CLASE 16 - Viernes 22/05/2021

La clase pasada vimos:

En RPA:

Función polarización

Plasmón a $q = 0$

Contínuo de excitaciones partícula-hueco

En esta clase veremos:

Apantallamiento (screening) del plasma

REPASO

$$\frac{d}{dt}\langle a^\dagger_{\mathbf{k}-\mathbf{q}} a_{\mathbf{k}}\rangle \simeq i(\epsilon_{k-q} - \epsilon_k)\langle a^\dagger_{\mathbf{k}-\mathbf{q}} a_{\mathbf{k}}\rangle + \frac{iV_q}{\hbar}(f_{\mathbf{k}} - f_{\mathbf{k}-\mathbf{q}})\sum_{\mathbf{p}'}\langle a^\dagger_{\mathbf{p}'-\mathbf{q}} a_{\mathbf{p}'}\rangle$$

RPA

Hacemos el ansatz : $\langle a^\dagger_{\mathbf{k}-\mathbf{q}} a_{\mathbf{k}}\rangle(t) = e^{-i(\omega+i\delta)t}\langle a^\dagger_{\mathbf{k}-\mathbf{q}} a_{\mathbf{k}}\rangle(0)$

$$\Rightarrow \quad V_q \sum_{\mathbf{k}} \frac{f_{\mathbf{k}-\mathbf{q}} - f_{\mathbf{k}}}{\hbar(\omega_q + \epsilon_{k-q} - \epsilon_k)} = 1 \qquad (8.25)$$

Buscamos soluciones de $\omega(q)$ que satisfagan esta ecuación $\rightarrow$ modos normales

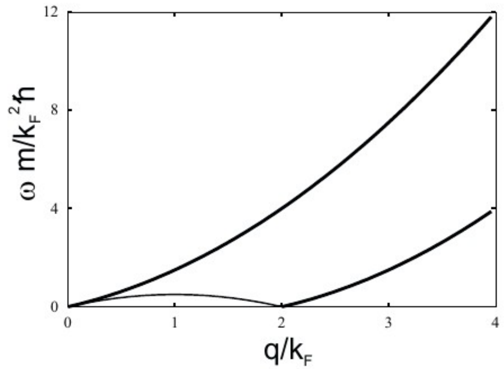
Soluciones con el ansatz

REPASO

Caso $\lambda \to \infty$ $\quad\Rightarrow\quad$ $$\omega_0^2 = \frac{4\pi e^2 n}{\epsilon_0 m} = \omega_{pl}^2$$

$$\omega_{q\to 0} = \omega_0$$

Recuperamos el resultado obtenido con el modelo de Drude: $q \to 0\,,\ \omega_{q\to 0} = \omega_{pl}$

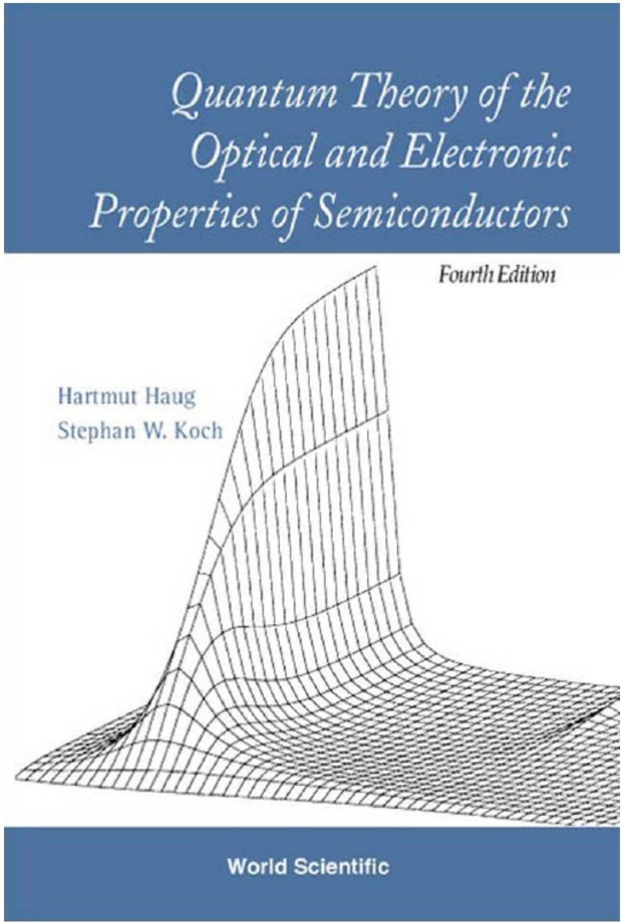


$$\omega_q^{max} = \frac{\hbar q k_F}{m} + \frac{\hbar q^2}{2m}$$

$$\omega_q^{min} = -\frac{\hbar q k_F}{m} + \frac{\hbar q^2}{2m}$$

Fig. 8.2 The thick lines show the boundary of the continuum of pair excitations at $T = 0\,K$, according to Eqs. (8.35) and (8.36), respectively. The thin line is the result of Eq. (8.37).

Estamos viendo: plasmones



Apantallamiento

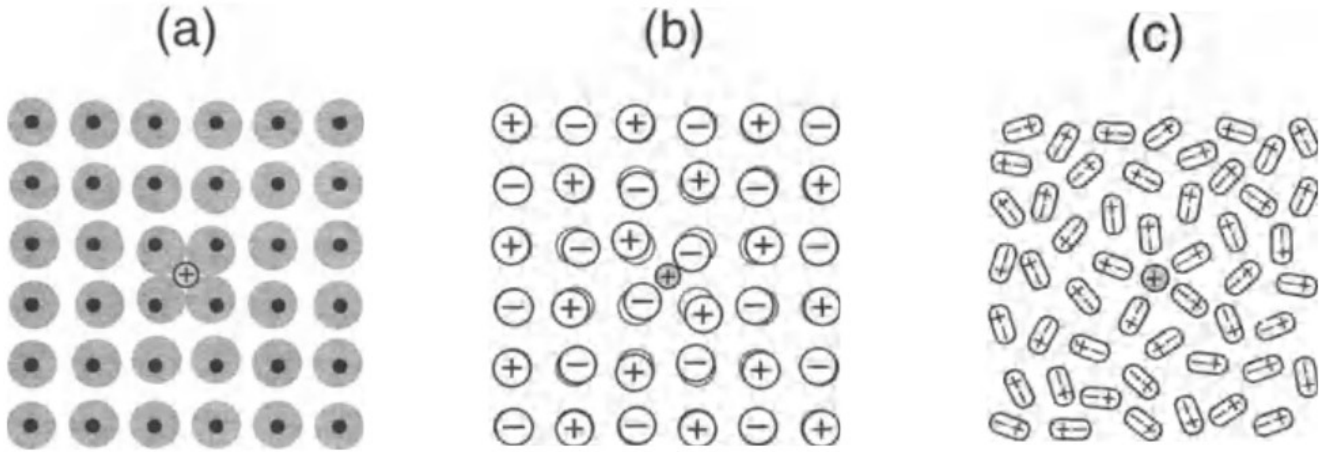


FIGURE Int.2. The three essential mechanisms for screening in insulators in the presence of a point charge (here positive point charge located at the center of the sample). (a) Polarization of the valence-electron clouds (electronic polarizability). The electronic clouds, schematized by the shaded areas, are displaced toward the positive charge. The ions, schematized by dark spots, stay at their unperturbed positions. (b) Displacement of the ionic cores (vibrational polarizability). The unperturbed positions of the ions appear as shaded circles. (c) Orientation of molecular dipoles in a polar liquid (orientational polarizability).

Jean-Noël Chazalviel

Coulomb Screening by Mobile Charges

Applications to Materials Science, Chemistry, and Biology

Apantallamiento del plasma

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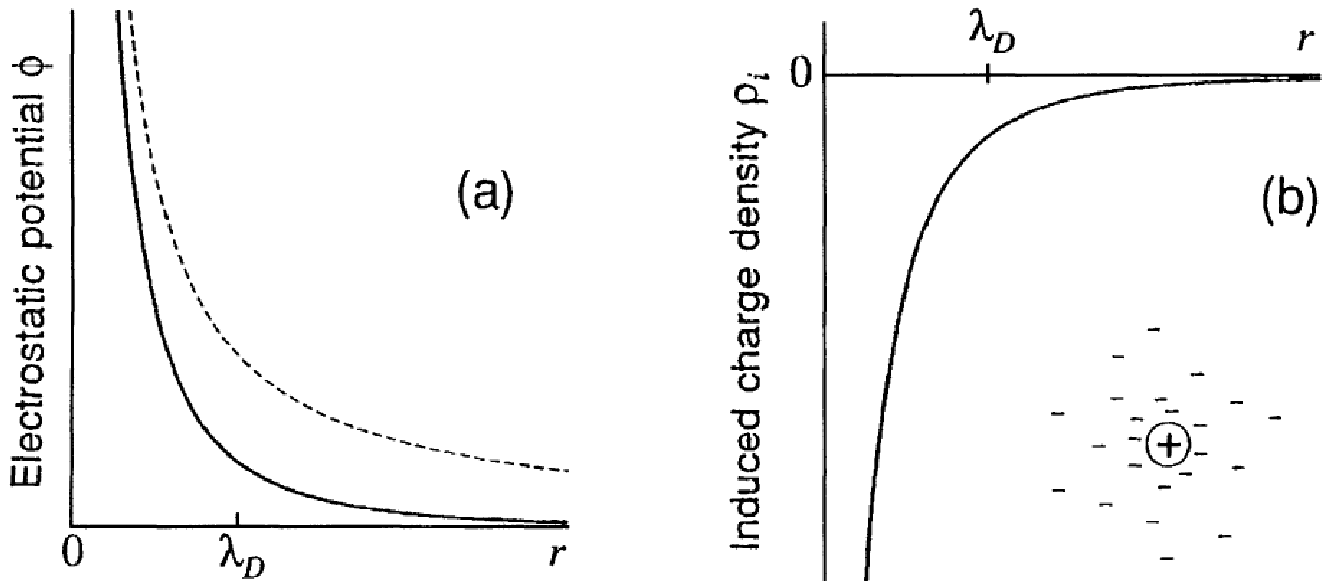


FIGURE Int.3. (a) Screened Coulomb potential around a point charge (here positive) in a Boltzmann gas of charged particles. The normal Coulomb potential is shown for comparison, as the dashed line. (b) Induced charge density in the screening cloud.

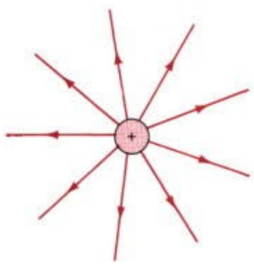
Apantallamiento del plasma

El apantallamiento o “screening” es un efecto importante en física de muchas partículas: nos permite hacer una primera simplificación de buena calidad con “bajo costo”.

$$\mathcal{H} = \int d^3r\, \psi^\dagger(\mathbf{r}) \left(-\frac{\hbar^2 \nabla^2}{2m} \right) \psi(\mathbf{r}) + \int d^3r\, V_{eff}(r) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Donde: $V_{eff}(r) = V(r) + V_{ind}(r)$

Potencial inducido por las cargas que apantallan



Potencial Coulombiano de una carga de prueba

Apantallamiento del plasma

$$\mathcal{H} = \int d^3r\, \psi^\dagger(\mathbf{r}) \left(-\frac{\hbar^2 \nabla^2}{2m} \right) \psi(\mathbf{r}) + \int d^3r\, V_{eff}(r) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Transf. Fourier

$$\mathcal{H} = \sum_{\mathbf{k}} E_k a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \sum_{\mathbf{p}} V_{eff}(p) \sum_{\mathbf{k}} a^\dagger_{\mathbf{k}+\mathbf{p}} a_{\mathbf{k}}$$

Planteamos la ecuación de movimiento:

$$\begin{aligned} \frac{d}{dt} a^\dagger_{\mathbf{k}-\mathbf{q}} a_{\mathbf{k}} &= \frac{i}{\hbar} [\mathcal{H}, a^\dagger_{\mathbf{k}-\mathbf{q}} a_{\mathbf{k}}] \\ &= i(\epsilon_{k-q} - \epsilon_k) a^\dagger_{\mathbf{k}-\mathbf{q}} a_{\mathbf{k}} \\ &- \frac{i}{\hbar} \sum_{\mathbf{p}} V_{eff}(p) (a^\dagger_{\mathbf{k}-\mathbf{q}} a_{\mathbf{k}-\mathbf{p}} - a^\dagger_{\mathbf{k}+\mathbf{p}-\mathbf{q}} a_{\mathbf{k}}) \end{aligned}$$

Apantallamiento del plasma

Ecuación de movimiento:

$$\frac{d}{dt}a^\dagger_{\mathbf{k}-\mathbf{q}}a_{\mathbf{k}} = \frac{i}{\hbar}[\mathcal{H}, a^\dagger_{\mathbf{k}-\mathbf{q}}a_{\mathbf{k}}]$$
$$= i(\epsilon_{k-q} - \epsilon_k)a^\dagger_{\mathbf{k}-\mathbf{q}}a_{\mathbf{k}}$$
$$- \frac{i}{\hbar}\sum_{\mathbf{p}} V_{eff}(p)(a^\dagger_{\mathbf{k}-\mathbf{q}}a_{\mathbf{k}-\mathbf{p}} - a^\dagger_{\mathbf{k}+\mathbf{p}-\mathbf{q}}a_{\mathbf{k}})$$

Planteamos la random-phase approximation en el término Coulombiano (**p** = **q**):

$$\frac{d}{dt}\langle a^\dagger_{\mathbf{k}-\mathbf{q}}a_{\mathbf{k}}\rangle = i(\epsilon_{k-q} - \epsilon_k)\langle a^\dagger_{\mathbf{k}-\mathbf{q}}a_{\mathbf{k}}\rangle - \frac{iV_{eff}(q)}{\hbar}(f_{\mathbf{k}-\mathbf{q}} - f_{\mathbf{k}})$$

Supongamos que la carga de prueba tiene una variación oscilatoria y un encendido adiabático: $\exp(-i(\omega+i\delta)t)$

Y de nuevo planteamos el ansatz: $\langle a^\dagger_{\mathbf{k}-\mathbf{q}}a_{\mathbf{k}}\rangle \propto e^{-i(\omega+i\delta)t}$

Apantallamiento del plasma

$$\frac{d}{dt}\langle a^{\dagger}_{\mathbf{k-q}}a_{\mathbf{k}}\rangle = i(\epsilon_{k-q}-\epsilon_k)\langle a^{\dagger}_{\mathbf{k-q}}a_{\mathbf{k}}\rangle - \frac{iV_{eff}(q)}{\hbar}(f_{\mathbf{k-q}}-f_{\mathbf{k}})$$

$$\langle a^{\dagger}_{\mathbf{k-q}}a_{\mathbf{k}}\rangle \propto e^{-i(\omega+i\delta)t}$$

$$\Rightarrow \quad \hbar(\omega+i\delta+\epsilon_{k-q}-\epsilon_k)\langle a^{\dagger}_{\mathbf{k-q}}a_{\mathbf{k}}\rangle = V_{eff}(q)(f_{\mathbf{k-q}}-f_{\mathbf{k}})$$

$$\Rightarrow \quad \langle a^{\dagger}_{\mathbf{k-q}}a_{\mathbf{k}}\rangle = V_{eff}(q)\frac{f_{\mathbf{k-q}}-f_{\mathbf{k}}}{\hbar(\omega+i\delta+\epsilon_{k-q}-\epsilon_k)}$$

$$\left(\frac{-|e|}{L^3}\right)\sum_{\mathbf{k}} \quad \Rightarrow \quad \langle\rho_q\rangle = -\frac{|e|}{L^3}V_{eff}(q)P^1(q,\omega)$$

$$P^1(q,\omega) = \sum_{\mathbf{k}}\frac{f_{\mathbf{k-q}}-f_{\mathbf{k}}}{\hbar(\omega+i\delta+\epsilon_{k-q}-\epsilon_k)}$$

Apantallamiento del plasma

$$\langle a^{\dagger}_{\mathbf{k}-\mathbf{q}} a_{\mathbf{k}} \rangle = V_{eff}(q) \frac{f_{\mathbf{k}-\mathbf{q}} - f_{\mathbf{k}}}{\hbar(\omega + i\delta + \epsilon_{k-q} - \epsilon_k)}$$

$$\left(\frac{-|e|}{L^3} \right) \sum_{\mathbf{k}}$$

$$\langle \rho_q \rangle = -\frac{|e|}{L^3} V_{eff}(q) P^1(q,\omega)$$

donde: $$P^1(q,\omega) = \sum_{\mathbf{k}} \frac{f_{\mathbf{k}-\mathbf{q}} - f_{\mathbf{k}}}{\hbar(\omega + i\delta + \epsilon_{k-q} - \epsilon_k)}$$

El potencial inducido por el reagrupamiento de cargas que produce el apantallamiento satisface la ecuación de Poisson:

$$\nabla^2 V_{ind}(r) = \frac{4\pi |e| \rho(r)}{\epsilon_0}$$

Apantallamiento del plasma

$$\nabla^2 V_{ind}(r) = \frac{4\pi|e|\rho(r)}{\epsilon_0}$$

Transf. Fourier:

$$V_{ind}(q) = -\frac{4\pi|e|}{\epsilon_0 q^2}\rho_q$$

$$\langle\rho_q\rangle = -\frac{|e|}{L^3}V_{eff}(q)P^1(q,\omega)$$

$$V_{ind}(q) = -\frac{4\pi|e|}{\epsilon_0 q^2}\rho_q = \frac{4\pi e^2}{\epsilon_0 q^2 L^3}V_{eff}(q)P^1(q,\omega)$$

$$= V_q\, V_{eff}(q)P^1(q,\omega)$$

Apantallamiento del plasma

$$V_{ind}(q) = V_q\, V_{eff}(q) P^1(q,\omega)$$

$$V_{eff}(q) = V_q + V_{ind}(q)$$

$$\Rightarrow \quad V_{eff}(q) = V_q[1 + V_{eff}(q)P^1(q,\omega)]$$

Despejando $V_{eff}(q)$: $\Rightarrow$ $$V_{eff}(q) = \frac{V_q}{1 - V_q P^1(q,\omega)} = \frac{V_q}{\epsilon(q,\omega)} \equiv V_s(q,\omega)$$

$V_s(q,\omega)$ Potencial Coulombiano apantallado dinámicamente

$$\boxed{\epsilon(q,\omega) = 1 - V_q P^1(q,\omega)}$$

Función dieléctrica dinámica

Apantallamiento del plasma

$$V_s(q,\omega) \equiv \frac{V_q}{\epsilon(q,\omega)}$$

$$\epsilon(q,\omega) = 1 - V_q P^1(q,\omega)$$

$$P^1(q,\omega) = \sum_{\mathbf{k}} \frac{f_{\mathbf{k}-\mathbf{q}} - f_{\mathbf{k}}}{\hbar(\omega + i\delta + \epsilon_{k-q} - \epsilon_k)}$$

$$\boldsymbol{\epsilon(q,\omega) = 1 - V_q \sum_{\mathbf{k}} \frac{f_{\mathbf{k}-\mathbf{q}} - f_{\mathbf{k}}}{\hbar(\omega + i\delta + \epsilon_{k-q} - \epsilon_k)}}$$

Fórmula de Lindhard para la función dieléctrica longitudinal

Apantallamiento del plasma

$$\epsilon(q,\omega) = 1 - V_q \sum_{\mathbf{k}} \frac{f_{\mathbf{k}-\mathbf{q}} - f_{\mathbf{k}}}{\hbar(\omega + i\delta + \epsilon_{k-q} - \epsilon_k)}$$

Fórmula de Lindhard

- Función dieléctrica compleja, retardada = polos debajo del eje x
- Dispersión espacial = dependencia de **q**
- Dispersión temporal = dependencia de ω
- Válida en 3 y 2 dimensiones
- La $f_{\mathbf{k}}$ acá es la distribución de Fermi-Dirac (en caso de equilibrio), pero también puede ser distribución fuera del equilibrio.

Apantallamiento del plasma

$$\epsilon(q,\omega) = 1 - V_q \sum_{\mathbf{k}} \frac{f_{\mathbf{k}-\mathbf{q}} - f_{\mathbf{k}}}{\hbar(\omega + i\delta + \epsilon_{k-q} - \epsilon_k)}$$

Fórmula de Lindhard

$$V_q \sum_{\mathbf{k}} \frac{f_{\mathbf{k}-\mathbf{q}} - f_{\mathbf{k}}}{\hbar(\omega_q + \epsilon_{k-q} - \epsilon_k)} = 1 \qquad (8.25)$$

$$\mathbf{Re}[\epsilon(q,\omega)] = 0 \quad \text{or} \quad 1 = V_q \mathbf{Re}[P^1(q,\omega)]$$

Modos normales de plasma longitudinales.

This equation is identical to the plasma eigenmode equation (8.25). Hence, our discussion of plasma screening of the Coulomb potential and of the collective plasma oscillations obtained from $\epsilon(q,\omega) = 0$, shows that screening and plasmons are intimately related phenomena.

Resumen de la clase 16

En RPA:

Apantallamiento (screening) del plasma