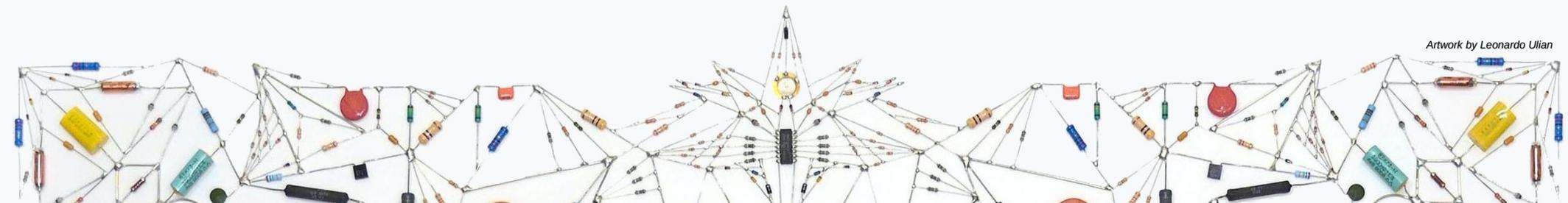


# Termodinámica estocástica en circuitos electrónicos

Nahuel Freitas

Temas avanzados de termodinámica y física estadística  
FCEyN – UBA

Septiembre 2023



Artwork by Leonardo Ulian

# Motivaciones

1) Termodinámica de la computación: Un *problema fundamental* pero con una gran *relevancia tecnológica*

## The Thermodynamics of Computation—a Review

Charles H. Bennett

*IBM Watson Research Center, Yorktown Heights, New York 10598*

*Received May 8, 1981*

NEWS FEATURE · 12 SEPTEMBER 2018 · CORRECTION 13 SEPTEMBER 2018

nature

### How to stop data centres from gobbling up the world's electricity

The energy-efficiency drive at the information factories that serve us Facebook, Google and Bitcoin.

### Bitcoin consumes 'more electricity than Argentina'

By Cristina Criddle  
Technology reporter

© 10 February

BBC

# Motivaciones

2) **Termodinámica estocástica:** hubo muchas contribuciones recientes al problema de la termodinámica de la información, pero en general solo se consideran **modelos de juguete**

## Teoría:

Editors' Suggestion

## Thermodynamics of Prediction

Susanne Still, David A. Sivak, Anthony J. Bell, and Gavin E. Crooks  
Phys. Rev. Lett. **109**, 120604 – Published 19 September 2012

PAPER

## Thermodynamic and logical reversibilities revisited

Takahiro Sagawa<sup>1</sup>

Published 26 March 2014 • © 2014 IOP Publishing Ltd and SISSA Medialab srl

[Journal of Statistical Mechanics: Theory and Experiment, Volume 2014, March 2014](#)

Citation Takahiro Sagawa *J. Stat. Mech.* (2014) P03025

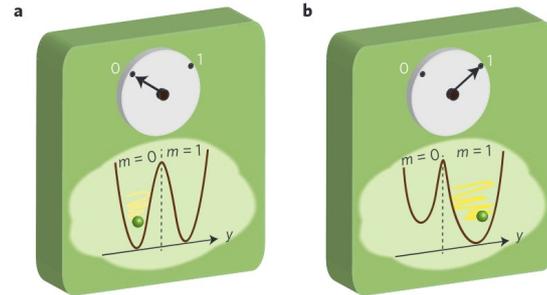
Published: 03 February 2015

## Thermodynamics of information

Juan M. R. Parrondo , Jordan M. Horowitz & Takahiro Sagawa

[Nature Physics](#) **11**, 131–139(2015) | [Cite this article](#)

6185 Accesses | 445 Citations | 49 Altmetric | [Metrics](#)



**Figure 2 | Toy model of a memory.** A Brownian particle in a double-well potential with position  $y$  can be stably trapped in either the left or right well, which represent the mesoscopic informational states  $m=0$  and  $m=1$  of a bit. The memory can either be symmetric (a) or asymmetric (b).

## Experimentos:

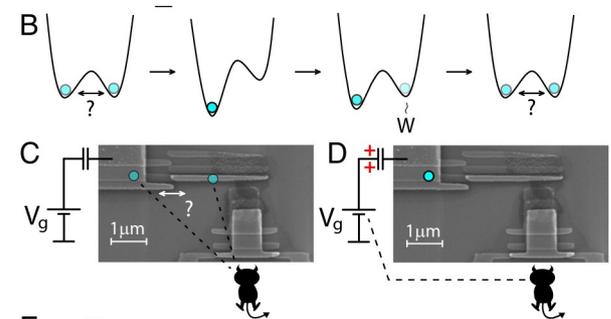
RESEARCH ARTICLE

## Experimental realization of a Szilard engine with a single electron

Jonne V. Koski, Ville F. Maisi, Jukka P. Pekola, and Dmitri V. Averin

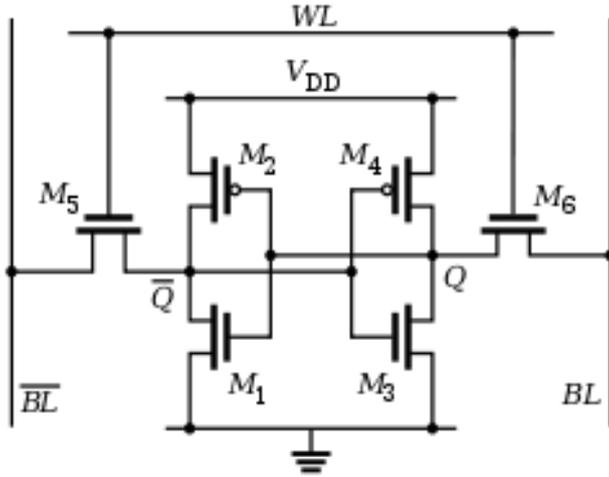
PNAS September 23, 2014 111 (38) 13786–13789; first published September 8, 2014; <https://doi.org/10.1073/pnas.1406966111>

Edited by Peter J. Rossky, Rice University, Houston, TX and approved August 12, 2014 (received for review April 17, 2014)



# Motivaciones

3) Existe una necesidad de **nuevos métodos** termodinámicamente consistentes en **ingeniería electrónica**



Para el tipo de ruido térmico que tienen los circuitos CMOS de baja potencia no se pueden usar:

- Ecuaciones de Langevin
- Ecuaciones de Fokker-Planck

En regímenes de baja potencia es necesario usar **cadena de Markov en tiempo continuo**.

## Fundamental Thermal Limits on Data Retention in Low-Voltage CMOS Latches and SRAM

Publisher: IEEE

[Cite This](#)

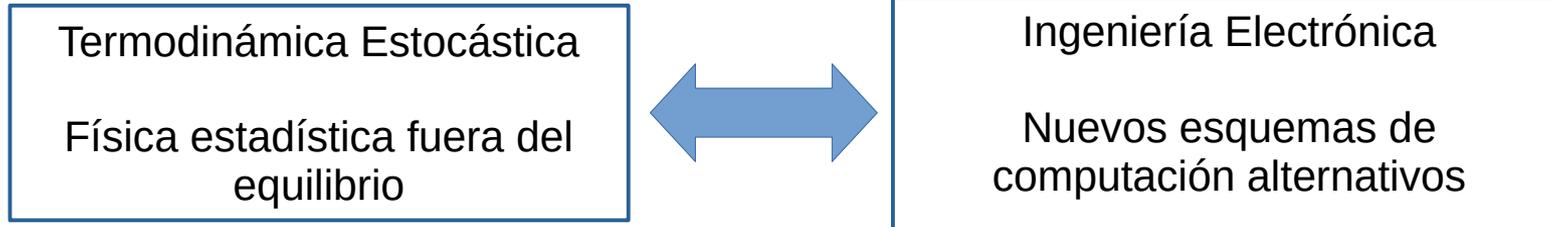
[PDF](#)

Elahe Rezaei ; Marco Donato ; William R. Patterson ; Alexander Zaslavsky ; R. Iris Bahar [All Authors](#)

# Objetivos

Quisieramos construir y estudiar **modelos termodinámicamente consistentes** de dispositivos electrónicos **tecnológicamente relevantes**.

**Establecer nuevos puentes** entre la ingeniería electrónica y la física estadística fuera del equilibrio.



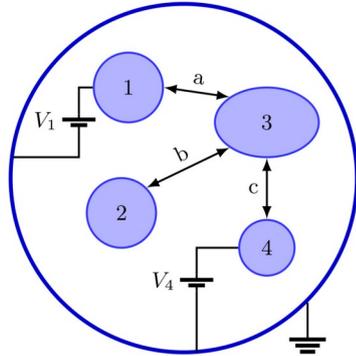
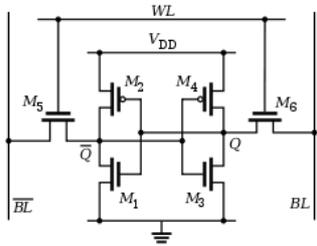
Stochastic Thermodynamics of Nonlinear Electronic Circuits:  
A Realistic Framework for Computing Around  $kT$

Nahuel Freitas, Jean-Charles Delvenne, and Massimiliano Esposito  
Phys. Rev. X **11**, 031064 – Published 22 September 2021



# Formalismo Básico

Back to F3: Un sistema de conductores y fuentes de tensión...



Los estados del sistema son las diferentes **configuraciones de carga**:

$$\mathbf{q}_0 = (q_1, \dots, q_{N_0})^T \quad \mathbf{q}_0 = \mathbf{C}_0 \mathbf{V}_0$$

$$\mathbf{V}_0 = (V_1, \dots, V_{N_0})^T$$

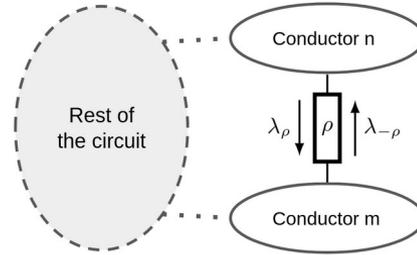
La **energía electrostática** está dada por una **forma cuadrática**:

$$E = \frac{1}{2} \mathbf{q}_0^T \mathbf{V}_0 = \frac{1}{2} \mathbf{V}_0^T \mathbf{C}_0 \mathbf{V}_0 = \frac{1}{2} \mathbf{q}_0^T \mathbf{C}_0^{-1} \mathbf{q}_0$$

Los conductores se pueden dividir en **libres y regulados**:

$$\begin{matrix} \mathbf{q}_0 & \longrightarrow & \mathbf{q} & \mathbf{q}_r \\ \mathbf{V}_0 & \longrightarrow & \mathbf{V} & \mathbf{V}_r \end{matrix} \quad \begin{bmatrix} \mathbf{q} \\ \mathbf{q}_r \end{bmatrix} = \begin{bmatrix} \mathbf{C} & \mathbf{C}_m \\ \mathbf{C}_m^T & \mathbf{C}_r \end{bmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{V}_r \end{bmatrix}$$

## Modelado de los dispositivos, o canales de conducción:



- Consideramos un **modelo de Poisson bidireccional** para cada dispositivo.
- Esto excluye dispositivos que no muestran **shot noise**

$$\begin{aligned} \text{Forward transition: } & \mathbf{q} \rightarrow \mathbf{q} + q_e \Delta_\rho \quad \longrightarrow \quad \lambda_\rho(\mathbf{q}, t) \\ \text{Reverse transition: } & \mathbf{q} \rightarrow \mathbf{q} + q_e \Delta_{-\rho} \quad \longrightarrow \quad \lambda_{-\rho}(\mathbf{q}, t) \end{aligned}$$

## Condiciones de Balance Detallado Local (LDB):

$$\log \left( \frac{\lambda_\rho(\mathbf{q}, t)}{\lambda_{-\rho}(\mathbf{q} + q_e \Delta_\rho, t)} \right) = -\beta_\rho \delta Q_\rho(\mathbf{q})$$

Calor intercambiado con el entorno en una dada transición

$$\delta Q_\rho(\mathbf{q}) = \Phi(\mathbf{q} + q_e \Delta_\rho) - \Phi(\mathbf{q}) + q_e \mathbf{V}_r^T \Delta_\rho^r$$

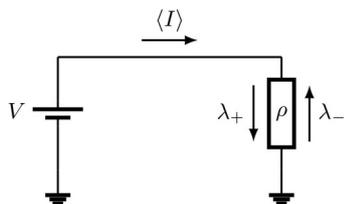
$$\Phi(\mathbf{q}) = E(\mathbf{q}) - \mathbf{V}_r^T \mathbf{C}_m^T \mathbf{C}^{-1} \mathbf{q}$$

Trabajo no-conservativo

Potencial electrostático en presencia de conductores regulados

# Formalismo Básico

La curva I-V y el modelo bi-Poisson:



Los **rates de Poisson** están completamente determinados por la **curva I-V del dispositivo**, y las **condiciones LDB**:

$$\langle I \rangle = \langle q(\Delta t) / \Delta t \rangle = q_e (\tilde{\lambda}_+(V) - \tilde{\lambda}_-(V)) \quad \log \left( \frac{\tilde{\lambda}_+(V)}{\tilde{\lambda}_-(V)} \right) = \beta q_e V$$

Esto implica **shot noise**:

$$\sigma_I^2(\Delta t) = \frac{q_e}{\Delta t} \langle I(\Delta t) \rangle \coth(\beta q_e V / 2)$$

Efectos de carga y la regla del punto medio:

**Regla del punto medio:** para respetar las condiciones **LDB** cuando un dispositivo se conecta en un circuito arbitrario, los **rates de Poisson a voltaje fijo** deben ser evaluados en el **voltaje promedio antes y después de una transición**.

$$\lambda_{\pm\rho}(\mathbf{q}) = \tilde{\lambda}_{\pm} \left( \overline{\Delta V}_{nm}^{\pm\rho}(\mathbf{q}) \right)$$

Ecuación Maestra:

$$d_t P(\mathbf{q}, t) = \sum_{\rho} \{ J_{-\rho}(\mathbf{q} + q_e \Delta_{\rho}, t) - J_{\rho}(\mathbf{q}, t) \} \quad J_{\rho}(\mathbf{q}, t) = \lambda_{\rho}(\mathbf{q}, t) P(\mathbf{q}, t)$$

Balance de Energía:

$$d_t \langle \Phi \rangle = \langle \dot{W}_{\Phi} \rangle + \sum_{\rho>0} \langle \dot{W}_{\rho}^r \rangle + \sum_{\rho>0} \langle \dot{Q}_{\rho} \rangle$$

Producción de Entropía:

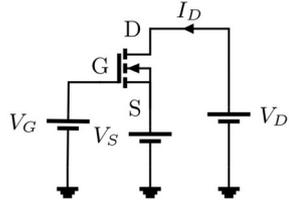
$$\langle \dot{\Sigma} \rangle \equiv d_t \langle S \rangle + \langle \dot{\Sigma}_e \rangle \geq 0$$

Teoremas de Fluctuación:

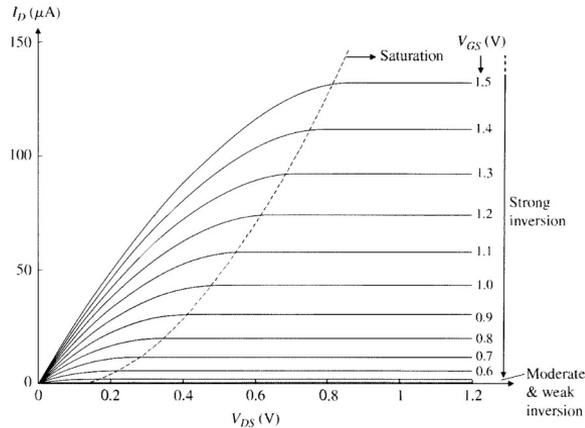
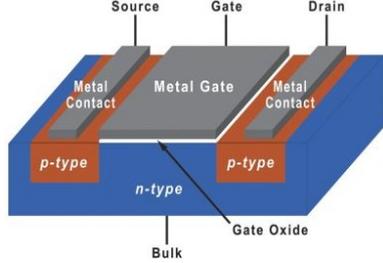
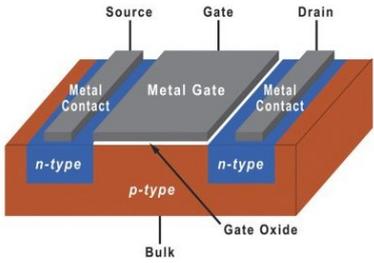
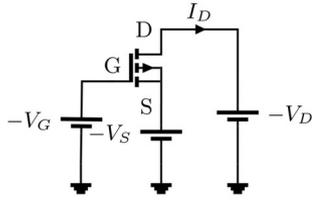
$$\frac{P(\{W_{n_f}\}, W_{\Psi})}{P^{\dagger}(\{-W_{n_f}\}, -W_{\Psi})} = \exp((W_{\Psi} + \sum_{n_f} W_{n_f}) / (k_b T))$$

# La tecnología CMOS en 2 minutos

## nMOSFET

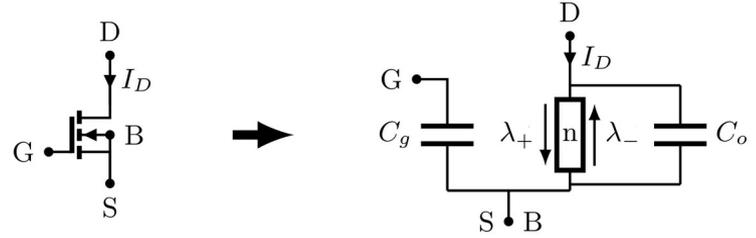


## pMOSFET



## Modelo efectivo:

En configuraciones típicas los transistores MOS se pueden considerar **dispositivos de dos terminales con un control externo**:



$$\lambda_+(\Delta V_{GS}, \Delta V_{DS})$$

$$\lambda_-(\Delta V_{GS}, \Delta V_{DS})$$

Parámetro de control

Dif. de voltaje

En el régimen de operación '**sub-threshold**':

$$\lambda_+ = (I_0/q_e) e^{(\Delta V_{GS} - V_{th})/(nV_T)}$$

$$\lambda_- = (I_0/q_e) e^{(\Delta V_{GS} - V_{th})/(nV_T)} e^{-(\Delta V_{DS})/V_T}$$

**Parámetros fenomenológicos**

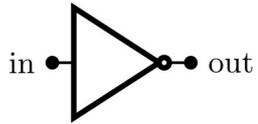
$$I_0, n, V_{th}$$

**Voltaje Térmico**

$$V_T = (\beta q_e)^{-1} = k_b T / q_e$$

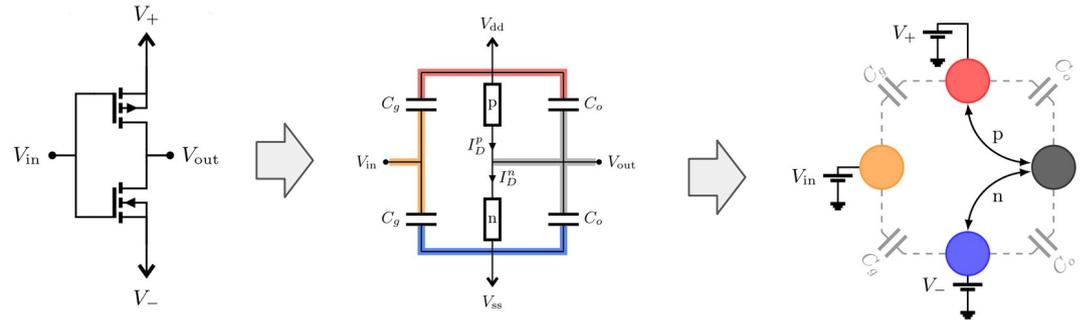
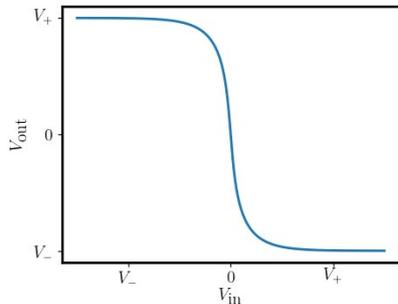
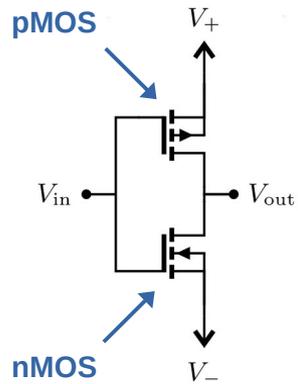
# Aplicación 1: Inversor CMOS, o compuerta NOT

## Compuerta NOT



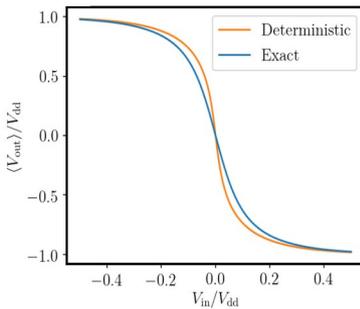
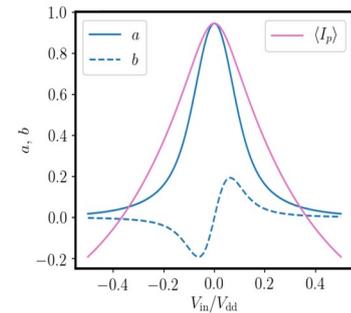
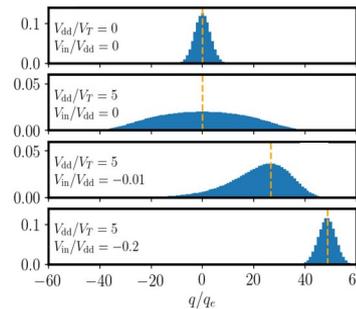
in	out
True	False
False	True

## Implementación CMOS



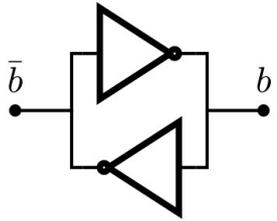
Construyendo la ecuación maestra y calculando el estado estacionario...

$$P_{st}(q) = \frac{\alpha_p + \alpha_n \gamma e^{-(q-q_e)/q_T}}{\alpha_n + \alpha_p \gamma e^{q/q_T}} P_{st}(q - q_e)$$



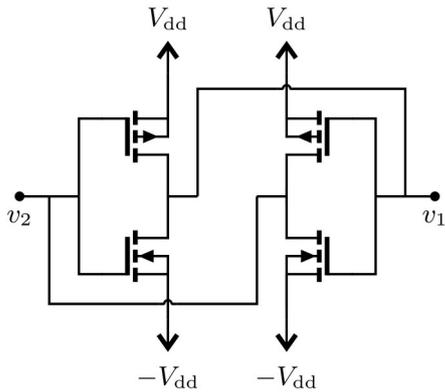
# Aplicación 2: Una Memoria

## Un bit: memoria SRAM



$b$	$\bar{b}$
True	False
False	True

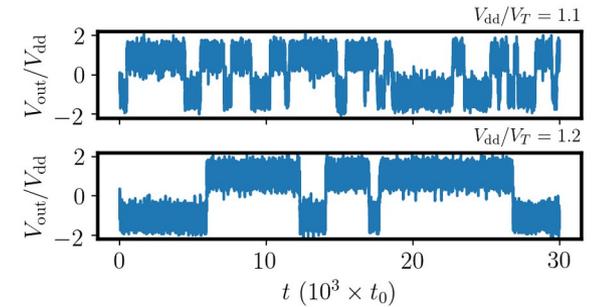
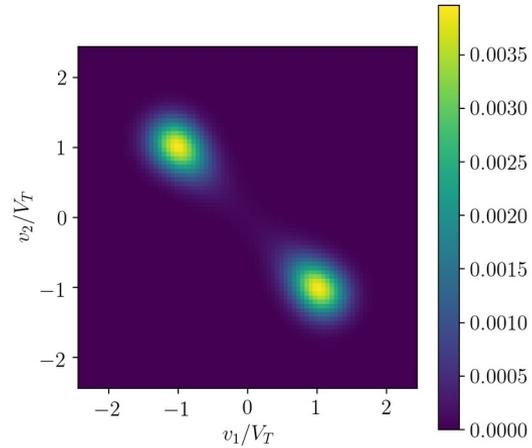
## Implementación CMOS



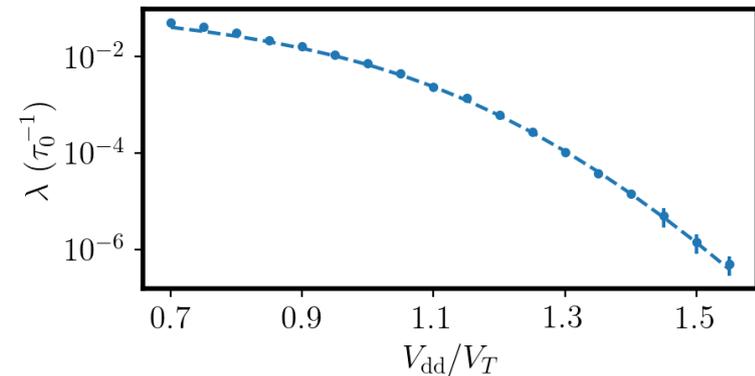
$v_1$	$v_2$
$+V_{dd}$	$-V_{dd}$
$-V_{dd}$	$+V_{dd}$

Los dos **estados lógicos estables** se corresponden con dos **estados estacionarios fuera del equilibrio (NESSs) metastables**

## Estado estacionario y trayectorias

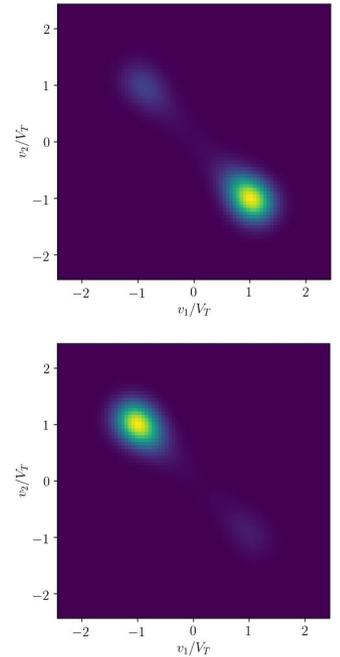
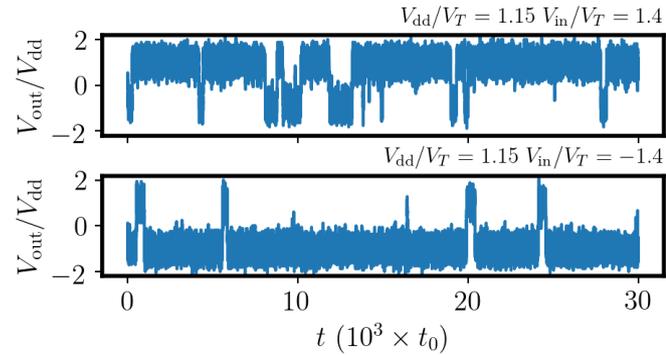
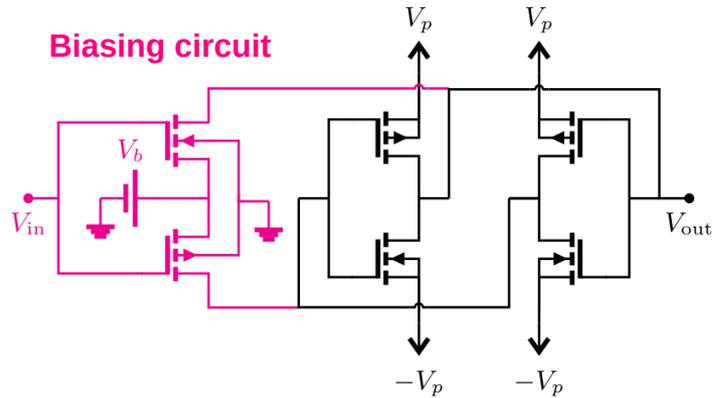


Para reducir los errores lógicos hay que aumentar el voltaje de operación...



# Aplicación 3: Una neurona estocástica, o bit probabilístico

¿Qué se puede hacer con una mala memoria?

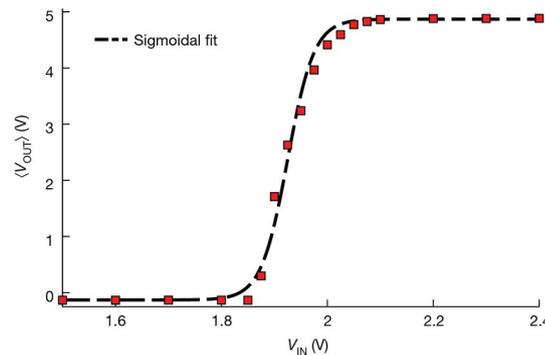
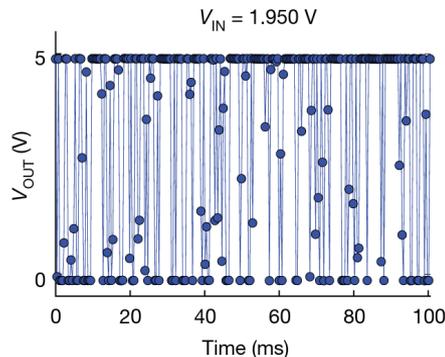
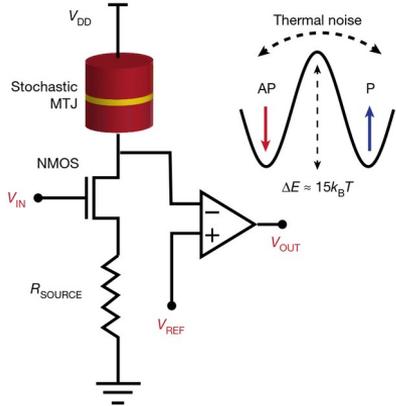


¿por qué esto es útil?

Letter | Published: 18 September 2019

## Integer factorization using stochastic magnetic tunnel junctions

William A. Borders, Ahmed Z. Pervaiz, Shunsuke Fukami, Kerem Y. Camsari, Hideo Ohno & Supriyo Datta



En principio se pueden construir

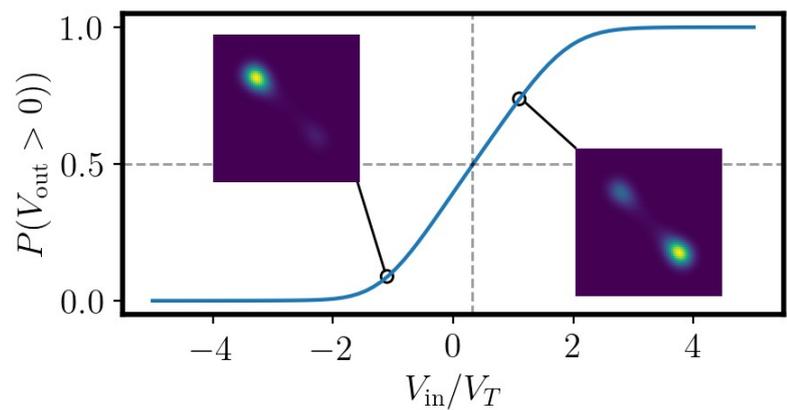
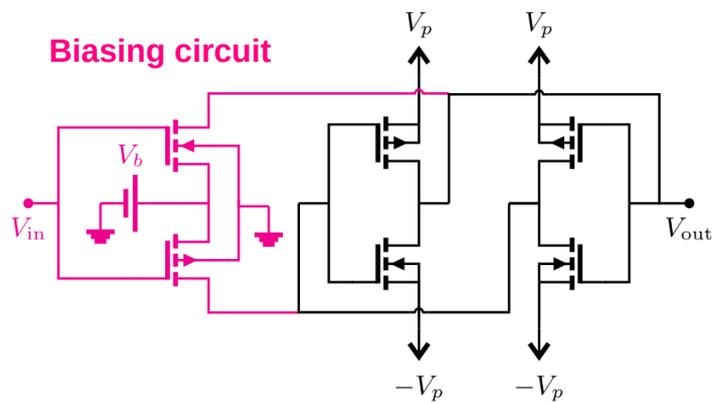
- “annealers” estocásticos
- redes neuronales estocásticas,

**Usando las fluctuaciones térmicas como recurso !!**

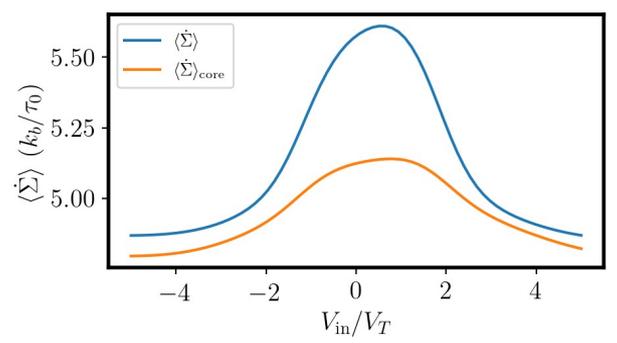
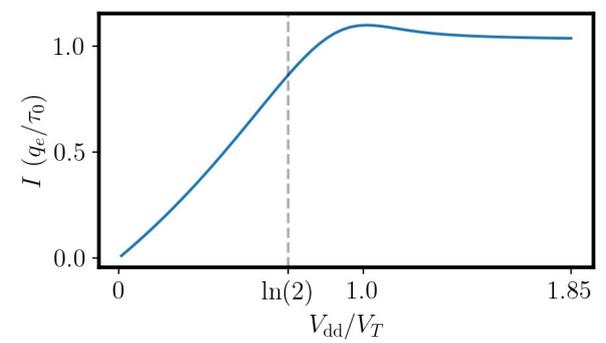
# Aplicación 3: Una neurona estocástica, o bit probabilístico

¿Qué se puede hacer con una mala memoria?

Función de activación



Consumo de energía



Para procesos de fabricación modernos (7nm) es posible lograr **valores de consumo tan bajos como 10 aJ por bit**.  
Dos órdenes de magnitud por debajo de propuestas anteriores.

## Algunas conclusiones:

Una descripción rigurosa del ruido térmico en circuitos electrónicos es crucial para:

- **Reducir el consumo energético**
- **Estudiar la confiabilidad**
- **Nuevos esquemas de cómputo**

### Fundamental Thermal Limits on Data Retention in Low-Voltage CMOS Latches and SRAM

Publisher: IEEE

Cite This

PDF

Elahe Rezaei ; Marco Donato ; William R. Patterson ; Alexander Zaslavsky ; R. Iris Bahar All Authors

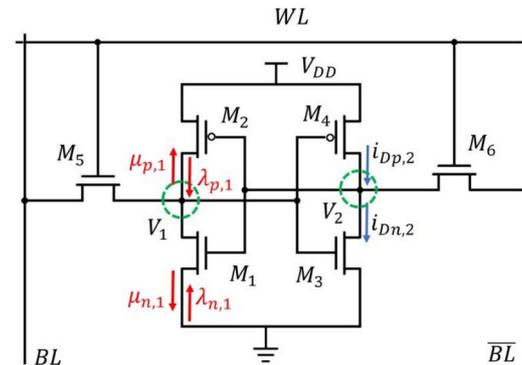
“If we start with  $\Delta v(t) = V_{DD} = 180$  mV, we can try to capture a bit-flip error by simulating until  $\Delta v(t) < 0$ . However, the minimum  $\Delta v(t)$  captured by continuously computing the numerical solution of (7), after a month of continuous simulation, was 118 mV, as shown in Fig. 3(a). Thus, reaching”

Pero una descripción rigurosa es muy demandante. Hay dos opciones principales.

- **Ecuaciones maestras y métodos espectrales:**

$$d_t p_i = \sum_{\nu, j} \lambda_{ij}^{\nu} p_j$$

- **Simulaciones estocásticas**  
(Por ejemplo, el algoritmo de Gillespie)

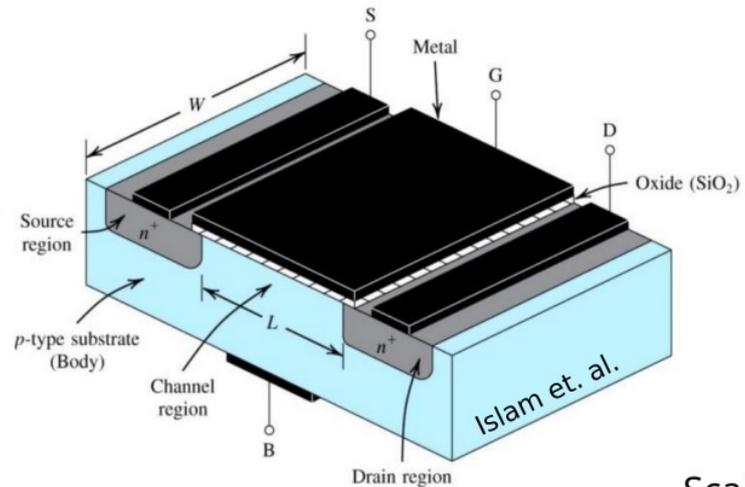
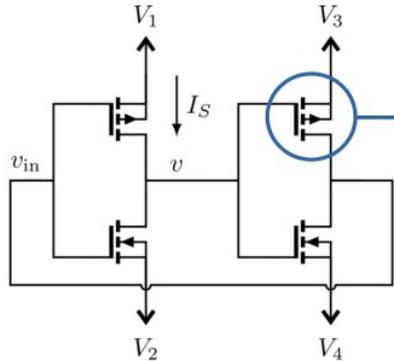


$$\begin{aligned} \mu_n &= \frac{I_0}{q} \exp\left(\frac{qBV_{ds}}{kT}\right) \exp\left(\frac{qV_{gs}}{mkT}\right) \\ \lambda_n &= \mu_n \exp\left(\frac{-qV_{ds}}{kT}\right) \\ \mu_p &= \frac{I_0}{q} \exp\left(\frac{qBV_{sd}}{kT}\right) \exp\left(\frac{qV_{sg}}{mkT}\right) \\ \lambda_p &= \mu_p \exp\left(\frac{-qV_{sd}}{kT}\right), \end{aligned}$$

**Una alternativa:  
Teoría de Grandes Desviaciones**

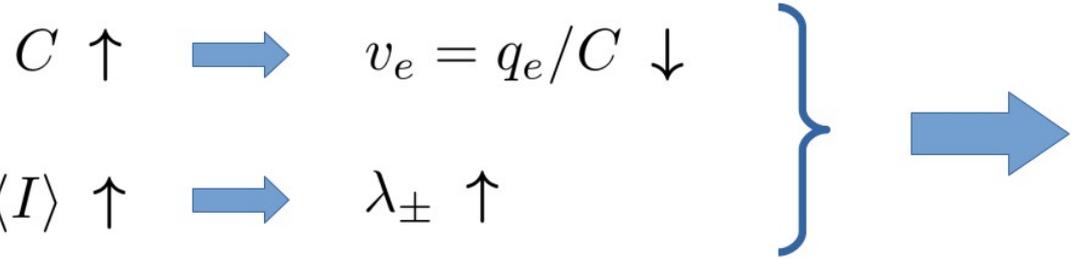


# The macroscopic limit



What happens if we increase the physical dimensions of the transistors?

Keeping L fixed and increasing W, we have:



Scale parameter

$$\Omega = v_e^{-1} \propto W$$

Rate function

$$P_t(\mathbf{v}) \asymp e^{-\Omega f(\mathbf{v}, t)}$$

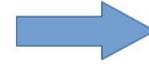
**A large deviations principle is satisfied**

*Freitas, Delvenne, Esposito PRX 2021*  
*Freitas, Proesmans, Esposito PRE 2022*  
*Gopal, Esposito, Freitas PRB 2022*  
*Freitas, Esposito Nat. Comm. 2022*

## The macroscopic limit

$$P_t(\mathbf{v}) \simeq e^{-\Omega f(\mathbf{v}, t)}$$

$$d_t P_t(\mathbf{v}) = \sum_{\rho} \lambda_{\rho}(\mathbf{v} - \Delta_{\rho} v_e) P_t(\mathbf{v} - \Delta_{\rho} v_e) - \lambda_{\rho}(\mathbf{v}) P_t(\mathbf{v})$$

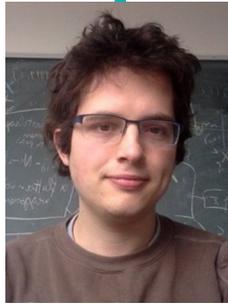


$$d_t f(\mathbf{v}, t) = \sum_{\rho} \omega_{\rho}(\mathbf{v}) \left[ 1 - e^{\Delta_{\rho} \cdot \nabla f(\mathbf{v}, t)} \right]$$

- The minimum of  $f(\mathbf{v}, t)$  follows the deterministic dynamics
- A quadratic expansion around it gives us a Gaussian approximation, increasingly valid for larger  $\Omega$

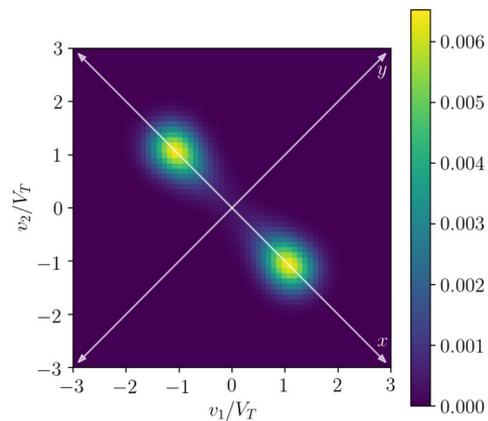
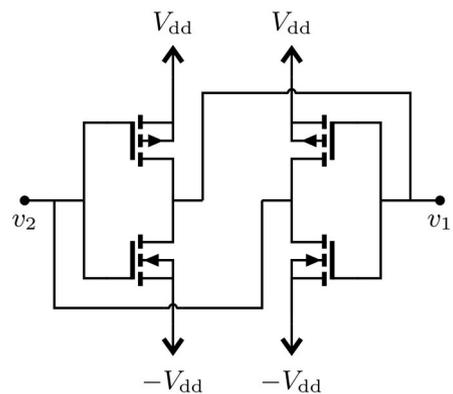
# Reliability and entropy production in non-equilibrium electronic memories

Nahuel Freitas, Karel Proesmans, and Massimiliano Esposito



**Phys. Rev. E 105, 034107**

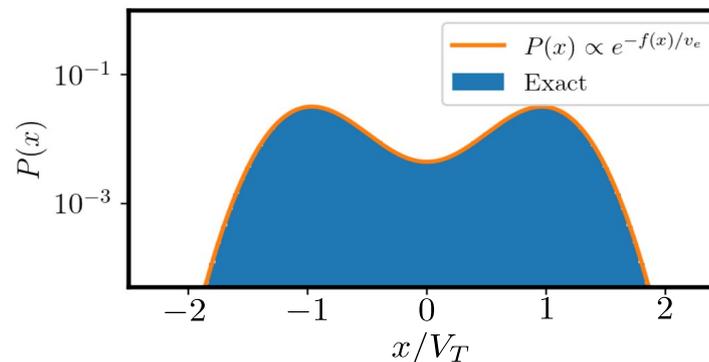
# Taking advantage of the macroscopic limit:



$$f(x, y) = g(x) + h(y)$$

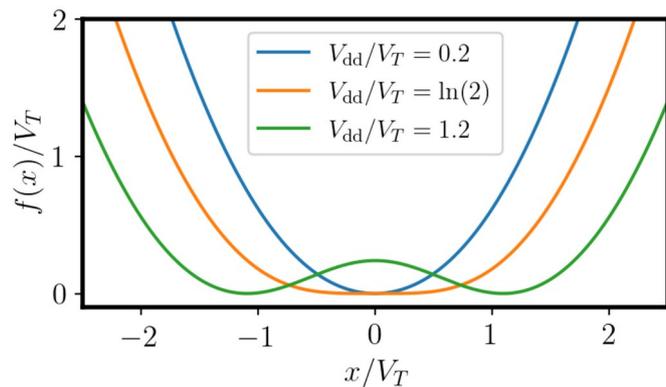
$$L(x, V_{dd}) = \text{Li}_2(-\exp((V_{dd} + x(1 + 2/n))/V_T))$$

$$g(x) = \frac{x^2 + 2V_{dd}x}{V_T} + \frac{2nV_T}{n+2} [L(x, V_{dd}) - L(x, -V_{dd})]$$



$$v_e/V_T = 0.1 !!$$

## Reliability in terms of the entropy production



$$\lambda_{\text{err}} = 4 \sum_{v_2} B(0, v_2) P(-v_2/2) Q(v_2/2) \propto e^{-\frac{4}{n+2} \frac{V_{dd}^2}{k_b T/C}}$$

