

Teoría de Campos 2020 - Práctica

Bilineales de Dirac.

Bilineales de Dirac

Formar escalares que transformen de manera covariante a partir de Ψ

Motivación:

- Obtención de un Lagrangiano
- Observables de la teoría

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$$\psi \in \mathbb{C}^4$$

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$$\bar{\Psi}\gamma^{[\mu}\gamma^{\nu]}\Psi \xrightarrow{\text{boost}} \Lambda^{\mu}_{\rho}\Lambda^{\nu}_{\sigma}\bar{\Psi}\gamma^{[\rho}\gamma^{\sigma]}\Psi \quad \text{Tensor antisimétrico de orden 2}$$

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$$(i\gamma^\mu \partial_\mu - m)\Psi = 0 \xrightarrow{P}$$

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$$S_P = \gamma^0$$

$$\gamma^5 S_P = i\gamma^0\gamma^1\gamma^2\gamma^3 S_P$$

$$\gamma^5 S_P = i S_P (S_P^{-1} \gamma^0 S_P) (S_P^{-1} \gamma^1 S_P) (S_P^{-1} \gamma^2 S_P) (S_P^{-1} \gamma^3 S_P)$$

$$\gamma^5 S_P = (-1)^3 i S_P \gamma^0 \gamma^1 \gamma^2 \gamma^3$$

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$\bar{\Psi} \gamma^\mu \gamma^5 \Psi$ pseudo-vector

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Bilineales de Dirac: $\bar{\Psi}\Gamma\Psi$, Γ una matriz de 4×4

$\bar{\Psi}\Psi$ Escalar

$\bar{\Psi}\gamma^\mu\Psi$ Vector

$\bar{\Psi}\gamma^{[\mu}\gamma^{\nu]}\Psi$ Tensor de orden 2

$\bar{\Psi}\gamma^{[\mu}\gamma^\nu\gamma^{\rho]}\Psi$ Tensor de orden 3

$\bar{\Psi}\gamma^{[\mu}\gamma^\nu\gamma^\rho\gamma^{\sigma]}\Psi$ Tensor de orden 4

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