Fenómenos Colectivos en Sólidos – Pablo I. Tamborenea
DF-FCEN-UBA, Primer cuatrimestre 2021
Charla final (6/7/2021)

Función de Green

Máximo Riso

Pre Intro:
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Dado un operador lineal diferencial $\mathcal{L}$ y una función $f(\mathbf{r})$, la función de Green $G(\mathbf{r},\mathbf{s})$ es una solución de la ecuación

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viene dada por $\phi(\mathbf{r}) = \int f(\mathbf{s})\, G(\mathbf{r},\mathbf{s})\, ds$

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multiplicamos por exponenciales e integramos en τ.

Inhomogeneidad sólo depende de τ=t-t', entonces *G* también

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escribimos G por su transformada

$$G(\tau) = \frac{1}{2\pi} \sum_{\omega'} G(\omega')\, e^{-i\omega'\tau}$$

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despejamos

$$G(\omega) = \frac{-1}{m_0}\,\frac{1}{(\omega^2 + 2\gamma i\omega - \omega_0^2)}$$

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despejamos y reescribimos

$$G(\omega) = \frac{-1}{2m_0\,\omega_0'}\left(\frac{1}{\omega-(\omega_0'-i\gamma)} - \frac{1}{\omega-(-\omega_0'-i\gamma)}\right)$$

donde $(\omega_0')^2 = \omega_0^2 - \gamma^2$

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Volviendo a $G(\tau)$ como antitransformada

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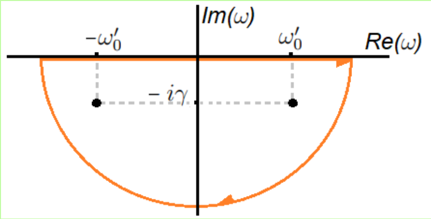
Puede reemplazarse G(ω) e integrarse



$$G(\tau) = -\frac{i\Theta(\tau)}{2m_0\omega_0'}\left(e^{-(i\omega_0'+\gamma)\tau} - e^{(i\omega_0'-\gamma)\tau}\right)$$

Se agrega $\Theta(\tau)$ porque debe ser $G^r(\tau) = 0$ si $\tau < 0$ ($t < t'$), esto equivale que sea analítica en Im(ω) > 0

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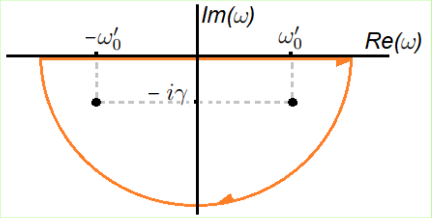
función de Green del oscilador



Se agrega $\Theta(\tau)$ porque debe ser $G^r(\tau) = 0$ si $\tau < 0$ ($t < t'$), esto equivale que sea analítica en $Im(\omega) > 0$

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Volviendo a la ec. del incio

A modo de ejemplo de aplicación

$$m_0\left(\frac{\partial^2}{\partial t^2}+2\gamma\frac{\partial}{\partial t}+\omega_0^2\right)x(t)=e\,\mathcal{E}(t)$$

$$x(t)=\int_{-\infty}^{+\infty}G(t-t')\,e\,\mathcal{E}(t')\,dt'$$

Sabíamos que puede calcularse x(t) así

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multiplicamos por exponencial e integramos en t

$$\int x(t)\, e^{i\omega t}\, dt = \int \int G(t-t')\, e^{i\omega t}\, e\,\mathcal{E}(t')\, dt'\, dt =$$
$$= \int G(t-t')\, e^{i\omega(t-t')}\, dt \int e\,\mathcal{E}(t') e^{i\omega t'}\, dt'$$

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$x(\omega) = e\, G(\omega)\, \mathcal{E}(\omega)$

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de las relaciones
$P(\omega) = e\, n_0\, x(\omega)$
$P(\omega) = \chi(\omega)\, \mathcal{E}(\omega)$
resulta
$\chi(\omega) = e^2\, n_0\, G(\omega)$

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$\chi(\omega)$ = e^2 n_0 $G(\omega)$

se obtiene la susceptibilidad

$$\chi(\omega) = -\frac{e^2 n_0}{2 m_0\, \omega_0'} \left(\frac{1}{\omega - (\omega_0' - i\gamma)} - \frac{1}{\omega - (-\omega_0' - i\gamma)} \right)$$

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Definición: función de Green

$$G^r_{ss'}(\mathbf{r}t, \mathbf{r}'t') = -\frac{i}{\hbar}\Theta(t - t') \left\langle \{\psi_s(\mathbf{r}, t), \psi^\dagger_{s'}(\mathbf{r}', t')\} \right\rangle$$

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Más explícita

$$G^r_{ss'}(\mathbf{r}t, \mathbf{r}'t') = \Theta(t-t')\Big(-\frac{i}{\hbar}\left\langle\psi_s(\mathbf{r},t), \psi^\dagger_{s'}(\mathbf{r}',t')\right\rangle - \frac{i}{\hbar}\left\langle\psi^\dagger_{s'}(\mathbf{r}',t'), \psi_s(\mathbf{r},t)\right\rangle\Big)$$

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derivamos

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Propiedades de conmutadores a t=t’

ecuación de Heisenberg

$$i\hbar\frac{\partial\psi}{\partial t} = [\psi,\mathcal{H}]$$

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Ecuación de movimiento para la función de Green retardada

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Sistemas de muchos cuerpos: electrones no interactuantes

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$$\mathcal{H}_c = \sum_s \int \psi_s^\dagger(\mathbf{r}) \Big(-\frac{\hbar^2 \nabla^2}{2m} \Big) \psi_s(\mathbf{r})\, d^3r$$

Hamiltoniano de partícula libre

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Reemplazamos y usamos la def de *G*

$$i\hbar\frac{\partial}{\partial t}G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\,\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\rangle$$

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Es la función de Green

$$i\hbar \frac{\partial}{\partial t} G^r_{ss'}(\mathbf{r}t, \mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t') \langle \{ [\psi_s(\mathbf{r},t), \mathcal{H}], \psi_{s'}^\dagger(\mathbf{r}',t') \} \rangle$$

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$$\left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2\nabla^2}{2m}\right)G^r_{0ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'}$$

de partícula libre

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Inhomogeneidades sólo dependen de $\boldsymbol{\rho}=\mathbf{r}-\mathbf{r}'$ y $\tau=t-t'$ entonces *G* también

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Inhomogeneidades sólo dependen de $\boldsymbol{\rho}=\mathbf{r}-\mathbf{r}'$ y $\tau=t-t'$ entonces *G* también

Escribimos G por su transformada en $\boldsymbol{\rho}$ y τ, aplicamos los operadores, multiplicamos por exponenciales e integramos en $\boldsymbol{\rho}$ y τ.

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Inhomogeneidades sólo dependen de **ρ**=**r**-**r**' y τ=t-t' entonces *G* también

$$\left(\hbar\omega - \frac{\hbar^2k^2}{2m}\right)G^r_{0ss'}(\mathbf{k},\omega) = \delta_{ss'}$$

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$$\left(i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2\nabla^2}{2m} \right) G^r_{0ss'}(\mathbf{r}t, \mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'}$$

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Escribimos G por su transformada en **ρ** y τ, aplicamos los operadores, multiplicamos por exponenciales e integramos en **ρ** y τ y despejamos.

$$\boxed{G^r_{0ss'}(\mathbf{k}, \omega) = \delta_{ss'} \frac{1}{\hbar(\omega - (\epsilon_k - i\delta))}}$$

$$\hbar\epsilon_k = \frac{\hbar^2 k^2}{2m}$$

Sistemas de muchos cuerpos: electrones no interactuantes

$$G^r_{0ss'}(\mathbf{k},\omega) = \delta_{ss'}\frac{1}{\hbar(\omega - (\epsilon_k - i\delta))}$$

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$$G^r_{0ss'}(\mathbf{k},\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} G^r_{0ss'}(\mathbf{k},\omega)\, e^{-i\omega\tau}\, d\omega$$

Escribimos G(**k**,ω) como antitransformada en ω

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$$\begin{aligned}\int_{-\infty}^{+\infty} G^r_{0ss'}(\mathbf{k},\omega)\, e^{-i\omega\tau}\, d\omega &= \int_{-\infty}^{+\infty}\frac{\delta_{ss'}}{\hbar(\omega-(\epsilon_k - i\delta))}\, e^{-i\omega\tau}\, d\omega \\ &= \frac{\delta_{ss'}}{\hbar}\left(-2\pi i\, e^{-i(\epsilon_k - i\delta)\tau}\right)\end{aligned}$$

reemplazamos G(**k**,ω) e integramos

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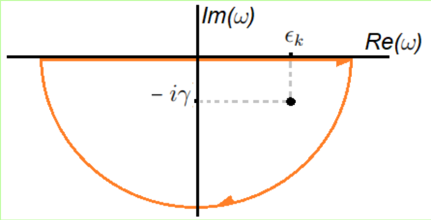
Nuevamente se hace sobre el semiplano complejo inferior y se tiene en cuenta el retardo consistentemente con Θ(τ)



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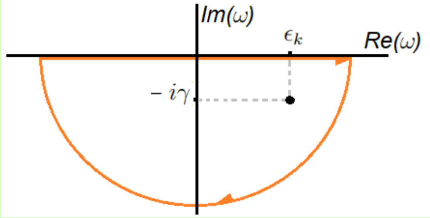

$$G^r_{0ss'}(\mathbf{k},\tau) = -\frac{i\Theta(\tau)}{\hbar}\, e^{-i(\epsilon_k - i\delta)\tau}$$

Sistemas de muchos cuerpos: electrones no interactuantes

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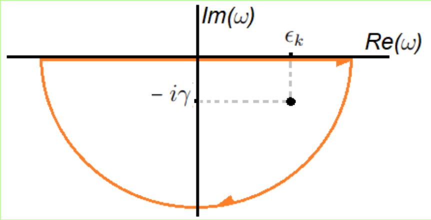

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función de Green retardada de electrones libres

Sistemas de muchos cuerpos:
electrones con interacción

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar \frac{\partial}{\partial t} G^r_{ss'}(\mathbf{r}t, \mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t') \left\langle \{[\psi_s(\mathbf{r},t), \mathcal{H}], \psi^\dagger_{s'}(\mathbf{r}',t')\} \right\rangle}$$

Ecuación de movimiento para la función de Green retardada

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Hamiltoniano

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Ecuación de movimiento para la función de Green retardada

Hamiltoniano

H = H_c + $H_{el\text{-}el}$ + $H_{el\text{-}f}$ + $H_{f\text{-}f}$

$$\mathcal{H}_c = \sum_s \int \psi^{\dagger}_s(\mathbf{r})\left(-\frac{\hbar^2\nabla^2}{2m}\right)\psi_s(\mathbf{r})\, d^3r$$

$$\mathcal{H}_{el-el} = \frac{1}{2}\sum_{ss'}\int \psi^{\dagger}_s(\mathbf{r})\psi^{\dagger}_{s'}(\mathbf{r}')\frac{e^2}{\epsilon_0|\mathbf{r}-\mathbf{r}'|}\,\psi_{s'}(\mathbf{r}')\psi_s(\mathbf{r})\, d^3r d^3r'$$

$$\mathcal{H}_{el-f} = -\sum_s \int \psi^{\dagger}_s(\mathbf{r})\frac{e^2 n(\mathbf{r}')}{\epsilon_0|\mathbf{r}-\mathbf{r}'|}\psi_s(\mathbf{r})\, d^3r d^3r'$$

$$\mathcal{H}_{f-f} = \frac{e^2}{2}\int \frac{n(\mathbf{r})n(\mathbf{r}')}{\epsilon_0|\mathbf{r}-\mathbf{r}'|}\, d^3r d^3r'$$

Sistemas de muchos cuerpos: electrones con interacción

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conmutamos

$$\mathcal{H}_c = \sum_s \int \psi^\dagger_s(\mathbf{r})\Big(-\frac{\hbar^2\nabla^2}{2m}\Big)\psi_s(\mathbf{r})\,d^3r \longrightarrow$$ ya sabemos

$$\mathcal{H}_{el-el} = \frac{1}{2}\sum_{ss'}\int \psi^\dagger_s(\mathbf{r})\psi^\dagger_{s'}(\mathbf{r}')\frac{e^2}{\epsilon_0|\mathbf{r}-\mathbf{r}'|}\psi_{s'}(\mathbf{r}')\psi_s(\mathbf{r})\,d^3r d^3r'$$

$$\mathcal{H}_{el-f} = -\sum_s \int \psi^\dagger_s(\mathbf{r})\frac{e^2 n(\mathbf{r}')}{\epsilon_0|\mathbf{r}-\mathbf{r}'|}\psi_s(\mathbf{r})\,d^3r d^3r'$$

$$\mathcal{H}_{f-f} = \frac{e^2}{2}\int \frac{n(\mathbf{r})n(\mathbf{r}')}{\epsilon_0|\mathbf{r}-\mathbf{r}'|}\,d^3r d^3r' \longrightarrow$$ no aporta

Sistemas de muchos cuerpos: electrones con interacción

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conmutamos

$$\begin{aligned}
[\psi_s(\mathbf{r},t), \mathcal{H}] = -\tfrac{\hbar^2\nabla^2}{2m}\psi_s(\mathbf{r},t) \\
+ \textstyle\sum_{s''} \int V(|\mathbf{r}-\mathbf{r}''|)\psi^\dagger_{s''}(\mathbf{r}'',t)\psi_{s''}(\mathbf{r}'',t)\psi_s(\mathbf{r},t)\, d^3r'' \\
- \textstyle\int V(|\mathbf{r}-\mathbf{r}''|) n(\mathbf{r}'')\psi_s(\mathbf{r},t)\, d^3r''
\end{aligned}$$

Sistemas de muchos cuerpos: electrones con interacción

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$$\begin{aligned}
[\psi_s(\mathbf{r},t),\mathcal{H}] = -\tfrac{\hbar^2\nabla^2}{2m}\psi_s(\mathbf{r},t) & \\
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-\textstyle\int V(|\mathbf{r}-\mathbf{r}''|)n(\mathbf{r}'')\psi_s(\mathbf{r},t)\,d^3r'' &
\end{aligned}$$

anticonmutamos

$$\begin{aligned}
\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\} = -\tfrac{\hbar^2\nabla^2}{2m}\{\psi_s,\psi^\dagger_{s'}\} & \\
+\textstyle\sum_{s''}\int V(|\mathbf{r}-\mathbf{r}''|)\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\,d^3r'' & \\
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$$\begin{aligned}[\psi_s(\mathbf{r},t),\mathcal{H}] = -\tfrac{\hbar^2\nabla^2}{2m}\psi_s(\mathbf{r},t) \\ + \textstyle\sum_{s''} \int V(|\mathbf{r}-\mathbf{r}''|)\psi^\dagger_{s''}(\mathbf{r}'',t)\psi_{s''}(\mathbf{r}'',t)\psi_s(\mathbf{r},t)\, d^3r'' \\ - \textstyle\int V(|\mathbf{r}-\mathbf{r}''|)n(\mathbf{r}'')\psi_s(\mathbf{r},t)\, d^3r''\end{aligned}$$

anticonmutamos

$$\boxed{\begin{aligned}\{[\psi_s(\mathbf{r},t),\mathcal{H}], \psi^\dagger_{s'}(\mathbf{r}',t')\} = -\tfrac{\hbar^2\nabla^2}{2m}\{\psi_s, \psi^\dagger_{s'}\} \\ + \textstyle\sum_{s''} \int V(|\mathbf{r}-\mathbf{r}''|)\{\psi^\dagger_{s''}\psi_{s''}\psi_s, \psi^\dagger_{s'}\}\, d^3r'' \\ - \textstyle\int V(|\mathbf{r}-\mathbf{r}''|)n(\mathbf{r}'')\{\psi_s, \psi^\dagger_{s'}\}\, d^3r''\end{aligned}}$$

$$\langle\{\psi^\dagger_{s''}(\mathbf{r}'',t)\psi_{s''}(\mathbf{r}'',t)\psi_s(\mathbf{r},t), \psi^\dagger_{s'}(\mathbf{r}',t')\}\rangle \equiv \langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s, \psi^\dagger_{s'}\}\rangle$$

notación abreviada

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar\frac{\partial}{\partial t}G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\boxed{\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\rangle}}$$

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-\textstyle\int V(|\mathbf{r}-\mathbf{r}''|)n(\mathbf{r}'')\psi_s(\mathbf{r},t)\,d^3r''
\end{aligned}$$

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\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\} = -\tfrac{\hbar^2\nabla^2}{2m}\{\psi_s,\psi^\dagger_{s'}\} \\
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-\textstyle\int V(|\mathbf{r}-\mathbf{r}''|)n(\mathbf{r}'')\{\psi_s,\psi^\dagger_{s'}\}\,d^3r''
\end{aligned}$$

tomamos valor de expectación

$$\boxed{\begin{aligned}
\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\rangle &= -\tfrac{\hbar^2\nabla^2}{2m}\langle\{\psi_s,\psi^\dagger_{s'}\}\rangle + \\
&+\textstyle\int V(|\mathbf{r}-\mathbf{r}''|)(\sum_{s''}\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle - n(\mathbf{r}'')\langle\{\psi_s,\psi^\dagger_{s'}\}\rangle)\,d^3r''
\end{aligned}}$$

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar \frac{\partial}{\partial t} G^r_{ss'}(\mathbf{r}t, \mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\,\langle\{[\psi_s(\mathbf{r},t), \mathcal{H}], \psi^\dagger_{s'}(\mathbf{r}',t')\}\rangle}$$

$$\begin{aligned}\langle\{[\psi_s(\mathbf{r},t), \mathcal{H}], \psi^\dagger_{s'}(\mathbf{r}',t')\}\rangle \; &= -\tfrac{\hbar^2\nabla^2}{2m}\langle\{\psi_s, \psi^\dagger_{s'}\}\rangle + \\ &+ \textstyle\int V(|\mathbf{r}-\mathbf{r}''|)\left(\sum_{s''}\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s, \psi^\dagger_{s'}\}\rangle - n(\mathbf{r}'')\langle\{\psi_s, \psi^\dagger_{s'}\}\rangle\right) d^3r''\end{aligned}$$

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar\frac{\partial}{\partial t}G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \underbrace{\frac{i}{\hbar}\Theta(t-t')}\,\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\rangle}$$

Es la función de Green

$$\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\rangle = -\frac{\hbar^2\nabla^2}{2m}\overbrace{\langle\{\psi_s,\psi^\dagger_{s'}\}\rangle} +$$

$$+\int V(|\mathbf{r}-\mathbf{r}''|)\left(\sum_{s''}\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle - n(\mathbf{r}'')\overbrace{\langle\{\psi_s,\psi^\dagger_{s'}\}\rangle}\right)d^3r''$$

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar \frac{\partial}{\partial t} G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\,\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\rangle}$$

$$\left(i\hbar\,\tfrac{\partial}{\partial t} + \overbrace{\tfrac{\hbar^2\nabla^2}{2m}\right) G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')} = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$

$$+\int V(|\mathbf{r}-\mathbf{r}''|)\left(\sum_{s''}\tfrac{i}{\hbar}\Theta(t-t')\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle - \overbrace{n(\mathbf{r}'')G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')}\right) d^3r''$$

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar \frac{\partial}{\partial t} G^r_{ss'}(\mathbf{r}t, \mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t') \left\langle \{[\psi_s(\mathbf{r},t), \mathcal{H}], \psi^\dagger_{s'}(\mathbf{r}',t')\} \right\rangle}$$

$$\left(i\hbar \tfrac{\partial}{\partial t} + \tfrac{\hbar^2 \nabla^2}{2m}\right) G^r_{ss'}(\mathbf{r}t, \mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$

$$+ \int V(|\mathbf{r}-\mathbf{r}''|)\left(\sum_{s''} \tfrac{i}{\hbar}\Theta(t-t')\boxed{\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s, \psi^\dagger_{s'}\}\rangle} - n(\mathbf{r}'')G^r_{ss'}(\mathbf{r}t, \mathbf{r}'t')\right) d^3r''$$

esta es la parte divertida

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar\frac{\partial}{\partial t}G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\left\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\right\rangle}$$

$$\left(i\hbar\,\tfrac{\partial}{\partial t}+\tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+\int V(|\mathbf{r}-\mathbf{r}''|)\left(\textstyle\sum_{s''}\tfrac{i}{\hbar}\Theta(t-t')\boxed{\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle}-n(\mathbf{r}'')G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')\right)d^3r''$$

$$\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\} = \psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi^\dagger_{s''}\psi_{s''}\psi_s =$$
$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s + \psi^\dagger_{s'}\psi^\dagger_{s''}\psi_{s''}\psi_s$$
$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - (1-\psi_{s''}\psi^\dagger_{s''})\psi^\dagger_{s'}\psi_s + \psi^\dagger_{s'}(1-\psi_{s''}\psi^\dagger_{s''})\psi_s$$
$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - \psi^\dagger_{s'}\psi_s + \psi_{s''}\psi^\dagger_{s''}\psi^\dagger_{s'}\psi_s + \psi^\dagger_{s'}\psi_s - \psi^\dagger_{s'}\psi_{s''}\psi^\dagger_{s''}\psi_s$$
$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - \psi_{s''}\psi^\dagger_{s'}\psi^\dagger_{s''}\psi_s - \psi^\dagger_{s'}\psi_{s''}\psi^\dagger_{s''}\psi_s$$
$$\psi^\dagger_{s''}\psi_{s''}(\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_s) - (\psi_{s''}\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_{s''})\psi^\dagger_{s''}\psi_s$$

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar\frac{\partial}{\partial t}G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\left\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\right\rangle}$$

$$\left(i\hbar\,\tfrac{\partial}{\partial t} + \tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+\int V(|\mathbf{r}-\mathbf{r}''|)\left(\textstyle\sum_{s''}\tfrac{i}{\hbar}\Theta(t-t')\boxed{\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle} - n(\mathbf{r}'')G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')\right)d^3r''$$

$$\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\} = \psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi^\dagger_{s''}\psi_{s''}\psi_s =$$
$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s + \psi^\dagger_{s'}\psi^\dagger_{s''}\psi_{s''}\psi_s$$
$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - (1-\psi_{s''}\psi^\dagger_{s''})\psi^\dagger_{s'}\psi_s + \psi^\dagger_{s'}(1-\psi_{s''}\psi^\dagger_{s''})\psi_s$$
$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - \psi^\dagger_{s'}\psi_s + \psi_{s''}\psi^\dagger_{s''}\psi^\dagger_{s'}\psi_s + \psi^\dagger_{s'}\psi_s - \psi^\dagger_{s'}\psi_{s''}\psi^\dagger_{s''}\psi_s$$
$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - \psi_{s''}\psi^\dagger_{s'}\psi^\dagger_{s''}\psi_s - \psi^\dagger_{s'}\psi_{s''}\psi^\dagger_{s''}\psi_s$$
$$\psi^\dagger_{s''}\psi_{s''}(\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_s) - (\psi_{s''}\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_{s''})\psi^\dagger_{s''}\psi_s$$

$$\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle = \langle\psi^\dagger_{s''}\psi_{s''}(\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_s)\rangle - \langle(\psi_{s''}\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_{s''})\psi^\dagger_{s''}\psi_s\rangle$$

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar\frac{\partial}{\partial t}G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\left\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\right\rangle}$$

$$\left(i\hbar\,\tfrac{\partial}{\partial t}+\tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+\int V(|\mathbf{r}-\mathbf{r}''|)\left(\textstyle\sum_{s''}\tfrac{i}{\hbar}\Theta(t-t')\boxed{\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle} - n(\mathbf{r}'')G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')\right)d^3r''$$

$$\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\} = \psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi^\dagger_{s''}\psi_{s''}\psi_s =$$
$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s + \psi^\dagger_{s'}\psi^\dagger_{s''}\psi_{s''}\psi_s$$
$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - (1-\psi_{s''}\psi^\dagger_{s''})\psi^\dagger_{s'}\psi_s + \psi^\dagger_{s'}(1-\psi_{s''}\psi^\dagger_{s''})\psi_s$$
$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - \psi^\dagger_{s'}\psi_s + \psi_{s''}\psi^\dagger_{s''}\psi^\dagger_{s'}\psi_s + \psi^\dagger_{s'}\psi_s - \psi^\dagger_{s'}\psi_{s''}\psi^\dagger_{s''}\psi_s$$
$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - \psi_{s''}\psi^\dagger_{s'}\psi^\dagger_{s''}\psi_s - \psi^\dagger_{s'}\psi_{s''}\psi^\dagger_{s''}\psi_s$$
$$\psi^\dagger_{s''}\psi_{s''}(\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_s) - (\psi_{s''}\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_{s''})\psi^\dagger_{s''}\psi_s$$

$$\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle = \langle\psi^\dagger_{s''}\psi_{s''}(\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_s)\rangle - \langle(\psi_{s''}\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_{s''})\psi^\dagger_{s''}\psi_s\rangle$$

$$\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle = \langle\psi^\dagger_{s''}\psi_{s''}\rangle\langle\{\psi_s,\psi^\dagger_{s'}\}\rangle - \langle\{\psi_{s''},\psi^\dagger_{s'}\}\rangle\langle\psi^\dagger_{s''}\psi_s\rangle$$

Random Phase Approximation

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar\frac{\partial}{\partial t}G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\left\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\right\rangle}$$

$$\left(i\hbar\tfrac{\partial}{\partial t} + \tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$

$$+\int V(|\mathbf{r}-\mathbf{r}''|)\left(\sum_{s''}\underbrace{\tfrac{i}{\hbar}\Theta(t-t')}\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle - n(\mathbf{r}'')G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')\right)d^3r''$$

$$\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\} = \psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi^\dagger_{s''}\psi_{s''}\psi_s =$$

$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s + \psi^\dagger_{s'}\psi^\dagger_{s''}\psi_{s''}\psi_s$$

$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - (1-\psi_{s''}\psi^\dagger_{s''})\psi^\dagger_{s'}\psi_s + \psi^\dagger_{s'}(1-\psi_{s''}\psi^\dagger_{s''})\psi_s$$

$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - \psi^\dagger_{s'}\psi_s + \psi_{s''}\psi^\dagger_{s''}\psi^\dagger_{s'}\psi_s + \psi^\dagger_{s'}\psi_s - \psi^\dagger_{s'}\psi_{s''}\psi^\dagger_{s''}\psi_s$$

$$\psi^\dagger_{s''}\psi_{s''}\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s''}\psi_{s''}\psi_{s''}\psi^\dagger_{s'}\psi_s - \psi_{s''}\psi^\dagger_{s'}\psi^\dagger_{s''}\psi_s - \psi^\dagger_{s'}\psi_{s''}\psi^\dagger_{s''}\psi_s$$

$$\psi^\dagger_{s''}\psi_{s''}(\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_s) - (\psi_{s''}\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_{s''})\psi^\dagger_{s''}\psi_s$$

$$\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle = \langle\psi^\dagger_{s''}\psi_{s''}(\psi_s\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_s)\rangle - \langle(\psi_{s''}\psi^\dagger_{s'} + \psi^\dagger_{s'}\psi_{s''})\psi^\dagger_{s''}\psi_s\rangle$$

$$\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle = \langle\psi^\dagger_{s''}\psi_{s''}\rangle\underbrace{\langle\{\psi_s,\psi^\dagger_{s'}\}\rangle} - \underbrace{\langle\{\psi_{s''},\psi^\dagger_{s'}\}\rangle}\langle\psi^\dagger_{s''}\psi_s\rangle$$

Random Phase Approximation

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar\frac{\partial}{\partial t}G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\left\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\right\rangle}$$

$$\left(i\hbar\tfrac{\partial}{\partial t}+\tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+\int V(|\mathbf{r}-\mathbf{r}''|)\left(\textstyle\sum_{s''}\frac{i}{\hbar}\Theta(t-t')\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle - n(\mathbf{r}'')G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')\right)d^3r''$$

$$\left(i\hbar\tfrac{\partial}{\partial t}+\tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+\int V(|\mathbf{r}-\mathbf{r}''|)\Big(\big(\textstyle\sum_{s''}\langle\psi^\dagger_{s''}\psi_{s''}\rangle - n(\mathbf{r}'')\big)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') -$$
$$-\textstyle\sum_{s''}\langle\psi^\dagger_{s''}\psi_s\rangle G^r_{s''s'}(\mathbf{r}''t,\mathbf{r}'t')\Big)\,d^3r''$$

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar\frac{\partial}{\partial t}G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\left\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\right\rangle}$$

$$\left(i\hbar\tfrac{\partial}{\partial t}+\tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+\int V(|\mathbf{r}-\mathbf{r}''|)\left(\sum_{s''}\tfrac{i}{\hbar}\Theta(t-t')\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle - n(\mathbf{r}'')G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')\right)d^3r''$$

$$\left(i\hbar\tfrac{\partial}{\partial t}+\tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+\int V(|\mathbf{r}-\mathbf{r}''|)\Big(\big(\sum_{s''}\langle\psi^\dagger_{s''}\psi_{s''}\rangle - n(\mathbf{r}'')\big)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') -$$
$$-\sum_{s''}\langle\psi^\dagger_{s''}\psi_s\rangle G^r_{s''s'}(\mathbf{r}''t,\mathbf{r}'t')\Big)\,d^3r''$$

definiendo la ***autoenergía retardada*** $\hbar\sum^r_{ss''}(\mathbf{r}t,\mathbf{r}''t'')$

$$\overbrace{\delta(t-t'')\Big(\delta(\mathbf{r}-\mathbf{r}'')\delta_{ss''}\int V(|\mathbf{r}-\tilde{r}|)\big(\sum_{\tilde{s}}\langle\psi^\dagger_{\tilde{s}}(\tilde{r},t)\psi_{\tilde{s}}(\tilde{r},t)\rangle - n(\tilde{r})\big)d\tilde{r}^3 - -V(|\mathbf{r}-\mathbf{r}''|)\langle\psi^\dagger_{s''}(\mathbf{r}'',t)\psi_s(\mathbf{r},t)\rangle\Big)}$$

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar\frac{\partial}{\partial t}G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\left\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\right\rangle}$$

$$\left(i\hbar\tfrac{\partial}{\partial t}+\tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+\int V(|\mathbf{r}-\mathbf{r}''|)\left(\sum_{s''}\tfrac{i}{\hbar}\Theta(t-t')\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle - n(\mathbf{r}'')G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')\right)d^3r''$$

$$\left(i\hbar\tfrac{\partial}{\partial t}+\tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+\int V(|\mathbf{r}-\mathbf{r}''|)\Big(\big(\sum_{s''}\langle\psi^\dagger_{s''}\psi_{s''}\rangle - n(\mathbf{r}'')\big)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') -$$
$$-\sum_{s''}\langle\psi^\dagger_{s''}\psi_s\rangle G^r_{s''s'}(\mathbf{r}''t,\mathbf{r}'t')\Big)\,d^3r''$$

definiendo la ***autoenergía retardada*** $\hbar\sum^r_{ss''}(\mathbf{r}t,\mathbf{r}''t'')$

$$\delta(t-t'')\Big(\delta(\mathbf{r}-\mathbf{r}'')\delta_{ss''}\int V(|\mathbf{r}-\tilde{r}|)\big(\sum_{\tilde{s}}\langle\psi^\dagger_{\tilde{s}}(\tilde{r},t)\psi_{\tilde{s}}(\tilde{r},t)\rangle - n(\tilde{r})\big)d\tilde{r}^3 -$$
$$-V(|\mathbf{r}-\mathbf{r}''|)\langle\psi^\dagger_{s''}(\mathbf{r}'',t)\psi_s(\mathbf{r},t)\rangle\Big)$$

Sistemas de muchos cuerpos: electrones con interacción

$$\boxed{i\hbar\frac{\partial}{\partial t}G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\left\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\right\rangle}$$

$$\left(i\hbar\tfrac{\partial}{\partial t}+\tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+\int V(|\mathbf{r}-\mathbf{r}''|)\left(\textstyle\sum_{s''}\frac{i}{\hbar}\Theta(t-t')\langle\{\psi^\dagger_{s''}\psi_{s''}\psi_s,\psi^\dagger_{s'}\}\rangle - n(\mathbf{r}'')G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')\right)d^3r''$$

$$\left(i\hbar\tfrac{\partial}{\partial t}+\tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+\int V(|\mathbf{r}-\mathbf{r}''|)\Big(\big(\textstyle\sum_{s''}\langle\psi^\dagger_{s''}\psi_{s''}\rangle - n(\mathbf{r}'')\big)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') -$$
$$-\textstyle\sum_{s''}\langle\psi^\dagger_{s''}\psi_s\rangle G^r_{s''s'}(\mathbf{r}''t,\mathbf{r}'t')\Big)\,d^3r''$$

$$\left(i\hbar\tfrac{\partial}{\partial t}+\tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$\boxed{+\textstyle\sum_{s''}\int\int\hbar\,\Sigma^r_{ss''}(\mathbf{r}t,\mathbf{r}''t'')G^r_{s''s'}(\mathbf{r}''t'',\mathbf{r}'t'))\,d^3r''dt''}$$

Sistemas de muchos cuerpos: electrones con interacción

$$i\hbar\frac{\partial}{\partial t}G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} - \frac{i}{\hbar}\Theta(t-t')\left\langle\{[\psi_s(\mathbf{r},t),\mathcal{H}],\psi^\dagger_{s'}(\mathbf{r}',t')\}\right\rangle$$

Ecuación de movimiento para la función de Green retardada

$$\left(i\hbar\,\tfrac{\partial}{\partial t} + \tfrac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+\textstyle\sum_{s''}\int\int\hbar\sum^r_{ss''}(\mathbf{r}t,\mathbf{r}''t'')G^r_{s''s'}(\mathbf{r}''t'',\mathbf{r}'t'))\,d^3r''dt''$$

Ecuación de Dyson para la función de Green retardada

Sistemas de muchos cuerpos: electrones con interacción

$$\left(i\hbar\,\frac{\partial}{\partial t} + \frac{\hbar^2\nabla^2}{2m}\right) G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'} +$$
$$+ \sum_{s''}\int\int \hbar \sum\nolimits^r_{ss''}(\mathbf{r}t,\mathbf{r}''t'')G^r_{s''s'}(\mathbf{r}''t'',\mathbf{r}'t'))\, d^3r''dt''$$

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$$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')=\delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'}+$$
$$+\sum_{s''}\int\int\hbar\sum\nolimits^r_{ss''}(\mathbf{r}t,\mathbf{r}''t'')G^r_{s''s'}(\mathbf{r}''t'',\mathbf{r}'t'))\,d^3r''dt''$$

Caso particular: 1) espín diagonal (simplicidad)
2) dependencia de $\rho=\mathbf{r}-\mathbf{r}'$ y $\tau=t-t'$

$$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2\nabla^2}{2m}\right)G^r_{ss}(\rho,\tau)=\delta(\tau)\delta(\rho)+$$
$$+\sum_{s''}\int\int\hbar\sum\nolimits^r_{ss''}(\mathbf{r}''-\mathbf{r}\,,t''-t\,)G^r_{s's'}(\mathbf{r}''-\mathbf{r}',t''-t')\,d^3r''dt''$$

Sistemas de muchos cuerpos: electrones con interacción

$$\left(i\hbar\,\frac{\partial}{\partial t}+\frac{\hbar^2\nabla^2}{2m}\right)G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')=\delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'}+$$
$$+\sum_{s''}\int\int\hbar\sum\nolimits^r_{ss''}(\mathbf{r}t,\mathbf{r}''t'')G^r_{s''s'}(\mathbf{r}''t'',\mathbf{r}'t'))\,d^3r''dt''$$

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$$\left(\hbar\omega-\frac{\hbar^2k^2}{2m}\right)G^r_{ss}(\mathbf{k},\omega)=1+\;\;\hbar\sum\nolimits^r_{ss}(\mathbf{k},\omega)\,G^r_{ss}(\mathbf{k},\omega)$$

transformamos Fourier

Sistemas de muchos cuerpos: electrones con interacción

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transformamos Fourier

$$\boxed{G^r_{ss}(\mathbf{k},\omega)=\frac{1}{\hbar(\omega-(\epsilon_k-i\delta+\sum^r_{ss}(\mathbf{k},\omega)))}}$$

función de Green retardada de electrones interactuantes

$$\tilde{\epsilon}_k=\epsilon_k+\sum\nolimits^r(\mathbf{k},\omega)=\frac{\hbar^2k^2}{2m}+\sum\nolimits^r(\mathbf{k},\omega)$$

se corre la energía de partícula libre

Resumiendo:

Resumiendo:

Función de Green retardada clásica de la ec. del oscilador inhomogenea

$$m_0 \left(\frac{\partial^2}{\partial t^2} + 2\gamma \frac{\partial}{\partial t} + \omega_0^2 \right) G(t-t') = \delta(t-t')$$

transformada de Fourier mediante

$$\boxed{G(\omega) = \frac{-1}{2m_0\,\omega_0'} \left(\frac{1}{\omega - (\omega_0' - i\gamma)} - \frac{1}{\omega - (-\omega_0' - i\gamma)} \right)}$$

$$\boxed{G(\tau) = -\frac{i\Theta(\tau)}{2m_0\omega_0'} \left(e^{-(i\omega_0'+\gamma)\tau} - e^{(i\omega_0'-\gamma)\tau} \right)}$$

Resumiendo:

Función de Green retardada clásica

de la ec. del oscilador inhomogenea

$$m_0\left(\frac{\partial^2}{\partial t^2}+2\gamma\frac{\partial}{\partial t}+\omega_0^2\right)G(t-t')=\delta(t-t')$$

transformada de Fourier mediante

$$\boxed{G(\omega)=\frac{-1}{2m_0\,\omega_0'}\left(\frac{1}{\omega-(\omega_0'-i\gamma)}-\frac{1}{\omega-(-\omega_0'-i\gamma)}\right)}$$

$$\boxed{G(\tau)=-\frac{i\Theta(\tau)}{2m_0\omega_0'}\left(e^{-(i\omega_0'+\gamma)\tau}-e^{(i\omega_0'-\gamma)\tau}\right)}$$

Función de Green muchas partículas

Definición: función de Green

$$G^r_{ss'}(\mathbf{r}t,\mathbf{r}'t')=-\frac{i}{\hbar}\Theta(t-t')\left\langle\{\psi_s(\mathbf{r},t),\psi^\dagger_{s'}(\mathbf{r}',t')\}\right\rangle$$

Resumiendo:

Función de Green retardada clásica

de la ec. del oscilador inhomogenea

$$m_0 \left(\frac{\partial^2}{\partial t^2} + 2\gamma \frac{\partial}{\partial t} + \omega_0^2 \right) G(t-t') = \delta(t-t')$$

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Función de Green muchas partículas

Definición: función de Green

$$G^r_{ss'}(\mathbf{r}t, \mathbf{r}'t') = -\frac{i}{\hbar}\Theta(t-t') \left\langle \{ \psi_s(\mathbf{r},t), \psi^\dagger_{s'}(\mathbf{r}',t') \} \right\rangle$$

a) Caso electrones no interactuantes

del Hamiltoniano y Heisenberg

$$\left(i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2 \nabla^2}{2m} \right) G^r_{0ss'}(\mathbf{r}t, \mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'}$$

transformada de Fourier mediante

$$\boxed{G^r_{0ss'}(\mathbf{k},\omega) = \delta_{ss'} \frac{1}{\hbar(\omega - (\epsilon_k - i\delta))}}$$

$$\hbar\epsilon_k = \frac{\hbar^2 k^2}{2m}$$

$$\boxed{G^r_{0ss'}(\mathbf{k},\tau) = -\frac{i\Theta(\tau)}{\hbar}\, e^{-i(\epsilon_k - i\delta)\tau}}$$

Resumiendo:

Función de Green retardada clásica

de la ec. del oscilador inhomogenea

$$m_0 \left(\frac{\partial^2}{\partial t^2} + 2\gamma \frac{\partial}{\partial t} + \omega_0^2 \right) G(t-t') = \delta(t-t')$$

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$$\boxed{G(\omega) = \frac{-1}{2m_0\,\omega_0'} \left(\frac{1}{\omega - (\omega_0' - i\gamma)} - \frac{1}{\omega - (-\omega_0' - i\gamma)} \right)}$$

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Función de Green muchas partículas

Definición: función de Green

$$G^r_{ss'}(\mathbf{r}t, \mathbf{r}'t') = -\frac{i}{\hbar}\Theta(t-t') \left\langle \{\psi_s(\mathbf{r},t), \psi^\dagger_{s'}(\mathbf{r}',t')\} \right\rangle$$

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$$\left(i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2\nabla^2}{2m} \right) G^r_{0ss'}(\mathbf{r}t, \mathbf{r}'t') = \delta(t-t')\delta(\mathbf{r}-\mathbf{r}')\delta_{ss'}$$

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b) Caso electrones con interacción

Caso particular

del Hamiltoniano y Heisenberg

$$\left(\hbar\omega - \frac{\hbar^2 k^2}{2m} \right) G^r_{ss}(\mathbf{k},\omega) = 1 + \hbar \textstyle\sum^r_{ss}(\mathbf{k},\omega)\, G^r_{ss}(\mathbf{k},\omega)$$

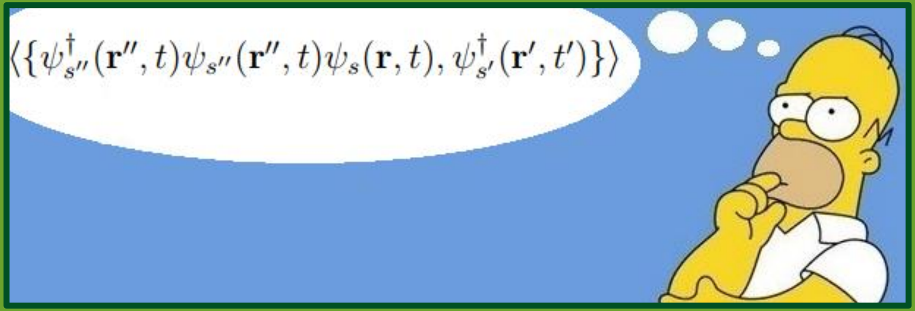
transformada de Fourier mediante

$$\boxed{G^r_{ss}(\mathbf{k},\omega) = \frac{1}{\hbar\left(\omega - \left(\epsilon_k - i\delta + \sum^r_{ss}(\mathbf{k},\omega)\right)\right)}}$$

$$\tilde{\epsilon}_k = \epsilon_k + \textstyle\sum^r(\mathbf{k},\omega) = \frac{\hbar^2 k^2}{2m} + \sum^r(\mathbf{k},\omega)$$

Muchísimas gracias por su atención.

¿Están ahí?



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