

# Teoría de Campos 2020 - Práctica

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Teoría de QED escalar.

# Teoría de QED escalar

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^\dagger(D_\mu\phi) - m^2\phi^\dagger\phi$$

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# Teoría de QED escalar

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$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0\phi)} = \partial^0\phi^* - ieA^0\phi^*$$

$$\mathcal{H} = \pi\dot{\phi} + \pi^*\dot{\phi}^* - \left( \dot{\phi}^*\dot{\phi} - \nabla\phi^*\nabla\phi - m^2\phi^*\phi \right) - \mathcal{L}^{\text{int}} + \mathcal{H}^{\text{EM}}$$

# Teoría de QED escalar

$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0\phi)} = \partial^0\phi^* - ieA^0\phi^*$$

$$\begin{aligned}\mathcal{H} &= \pi (\pi^* - ieA^0\phi) + \pi^* (\pi + ieA^0\phi^*) \\ &- ((\pi + ieA^0\phi^*) (\pi^* - ieA^0\phi) - \nabla\phi^*\nabla\phi - m^2\phi^*\phi) - \mathcal{L}^{\text{int}} + \mathcal{H}^{\text{EM}}\end{aligned}$$

# Teoría de QED escalar

$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0\phi)} = \partial^0\phi^* - ieA^0\phi^*$$

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# Teoría de QED escalar

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$$\mathcal{H}^{\text{int}} = -\mathcal{L}^{\text{int}} - e^2\phi^*\phi(A^0)^2$$

# Teoría de QED escalar

$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0\phi)} = \partial^0\phi^* - ieA^0\phi^*$$

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$$\mathcal{H}^{\text{int}} = ie\phi^*\overleftrightarrow{\partial_\mu}\phi A^\mu - e^2\phi^*\phi A^\mu A_\mu - e^2\phi^*\phi(A^0)^2$$

# Teoría de QED escalar

$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0\phi)} = \partial^0\phi^* - ieA^0\phi^*$$

$$\mathcal{H} = \mathcal{H}^{\text{int}} + \mathcal{H}^{\text{KG}} + \mathcal{H}^{\text{EM}}$$

$$\mathcal{H}^{\text{int}} = ie\phi^* \overleftrightarrow{\partial}_k \phi A^\mu + i\phi^* \partial_0 \phi A^0 - i\partial_0 \phi^* \phi A^0$$

$$-e^2\phi^*\phi A^\mu A_\mu - e^2\phi^*\phi (A^0)^2$$

# Teoría de QED escalar

$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0\phi)} = \partial^0\phi^* - ieA^0\phi^*$$

$$\mathcal{H} = \mathcal{H}^{\text{int}} + \mathcal{H}^{\text{KG}} + \mathcal{H}^{\text{EM}}$$

$$\begin{aligned}\mathcal{H}^{\text{int}} = & ie\phi^* \overleftrightarrow{\partial}_k \phi A^k + i\phi^* (\pi^* - ieA^0\phi) A^0 - i(\pi + ieA^0\phi^*) \phi A^0 \\ & - e^2\phi^*\phi A^\mu A_\mu - e^2\phi^*\phi(A^0)^2\end{aligned}$$

# Teoría de QED escalar

$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0\phi)} = \partial^0\phi^* - ieA^0\phi^*$$

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# Teoría de QED escalar

$$\pi = \frac{\partial \mathcal{L}}{\partial(\partial_0\phi)} = \partial^0\phi^* - ieA^0\phi^*$$

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$$\phi \rightarrow \hat{\phi}, \quad \pi \rightarrow \hat{\pi}, \quad A_\mu \rightarrow \hat{A}_\mu$$

$$[\hat{\phi}(t, \mathbf{x}), \hat{\pi}(t, \mathbf{x}')]=[\hat{\phi}(t, \mathbf{x}), \hat{\pi}(t, \mathbf{x}')]=i\delta(\mathbf{x}-\mathbf{x}')$$

# Teoría de QED escalar

$$\partial^0 \phi_I = \partial^0 (U \phi U^\dagger) = (\partial^0 U) \phi^H U^\dagger + U (\partial^0 \phi) U^\dagger + U \phi (\partial^0 U^\dagger)$$

$$\partial^0 U = e^{iH_S^0 t} (iH_S^0 - iH) e^{-iHt} = -ie^{iH_S^0 t} e^{-iHt} \left( e^{iHt} H_S^{\text{int}} e^{-iHt} \right) = -iU H_H^{\text{int}}$$

$$\partial^0 U^\dagger = iU H_H^{\text{int}}$$

$$i\partial^0 \phi_H = [\phi_H, H]$$

$$\partial^0 \phi_I = -iU H_H^{\text{int}} \phi_H U^\dagger - iU [\phi_H, H] U^\dagger + U \phi iU H_H^{\text{int}}$$

$$= -iU \left( [\phi_H, H] - [\phi_H, H_H^{\text{int}}] \right) U^\dagger = -iU [\phi_H, H_H^0] U^\dagger$$

$$[\phi_H, H_H^0] = \int [\phi_H, \mathcal{H}_H^{\text{KG}}] d^3x = \int [\phi_H(x), \pi_H^\dagger \pi_H(x')] d^3x' = i\pi_H^\dagger$$

$$\implies \boxed{\partial^0 \phi_I = -iUi\pi_H^\dagger U^\dagger = \pi_I^\dagger}$$

# Teoría de QED escalar

$$\mathcal{H}_I^{\text{int}} = ie\phi_I^\dagger \overleftrightarrow{\partial}_k \phi_I A_I^k + ie \left( \partial^0 \phi_I \phi_I^\dagger - \phi_I \partial^0 \phi_I^\dagger \right) A^0 - e^2 \phi_I^\dagger \phi_I A_I^\mu A_{\mu I} - e^2 \phi_I^\dagger \phi_I A_I^0 A_{0I}$$

# Teoría de QED escalar

$$\mathcal{H}_I^{\text{int}} = ie\phi^\dagger \overleftrightarrow{\partial_\mu} \phi A^\mu - e^2 \phi^\dagger \phi A^\mu A_\mu - e^2 \phi^\dagger \phi A^0 A_0$$

# Teoría de QED escalar

$$\mathcal{H}_I^{\text{int}} = ie\phi^\dagger \overleftrightarrow{\partial_\mu} \phi A^\mu - e^2 \phi^\dagger \phi A^\mu A_\mu - e^2 \phi^\dagger \phi A^0 A_0$$



# Teoría de QED escalar

$$\mathcal{H}_I^{\text{int}} = ie\phi^\dagger \overleftrightarrow{\partial_\mu} \phi A^\mu - e^2 \phi^\dagger \phi A^\mu A_\mu - e^2 \phi^\dagger \phi A^0 A_0$$

$$S = T \exp \left( \int dx \mathcal{H}^{\text{int}}(x) \right) = \sum_q e^q S^{(q)}$$

$$S^{(1)} = -i \int dx T \left( \phi^\dagger \overleftrightarrow{\partial_\mu} \phi A^\mu \right)$$

$$S^{(2)} = \frac{(-i)^2}{2!} \int dx dy T \left( \phi^\dagger(x) \overleftrightarrow{\partial_\mu} \phi(x) A^\mu(x) \phi^\dagger(y) \overleftrightarrow{\partial_\mu} \phi(y) A^\mu(y) \right) + \dots$$

# Teoría de QED escalar

$$\begin{aligned}\partial_\nu^y \Delta_F(x - y) &= \partial_\nu^y \langle 0 | T(\phi(x)\phi^\dagger(y)) | 0 \rangle \\&= \partial_\nu^y (\langle 0 | \phi(x)\phi^\dagger(y) | 0 \rangle \theta(x_0 - y_0) + \langle 0 | \phi(x)\phi^\dagger(y) | 0 \rangle \theta(y_0 - x_0)) \\&= \langle 0 | T(\phi(x)\partial_\nu\phi^\dagger(y)) | 0 \rangle - \eta_{\nu 0} \delta(x_0 - y_0) \langle 0 | [\phi(x), \phi^\dagger(y)] | 0 \rangle \\&\implies \boxed{\langle 0 | T(\phi(x)\partial_\nu\phi^\dagger(y)) | 0 \rangle = \partial_\nu^y \Delta_F(x - y)}\end{aligned}$$

# Teoría de QED escalar

$$\langle 0 | T (\phi(x) \partial_\nu \phi^\dagger(y)) | 0 \rangle = \partial_\nu^y \Delta_F(x - y)$$

$$\begin{aligned} \partial_\mu^x \partial_\nu^y \Delta_F(x - y) &= \partial_\mu^x \partial_\nu^y \langle 0 | T (\phi(x) \phi^\dagger(y)) | 0 \rangle \\ &= \langle 0 | T (\partial_\mu \phi(x) \partial_\nu \phi^\dagger(y)) | 0 \rangle + \eta_{\mu 0} \delta(x_0 - y_0) \langle 0 | [\phi(x), \partial_\nu \phi^\dagger(y)] | 0 \rangle \end{aligned}$$

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$$\langle 0 | T (\phi(x) \partial_\nu \phi^\dagger(y)) | 0 \rangle = \partial_\nu^y \Delta_F(x - y)$$

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$$\eta_{\mu 0} \delta(x_0 - y_0) \langle 0 | [\phi(x), \partial_\nu \phi^\dagger(y)] | 0 \rangle = \eta_{\mu 0} \delta(x_0 - y_0) \partial_\nu^y i \Delta(x - y)$$

# Teoría de QED escalar

$$\langle 0 | T (\phi(x) \partial_\nu \phi^\dagger(y)) | 0 \rangle = \partial_\nu^y \Delta_F(x - y)$$

$$\begin{aligned} \partial_\mu^x \partial_\nu^y \Delta_F(x - y) &= \partial_\mu^x \partial_\nu^y \langle 0 | T (\phi(x) \phi^\dagger(y)) | 0 \rangle \\ &= \langle 0 | T (\partial_\mu \phi(x) \partial_\nu \phi^\dagger(y)) | 0 \rangle + \eta_{\mu 0} \delta(x_0 - y_0) \langle 0 | [\phi(x), \partial_\nu \phi^\dagger(y)] | 0 \rangle \\ &\quad \eta_{\mu 0} \delta(x_0 - y_0) \langle 0 | [\phi(x), \partial_\nu \phi^\dagger(y)] | 0 \rangle = \\ &= \eta_{\mu 0} \delta(x_0 - y_0) \int \frac{d^3 p}{(2\pi)^3 2\omega_p} \left[ i p_\nu e^{ip(x-y)} - (-i p_\nu) e^{-ip(x-y)} \right] \end{aligned}$$

# Teoría de QED escalar

$$\langle 0 | T (\phi(x) \partial_\nu \phi^\dagger(y)) | 0 \rangle = \partial_\nu^y \Delta_F(x - y)$$

$$\begin{aligned} \partial_\mu^x \partial_\nu^y \Delta_F(x - y) &= \partial_\mu^x \partial_\nu^y \langle 0 | T (\phi(x) \phi^\dagger(y)) | 0 \rangle \\ &= \langle 0 | T (\partial_\mu \phi(x) \partial_\nu \phi^\dagger(y)) | 0 \rangle + \eta_{\mu 0} \delta(x_0 - y_0) \langle 0 | [\phi(x), \partial_\nu \phi^\dagger(y)] | 0 \rangle \end{aligned}$$

$$\eta_{\mu 0} \delta(x_0 - y_0) \langle 0 | [\phi(x), \partial_\nu \phi^\dagger(y)] | 0 \rangle =$$

$$= i \eta_{\mu 0} \delta(x_0 - y_0) \int \frac{d^3 p}{(2\pi)^3 2\omega_p} \left[ p_\nu e^{ip(x-y)} + p_\nu e^{-ip(x-y)} \right]$$

# Teoría de QED escalar

$$\langle 0 | T (\phi(x) \partial_\nu \phi^\dagger(y)) | 0 \rangle = \partial_\nu^y \Delta_F(x - y)$$

$$\begin{aligned} \partial_\mu^x \partial_\nu^y \Delta_F(x - y) &= \partial_\mu^x \partial_\nu^y \langle 0 | T (\phi(x) \phi^\dagger(y)) | 0 \rangle \\ &= \langle 0 | T (\partial_\mu \phi(x) \partial_\nu \phi^\dagger(y)) | 0 \rangle + \eta_{\mu 0} \delta(x_0 - y_0) \langle 0 | [\phi(x), \partial_\nu \phi^\dagger(y)] | 0 \rangle \end{aligned}$$

$$\eta_{\mu 0} \delta(x_0 - y_0) \langle 0 | [\phi(x), \partial_\nu \phi^\dagger(y)] | 0 \rangle =$$

$$= i \eta_{\mu 0} \eta_{\nu 0} \delta(x_0 - y_0) \int \frac{d^3 p}{(2\pi)^3 2\omega_p} 2\omega_p e^{ip(x-y)}$$

# Teoría de QED escalar

$$\langle 0 | T \left( \phi(x) \partial_\nu \phi^\dagger(y) \right) | 0 \rangle = \partial_\nu^y \Delta_F(x - y)$$

$$\begin{aligned} \partial_\mu^x \partial_\nu^y \Delta_F(x - y) &= \partial_\mu^x \partial_\nu^y \langle 0 | T \left( \phi(x) \phi^\dagger(y) \right) | 0 \rangle \\ &= \langle 0 | T \left( \partial_\mu \phi(x) \partial_\nu \phi^\dagger(y) \right) | 0 \rangle + \eta_{\mu 0} \delta(x_0 - y_0) \langle 0 | [\phi(x), \partial_\nu \phi^\dagger(y)] | 0 \rangle \end{aligned}$$

$$\eta_{\mu 0} \delta(x_0 - y_0) \langle 0 | [\phi(x), \partial_\nu \phi^\dagger(y)] | 0 \rangle =$$

$$= i \eta_{\mu 0} \eta_{\nu 0} \delta(x - y)$$

# Teoría de QED escalar

$$\langle 0 | T (\phi(x) \partial_\nu \phi^\dagger(y)) | 0 \rangle = \partial_\nu^y \Delta_F(x - y)$$

$$\langle 0 | T (\partial_\mu \phi(x) \partial_\nu \phi^\dagger(y)) | 0 \rangle = \partial_\mu^x \partial_\nu^y \Delta_F(x - y) - i \eta_{\mu 0} \eta_{\nu 0} \delta(x - y)$$

# Teoría de QED escalar

$$S^{(2)} = 2 \frac{(-i)^2}{2!} (i)^2 \int dx dy \phi^\dagger(x) \overleftrightarrow{\partial_\mu} \phi(x) A^\mu(x) \phi^\dagger(y) \overleftrightarrow{\partial_\nu} \phi(y) A^\nu(y) + \dots$$

# Teoría de QED escalar

$$S^{(2)} = (-i)^2 (i)^2 \int dx dy [\phi^\dagger(x) \overline{\partial_\mu \phi(x)} \phi(y) \partial_\nu \phi^\dagger(y) + \\ + \partial_\mu \phi^\dagger(x) \overline{\phi(x)} \phi(y) \partial_\nu \phi^\dagger(y) - \partial_\mu \phi^\dagger(x) \overline{\phi(x)} \phi(y) \partial_\nu \phi^\dagger(y) \\ - \phi^\dagger(x) \overline{\partial_\mu \phi(x)} \partial_\nu \phi(y) \phi^\dagger(y)] A^\mu(x) A^\nu(y) + \dots]$$

# Teoría de QED escalar

$$\begin{aligned} S^{(2)} = & (-i)^2 (i)^2 \int dx dy [\phi^\dagger(x) \partial_\nu \phi(y) \partial_\mu^x \Delta_F(x-y) + \\ & + \partial_\mu \phi^\dagger(x) \phi(y) \partial_\nu^y \Delta_F(x-y) - \partial_\mu \phi^\dagger(x) \partial_\mu \phi^\dagger(y) \Delta_F(x-y) + \\ & + \phi^\dagger(x) \phi^\dagger(y) (\partial_\mu^x \partial_\nu^y \Delta_F(x-y) - i \eta_{\mu 0} \eta_{\nu 0} \delta(x-y)) A^\mu(x) A^\nu(y)] + \dots \end{aligned}$$

# Teoría de QED escalar

$$\begin{aligned} S^{(2)} = & (-i)^2 (i)^2 \int dx dy (\phi^\dagger(x) \overleftrightarrow{\partial}_\mu^\lambda \Delta_F(x-y) \overleftrightarrow{\partial}_\mu^\lambda \phi(y) A^\mu(x) A^\nu(y) + \\ & + i \int dx \phi^\dagger(x) \phi(x) A^0(x) A^0(x) \\ & + i \int dx \phi^\dagger(x) \phi(x) \overline{A^0(x) A^0(x)} - i \int dx \phi^\dagger(x) \phi(x) \overline{A^0(x) A^0(x)} + \dots \end{aligned}$$

# Teoría de QED escalar

$$\tilde{\mathcal{H}}^{\text{int}} = ie\phi^\dagger \overleftrightarrow{\partial}_\mu \phi A^\mu - e^2 g_{\mu\nu} \phi^\dagger \phi A_\mu A_\nu$$

$$\partial_\mu^x \partial_\nu^y \tilde{T}(\phi(x)\phi^\dagger(y)) = T(\partial_\mu \phi(x) \partial_\nu \phi^\dagger(y))$$

$$S = \tilde{T} \exp \left( \int dx \tilde{\mathcal{H}}^{\text{int}}(x) \right)$$

$$\overline{\partial_\mu \phi(x) \partial_\nu \phi^\dagger(y)} = \partial_\mu^x \partial_\nu^y \Delta_F(x-y)$$

# Teoría de QED escalar



$$\begin{aligned} A = & (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqw} \frac{q^2(-g_{\mu\nu})}{i} \epsilon^\nu \\ & \times \int dz T \left( \phi^\dagger(z) \overleftrightarrow{\partial}_\mu \phi(z) A^\mu(z) \phi(y) \phi^\dagger(x) A^\nu(w) \right) \end{aligned}$$

# Teoría de QED escalar

$$A = (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz$$
$$\times T (\phi^\dagger(z) \partial_\mu \phi(z) \phi(y) \phi^\dagger(x) - \partial_\mu \phi^\dagger(z) \phi(z) \phi(y) \phi^\dagger(x))$$

# Teoría de QED escalar

$$A = (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz$$
$$\times \left[ \overline{\phi^\dagger(z) \partial_\mu \phi(z)} \overline{\phi(y) \phi^\dagger(x)} - \overline{\partial_\mu \phi^\dagger(z)} \overline{\phi(z) \phi(y) \phi^\dagger(x)} \right]$$

# Teoría de QED escalar

$$\begin{aligned} A &= (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz \\ &\quad \times [\Delta_F(y-z) \partial_\mu^z \Delta_F(z-x) - \partial_\mu^z \Delta_F(y-z) \Delta_F(z-x)] \\ &= A_1 - A_2 \end{aligned}$$

$$\begin{aligned} A_1 &= (-i)ie \epsilon^\mu \frac{(k^2 - m^2)}{i} \frac{(p^2 - m^2)}{i} \int dz \left( \int dy e^{ipy} \Delta_F(y-z) \right) \\ &\quad \times \left( \partial_\mu^z \int dx e^{-ikx} \Delta_F(z-x) \right) \end{aligned}$$

# Teoría de QED escalar

$$\begin{aligned} A &= (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz \\ &\quad \times [\Delta_F(y-z) \partial_\mu^z \Delta_F(z-x) - \partial_\mu^z \Delta_F(y-z) \Delta_F(z-x)] \\ &= A_1 - A_2 \end{aligned}$$

$$\begin{aligned} A_1 &= (-i)ie \epsilon^\mu \frac{(k^2 - m^2)}{i} \frac{(p^2 - m^2)}{i} \int dz \left( \int dy e^{ip(\tilde{y}+z)} \Delta_F(\tilde{y}) \right) \\ &\quad \times \left( \partial_\mu^z \int dx e^{-ik(z-\tilde{x})} \Delta_F(\tilde{x}) \right) \end{aligned}$$

# Teoría de QED escalar

$$\begin{aligned} A &= (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz \\ &\quad \times [\Delta_F(y-z) \partial_\mu^z \Delta_F(z-x) - \partial_\mu^z \Delta_F(y-z) \Delta_F(z-x)] \\ &= A_1 - A_2 \end{aligned}$$

$$A_1 = (-i)ie\epsilon^\mu \frac{(k^2 - m^2)}{i} \frac{(p^2 - m^2)}{i} \int dz e^{ipz} \frac{i}{p^2 - m^2} \left( \partial_\mu^z e^{-ikz} \frac{i}{k^2 - m^2} \right)$$

# Teoría de QED escalar

$$\begin{aligned} A &= (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz \\ &\quad \times [\Delta_F(y-z) \partial_\mu^z \Delta_F(z-x) - \partial_\mu^z \Delta_F(y-z) \Delta_F(z-x)] \\ &= A_1 - A_2 \end{aligned}$$

$$A_1 = (-i)ie\epsilon^\mu \int dz e^{ipz} e^{-iqz} e^{-ikz} (-ik_\mu)$$

# Teoría de QED escalar

$$\begin{aligned} A &= (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz \\ &\quad \times [\Delta_F(y-z) \partial_\mu^z \Delta_F(z-x) - \partial_\mu^z \Delta_F(y-z) \Delta_F(z-x)] \\ &= A_1 - A_2 \end{aligned}$$

$$A_1 = ie\epsilon^\mu \delta(p - k - q)(-k_\mu)$$

# Teoría de QED escalar

$$\begin{aligned} A &= (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz \\ &\quad \times [\Delta_F(y-z) \partial_\mu^z \Delta_F(z-x) - \partial_\mu^z \Delta_F(y-z) \Delta_F(z-x)] \\ &= ie\epsilon^\mu \delta(p-k-q)(-k_\mu) - A_2 \end{aligned}$$

$$\begin{aligned} A_2 &= (-i)ie\epsilon^\mu \frac{(k^2 - m^2)}{i} \frac{(p^2 - m^2)}{i} \int dz \left( \partial_\mu^z \int dy e^{ipy} \Delta_F(y-z) \right) \\ &\quad \left( \int dx e^{-ikx} \Delta_F(z-x) \right) \end{aligned}$$

# Teoría de QED escalar

$$\begin{aligned} A &= (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz \\ &\quad \times [\Delta_F(y - z) \partial_\mu^z \Delta_F(z - x) - \partial_\mu^z \Delta_F(y - z) \Delta_F(z - x)] \\ &= ie\epsilon^\mu \delta(p - k - q) (-k_\mu) - A_2 \end{aligned}$$

$$\begin{aligned} A_2 &= (-i)ie\epsilon^\mu \frac{(k^2 - m^2)}{i} \frac{(p^2 - m^2)}{i} \int dz \left( \partial_\mu^z \int dy e^{ip(\tilde{y}+z)} \Delta_F(\tilde{y}) \right) \\ &\quad \left( \int dx e^{-ik(z-\tilde{x})} \Delta_F(\tilde{x}) \right) \end{aligned}$$

# Teoría de QED escalar

$$\begin{aligned} A &= (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz \\ &\quad \times [\Delta_F(y-z) \partial_\mu^z \Delta_F(z-x) - \partial_\mu^z \Delta_F(y-z) \Delta_F(z-x)] \\ &= ie\epsilon^\mu \delta(p-k-q)(-k_\mu) - A_2 \end{aligned}$$

$$\begin{aligned} A_2 &= (-i)ie\epsilon^\mu \frac{(k^2 - m^2)}{i} \frac{(p^2 - m^2)}{i} \left( \partial_\mu^z \int dz e^{ipz} \frac{i}{p^2 - m^2} \right) \\ &\quad \left( e^{-ikz} \frac{i}{k^2 - m^2} \right) \end{aligned}$$

# Teoría de QED escalar

$$\begin{aligned} A &= (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz \\ &\quad \times [\Delta_F(y-z) \partial_\mu^z \Delta_F(z-x) - \partial_\mu^z \Delta_F(y-z) \Delta_F(z-x)] \\ &= ie\epsilon^\mu \delta(p-k-q)(-k_\mu) - A_2 \end{aligned}$$

$$A_2 = (-i)ie\epsilon^\mu \int dz e^{ipz} e^{-iqz} e^{-ikz} (ip_\mu)$$

# Teoría de QED escalar

$$\begin{aligned} A &= (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz \\ &\quad \times [\Delta_F(y-z) \partial_\mu^z \Delta_F(z-x) - \partial_\mu^z \Delta_F(y-z) \Delta_F(z-x)] \\ &= ie\epsilon^\mu \delta(p-k-q)(-k_\mu) - A_2 \end{aligned}$$

$$A_2 = ie\epsilon^\mu \delta(p-k-q)(p_\mu)$$

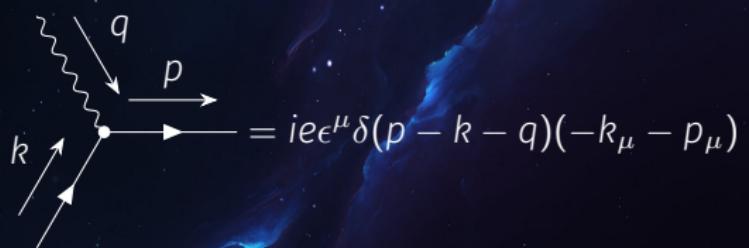
# Teoría de QED escalar

$$\begin{aligned} A &= (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz \\ &\quad \times [\Delta_F(y-z) \partial_\mu^z \Delta_F(z-x) - \partial_\mu^z \Delta_F(y-z) \Delta_F(z-x)] \\ &= ie\epsilon^\mu \delta(p-k-q)(-k_\mu) - ie\epsilon^\mu \delta(p-k-q)(p_\mu) \end{aligned}$$

# Teoría de QED escalar

$$\begin{aligned} A &= (-i)ie \int dy \int dx \int dw e^{ipy} \frac{(p^2 - m^2)}{i} e^{-ikx} \frac{(k^2 - m^2)}{i} e^{-iqz} \epsilon^\nu \int dz \\ &\quad \times [\Delta_F(y-z) \partial_\mu^z \Delta_F(z-x) - \partial_\mu^z \Delta_F(y-z) \Delta_F(z-x)] \\ &= ie\epsilon^\mu \delta(p-k-q)(-k_\mu - p_\mu) \end{aligned}$$

# Teoría de QED escalar



A Feynman diagram illustrating a vertex correction in scalar Quantum Electrodynamics (QED). The diagram shows a central vertex connected by three external lines: a horizontal solid line labeled  $p$ , a diagonal wavy line labeled  $q$ , and a diagonal solid line labeled  $k$ . The wavy line  $q$  has an arrow pointing downwards, while the solid lines  $p$  and  $k$  have arrows pointing to the right. To the right of the vertex, the expression  $= ie\epsilon^\mu \delta(p - k - q)(-k_\mu - p_\mu)$  is written, representing the vertex correction term.

# Teoría de QED escalar

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Realizar todos los diagramas de Feynman conectados y amputados.

# Teoría de QED escalar

Realizar todos los diagramas de Feynman conectados y amputados.

## Reglas de Feynman en el espacio de momentos

### 1. Patas externas

The image shows five Feynman diagrams illustrating the rules for external lines:

- A horizontal line with an arrow pointing right, ending in a black dot, labeled  $p$ , followed by  $= 1,$ .
- A horizontal line with an arrow pointing left, ending in a black dot, labeled  $\bar{p}$ , followed by  $= 1,$ .
- A wavy line ending in a black dot, labeled  $q$ , followed by  $= \epsilon^\mu(q),$ .
- A horizontal line with an arrow pointing right, ending in a black dot, labeled  $p$ , followed by  $= 1$ .
- A horizontal line with an arrow pointing left, ending in a black dot, labeled  $\bar{p}$ , followed by  $= 1$ .
- A wavy line ending in a black dot, labeled  $q$ , followed by  $= \epsilon^\mu(q)^*$ .

# Teoría de QED escalar

## 2. Líneas internas

$$\overrightarrow{p} = \frac{i}{p^2 - m^2 + i\epsilon}$$

$$\overbrace{\qquad\qquad\qquad}^q = \frac{-ig_{\mu\nu}}{q^2 + i\epsilon}$$

## Teoría de QED escalar

## 2. Líneas internas

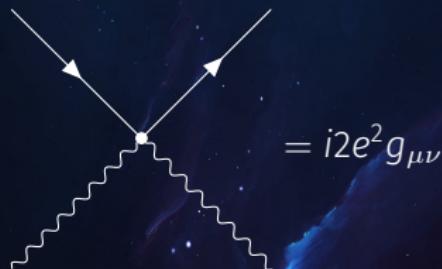
$$\frac{p}{\not{p}} = \frac{i}{p^2 - m^2 + i\epsilon}$$

$$\sim\!\!\!\sim\!\!\!\sim q = \frac{-ig_{\mu\nu}}{q^2 + i\epsilon}$$

### 3. Vértices



# Teoría de QED escalar



4. Conservación de momento sobre los vértices  
 $(2\pi)^4 \delta \left( \sum_{j \text{ saliente}} p'_j - \sum_{i \text{ entrante}} p_i \right).$

# Teoría de QED escalar



4. Conservación de momento sobre los vértices  
$$(2\pi)^4 \delta \left( \sum_{j \text{ saliente}} p'_j - \sum_{i \text{ entrante}} p_i \right).$$
5. Integraremos sobre el momento  $p$  de cada línea interna:  $\int \frac{d^4 p}{(2\pi)^4}$ .

# Teoría de QED escalar



4. Conservación de momento sobre los vértices  
 $(2\pi)^4 \delta \left( \sum_{j \text{ saliente}} p'_j - \sum_{i \text{ entrante}} p_i \right).$
5. Integraremos sobre el momento  $p$  de cada línea interna:  $\int \frac{d^4 p}{(2\pi)^4}$ .
6. Dividimos por el factor de simetría.