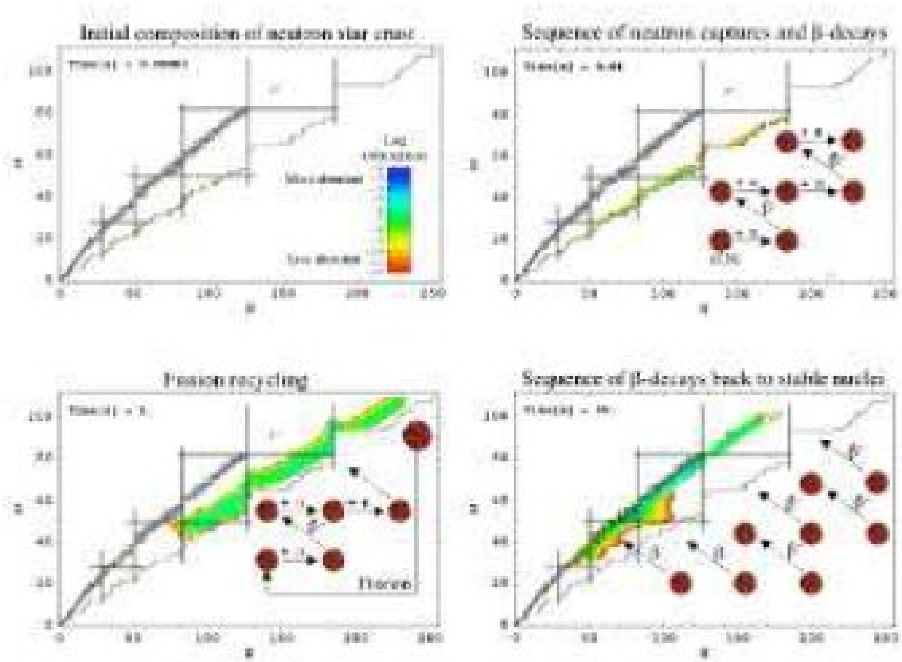


Fig. 3: The stable neutron-rich elements synthesized in the merger event are produced by a complicated sequence of reactions, starting form neutron captures and beta-decays onto light "seed" nuclei, and so producing heavier and heavier neutron-rich nuclei. When reaching the heaviest neutron- rich nuclei, fission reactions recycle the material towards lighter ones. These nuclear processes takes place forabout a second - as long as neutrons are available. Finally when all neutrons are captured, the nuclei decay back to stable nuclei. The principle reaction chains are shown here, where the colour coding indicates the abundance of the elements. The whole evolution is shown in this movie: divx (2.7 MB) or mp4 (1MB).

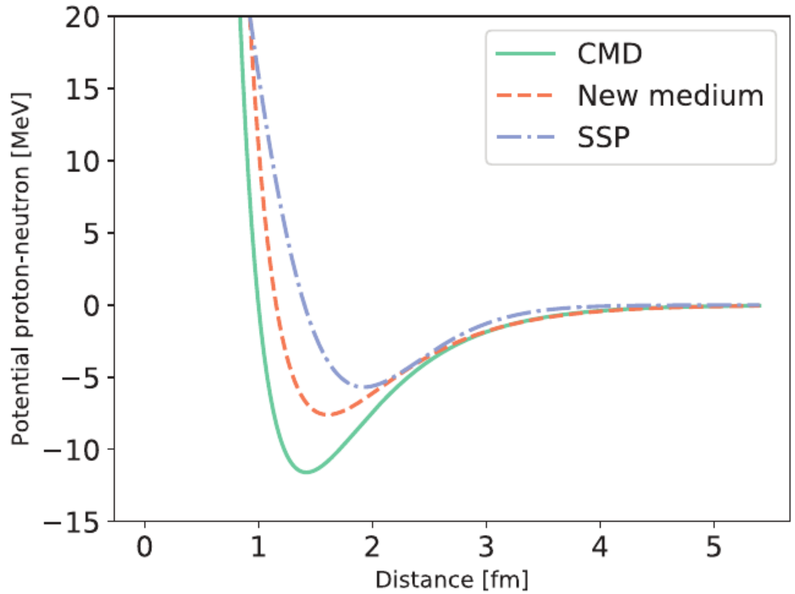


FIG. 1. Potential energy of the proton-neutron interaction of different models: SSP, CMD, and new medium.

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Dynamics of fragment formation in neutron-rich matter

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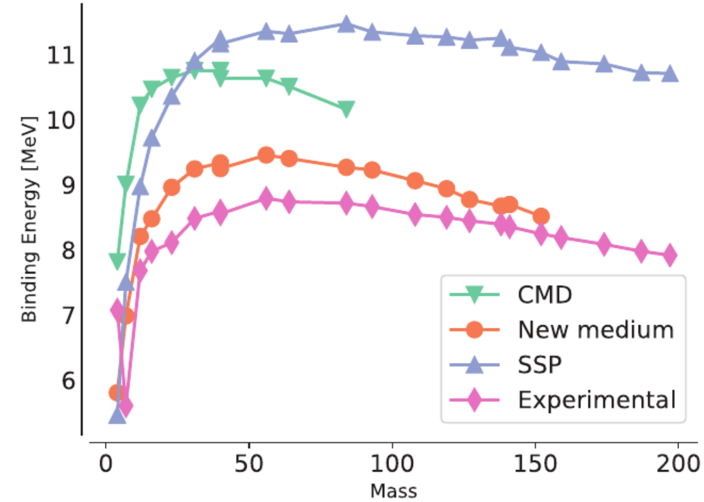


FIG. 2. Binding energies of ground-state nuclei obtained with CMD, SSP, and new medium models. Note that the new medium model yields results much closer to the experimental ones.

IV. EXPANSION

In order to expand the neutron-rich matter that simulates an infinite system with periodic boundary conditions, we follow the *microscopic big bang* method, as explained by Holian and Grady in Ref. [52] and used for the expansion of a infinite system [53]. It consists of an expansion of the simulation box at a constant isotropic rate:

$$L(t) = L_0\,(1 + \dot{\eta}\,t), \tag{2}$$

where L is the length of the simulation box in every direction and L_0 is the initial length. With only this box resizing, the system would expand dynamically. To simulate an expansion, we need to also give the particles an extra radial velocity that matches that of the box in the edges of the simulation:

$$\mathbf{v} = \mathbf{v_0} + \dot{\eta}\,\mathbf{r_0}. \tag{3}$$

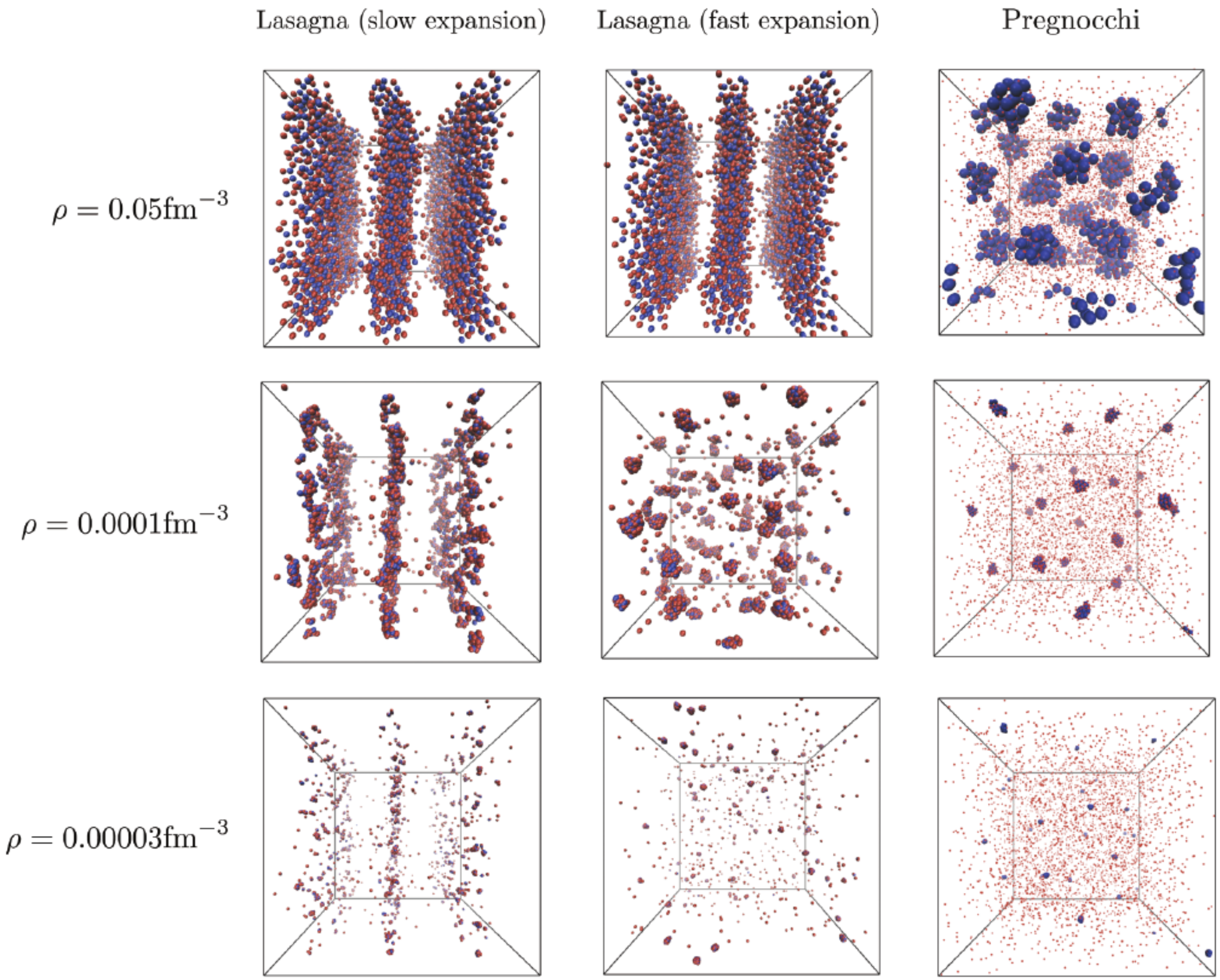


FIG. 9. Three different expansions of neutron star matter: lasagna (fast expansion): $x = 0.4$, $\dot{\eta} = 0.01\ c/\text{fm}$, $T = 0.8$ MeV; lasagna (slow expansion): $x = 0.4$, $\dot{\eta} = 0.0001\ c/\text{fm}$, $T = 0.8$ MeV; pregnocchi: $x = 0.1$, $\dot{\eta} = 0.0001\ c/\text{fm}$, $T = 0.1$ MeV.

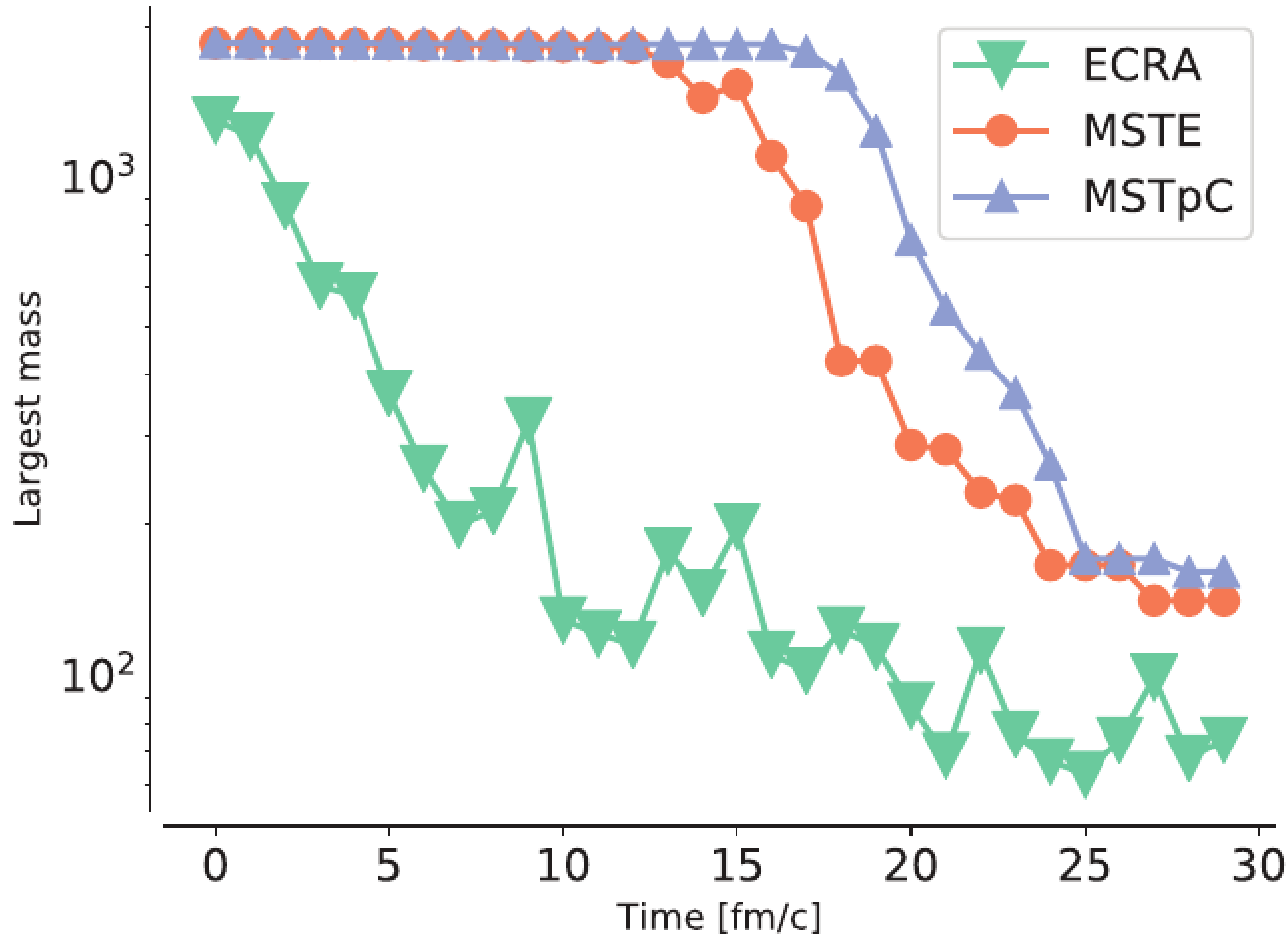
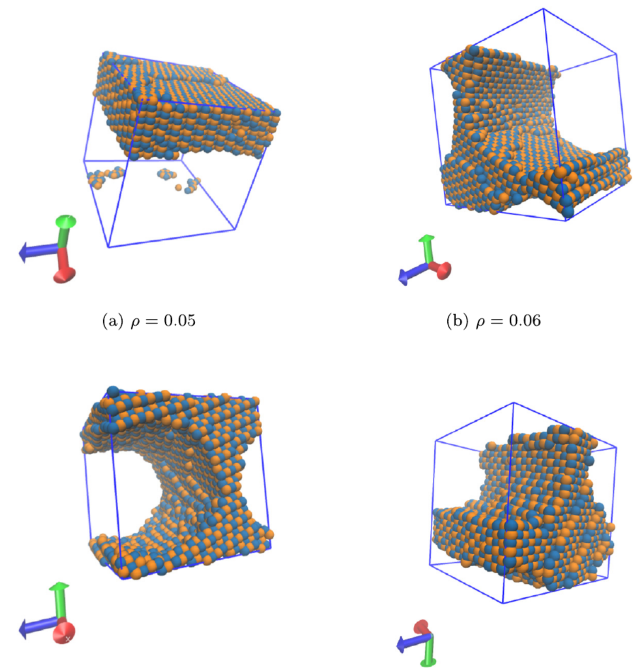


FIG. 14. Mass of the largest cluster for MSTE, MSTpC, and ECRA, for the early stages of the evolution, for the *lasagna* configuration. Note that, as in the previous case, the ECRA algorithm recognizes very early in the expansion the fracture of the clusters. In the asymptotic regime (for very large times, not shown in the figure) all three algorithms yield the same result.

f

$$\begin{cases} V_{np}(r) = \dfrac{V_r}{r} e^{-\mu_r r} - \dfrac{V_r}{r_c} e^{-\mu_r r_c} - \dfrac{V_a}{r} e^{-\mu_a r} + \dfrac{V_a}{r_c} e^{-\mu_a r_c} \\ V_{nn}(r) = \dfrac{V_0}{r} e^{-\mu_0 r} - \dfrac{V_0}{r_c} e^{-\mu_0 r_c} \end{cases}$$



(a) $\rho = 0.05$ (b) $\rho = 0.06$

(c) $\rho = 0.07$ (d) $\rho = 0.085$

Fig. 1. (Color online.) Pasta structures for nuclear matter systems with 6000 nucleons, $x = 0.5$ at $T = 0.2$ MeV and densities $\rho = 0.05$, 0.06, 0.07 and 0.085 fm^{-3}. Protons are represented in light color (orange), while neutrons are represented in darker color (blue). The red arrow (out of the page) corresponds to the x coordinate, the green arrow (vertical) to the y coordinate and the blue one (horizontal) to the z coordinate.

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Phase transitions and symmetry energy in nuclear pasta

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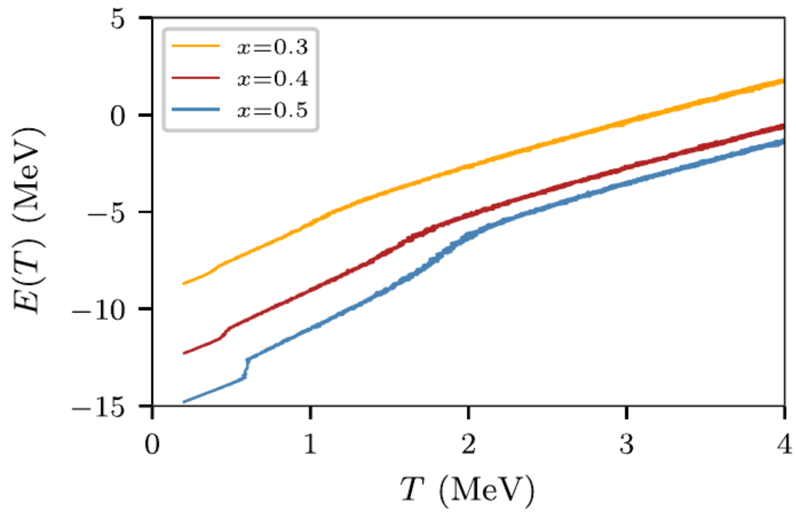
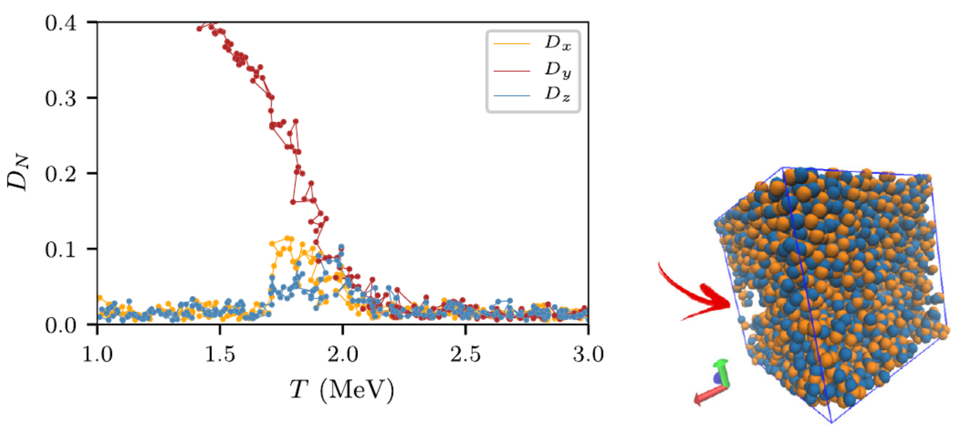


Fig. 4. The caloric curve for nuclear matter at $\rho = 0.05$ fm^{-3} and $x = 0.3$, 0.4, 0.5, respectively. The nucleons density was $\rho = 0.05$ fm^{-3}. The total number of nucleons was 6000.



(a) (b)

Fig. 11. (Color online.) (a) The Kolmogorov 1D statistic vs. temperature. Data corresponds to the position of 6000 nucleons at $\rho = 0.05$ fm^{-3} and $x = 0.5$ (the same configuration as in Fig. 1a). The simulation cell (with periodic boundary conditions) was slowly cooled from $T = 4$ MeV down to $T = 0.2$ MeV. D_x, D_y and D_z correspond to the x, y and z Kolmogorov statistics (sampled from the simulation cell), respectively (see text for details). (b) Visualization of the system analyzed in (a) at $T = 2$ MeV. The arrow points to the most noticeable bubble appearing in the picture.

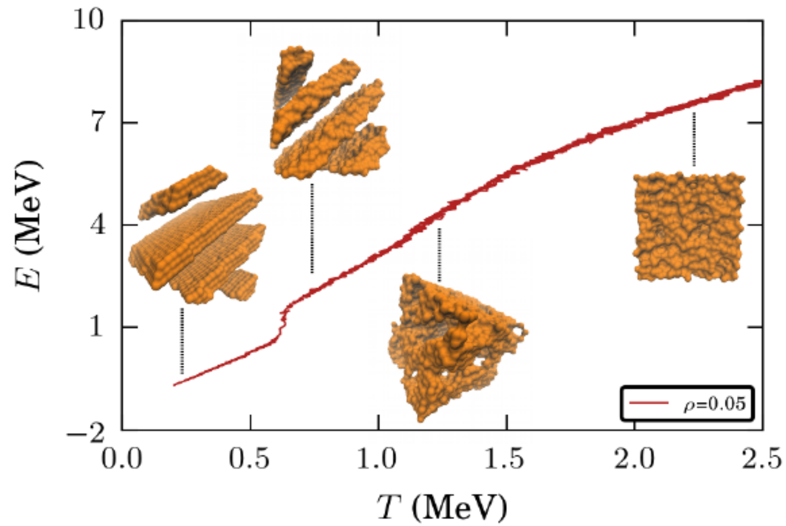


FIG. 3. (color online) Internal energy per nucleon for symmetric neutron star matter ($x = 0.5$) as a function of the bath temperature, for $\rho = 0.05\,\mathrm{fm}^{-3}$.

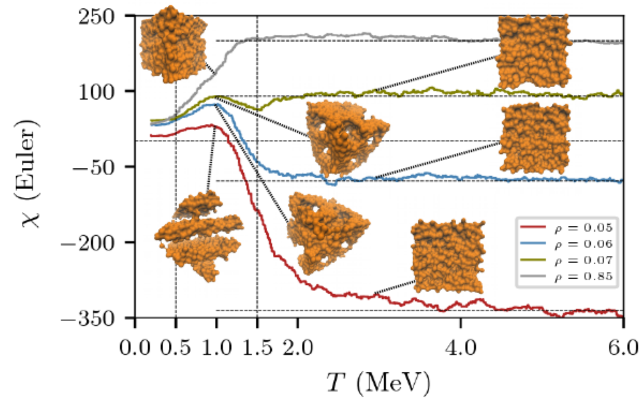


FIG. 7. (color online) The Euler functional χ obtained for isospin symmetric ($x = 0.5$) NSM systems as a function of temperature. The systems had the densities shown in the inset (in fm^{-3}) and were composed by $N = 4000$ nucleons interacting through the New Medium model. The corresponding binning distance is $d = 2.35\,\mathrm{fm}$. The data has been smoothed with a moving average procedure.

Symmetry energy in neutron star matter

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(Dated: September 11, 2018)

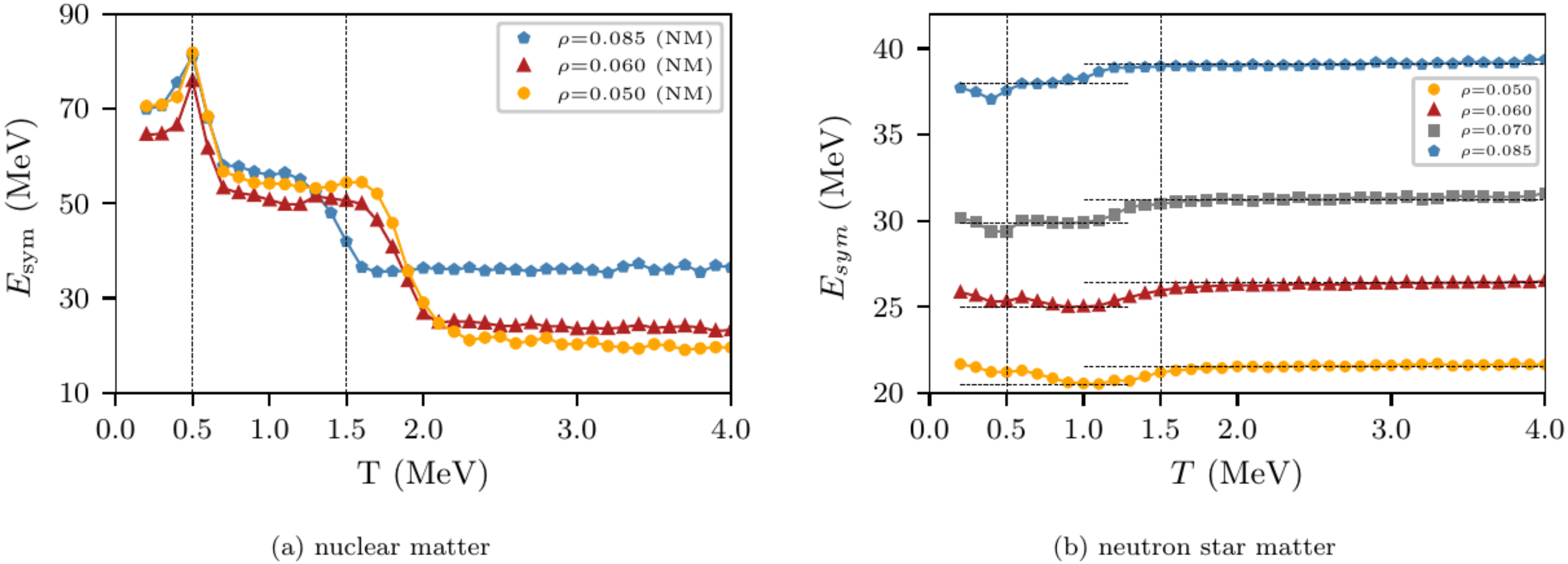

FIG. 18. (color online) Symmetry energy as a function of the temperature for the densities indicated in the insert in fm^{-3}. (a) Symmetry energy for nuclear matter (Pandharipande Medium Model) as a function of the temperature (extracted from Ref. [30]). The vertical dashed lines are a guide to the eye. (b) Symmetry energy for $N = 4000$ nucleons embedded in an electron cloud, as explained in Section II. The fitting procedures are detailed in Section III C and Appendix A. The horizontal and vertical dashed lines are a guide to the eye.