Selección Pathria

8.2 For a Fermi–Dirac gas, we may define a temperature T_0 at which the chemical potential of the gas is zero ($z = 1$). Express T_0 in terms of the Fermi temperature T_F of the gas.
[*Hint*: Use equation (E.16).]

8.7 Show that for an ideal Fermi gas

$$\langle u\rangle\left\langle\frac{1}{u}\right\rangle = \frac{4}{\pi}\frac{f_1(z)f_2(z)}{\{f_{3/2}(z)\}^2},$$

u being the speed of a particle. Further show that at low temperatures

$$\langle u\rangle\left\langle\frac{1}{u}\right\rangle \simeq \frac{9}{8}\left[1+\frac{\pi^2}{12}\left(\frac{kT}{\varepsilon_F}\right)^2\right];$$

8.10 Consider an ideal Fermi gas, with energy spectrum $\varepsilon \propto p^s$, contained in a box of "volume" V in a space of n dimensions. Show that, for this system,

(a) $PV = \frac{s}{n}U;$

(b) $\frac{C_V}{Nk} = \frac{n}{s}\left(\frac{n}{s}+1\right)\frac{f_{(n/s)+1}(z)}{f_{n/s}(z)} - \left(\frac{n}{s}\right)^2\frac{f_{n/s}(z)}{f_{(n/s)-1}(z)};$

(c) $\frac{C_P - C_V}{Nk} = \left(\frac{sC_V}{nNk}\right)^2\frac{f_{(n/s)-1}(z)}{f_{(n/s)}(z)};$

7.3. Combining equations (7.1.24) and (7.1.26), and making use of the first two terms of formula (D.9) in Appendix D, show that, as T approaches T_c from above, the parameter $\alpha(= -lnz)$ of the ideal Bose gas assumes the form

$$\alpha \approx \frac{1}{\pi}\left(\frac{3\zeta(3/2)}{4}\right)^2\left(\frac{T-T_c}{T_c}\right)^2.$$

7.9. Show that for an ideal Bose gas

$$\langle u\rangle\left\langle\frac{1}{u}\right\rangle = \frac{4}{\pi}\frac{g_1(z)g_2(z)}{\{g_{3/2}(z)\}^2},$$

u being the speed of a particle. Examine and interpret the limiting cases $z \to 0$ and $z \to 1$; compare with Problem 6.6.

7.12. Consider an ideal Bose gas in the grand canonical ensemble and study fluctuations in the total number of particles N and the total energy E. Discuss, in particular, the situation when the gas becomes highly degenerate.

7.14. Consider an n-dimensional Bose gas whose single-particle energy spectrum is given by $\varepsilon \propto p^s$, where s is some positive number. Discuss the onset of Bose–Einstein condensation in this system, especially its dependence on the numbers n and s. Study the thermodynamic behavior of this system and show that,

$$P = \frac{s}{n}\frac{U}{V}, \quad C_V(T\to\infty) = \frac{n}{s}Nk, \quad \text{and} \quad C_P(T\to\infty) = \left(\frac{n}{s}+1\right)Nk.$$

7.21. Show that the mean energy per photon in a blackbody radiation cavity is very nearly $2.7kT$.

7.23. The sun may be regarded as a black body at a temperature of 5800 K. Its diameter is about 1.4×10^9 m while its distance from the earth is about 1.5×10^{11} m.

(a) Calculate the total radiant intensity (in W/m^2) of sunlight at the surface of the earth.

(b) What pressure would it exert on a perfectly absorbing surface placed normal to the rays of the sun?

(c) If a flat surface on a satellite, which faces the sun, were an ideal absorber and emitter, what equilibrium temperature would it ultimately attain?

7.24. Calculate the photon number density, entropy density, and energy density of the 2.725 K cosmic microwave background.