

**La clase pasada vimos:**

Modelo de osciladores:

Función dieléctrica óptica

Interpretación física: absorción y refracción

**En esta clase veremos:**

Introducción a excitones

Polarización interbanda

Modelo de dos bandas

REPASO

## Función dieléctrica óptica

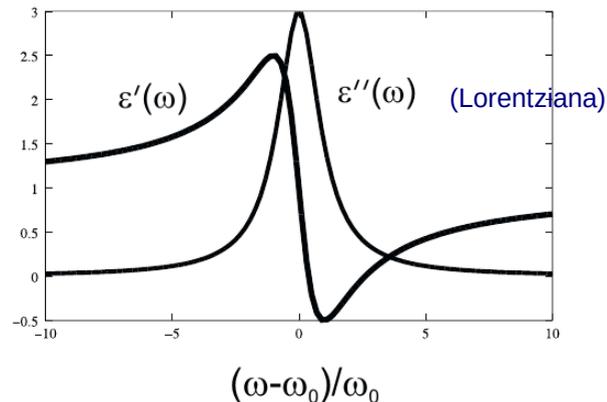
$$D(\omega) = \mathcal{E}(\omega) + 4\pi\mathcal{P}(\omega) = [1 + 4\pi\chi(\omega)]\mathcal{E}(\omega) = \epsilon(\omega)\mathcal{E}(\omega)$$

Campo desplazamiento

Campo eléctrico

(en cgs)

$$\epsilon(\omega) = 1 + 4\pi\chi(\omega) = 1 - \frac{\omega_{pl}^2}{2\omega'_0} \left( \frac{1}{\omega - \omega'_0 + i\gamma} - \frac{1}{\omega + \omega'_0 + i\gamma} \right)$$



REPASO

Función dieléctrica óptica

$$D(\omega) = \epsilon(\omega)\mathcal{E}(\omega)$$

$$\mathcal{E}(\mathbf{r}, \omega) = \mathcal{E}_0(\omega)e^{i[k(\omega)+i\kappa(\omega)]z}$$

Coeficiente de absorción:  $\alpha(\omega) = 2\kappa(\omega)$

Decaimiento de la intensidad:  $I \propto |\mathcal{E}|^2$

Longitud típica de decaimiento:  $1/\alpha$

$$n(\omega) = \sqrt{\frac{1}{2} \left[ \epsilon'(\omega) + \sqrt{\epsilon'^2(\omega) + \epsilon''^2(\omega)} \right]}$$

Índice de refracción

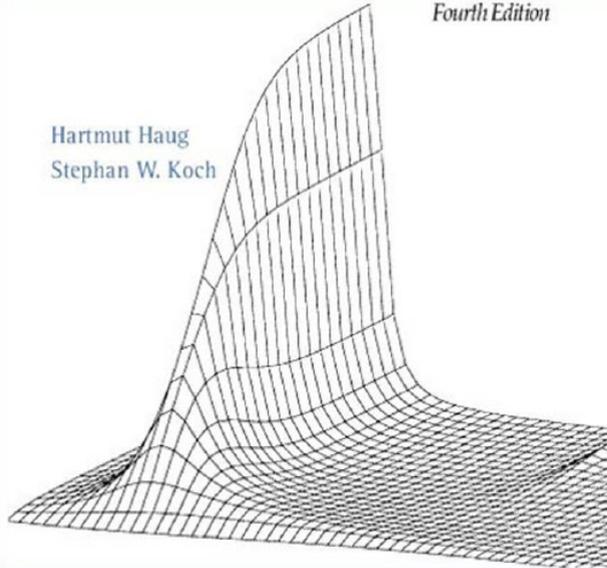
$$\alpha(\omega) = \frac{\omega}{n(\omega)c} \epsilon''(\omega)$$

Coeficiente de absorción

*Quantum Theory of the  
Optical and Electronic  
Properties of Semiconductors*

Fourth Edition

Hartmut Haug  
Stephan W. Koch

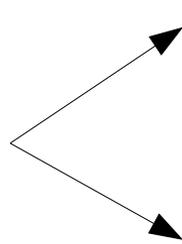


World Scientific

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# Introducción

Gas de electrones  
(jellium model)



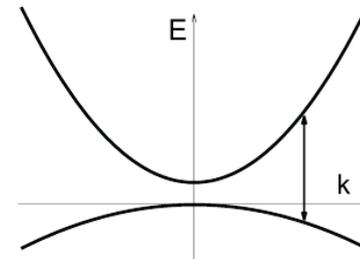
Metales – banda semillena

Semiconductores – interacción *intrabanda*

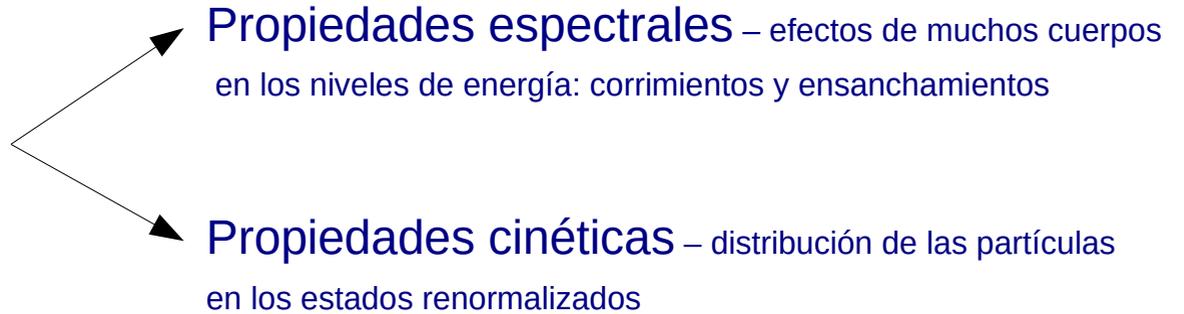
Propiedades ópticas



Semiconductores – transiciones *interbanda*



Sistema electrón-hueco  
en semiconductores



Propiedades cinéticas



## La polarización interbanda

En las dos clases anteriores vimos la susceptibilidad óptica en el modelo dieléctrico de osciladores.

En el capítulo 5 de Haug & Koch se introduce la polarización interbanda en un tratamiento de primera cuantización sin interacción electrón-electrón.

Los **excitones** aparecen cuando consideramos las transiciones interbanda teniendo en cuenta la interacción electrón-electrón.

# Polarización interbanda

## Polarización interbanda

Se aplica un campo eléctrico  $E(\mathbf{r},t)$  que inducirá una polarización eléctrica:

$$\mathbf{P}(t) = \sum_s \int d^3r \langle \hat{\psi}_s^\dagger(\mathbf{r}, t) e\mathbf{r} \hat{\psi}_s(\mathbf{r}, t) \rangle$$

Como antes, el promedio significa valor de expectación sobre el estado del sistema (N electrones) antes de que empiece a actuar el campo.

Introducimos la matriz densidad reducida de una partícula, dada por:

$$\rho_{ss'}(\mathbf{r}, \mathbf{r}', t) = \langle \hat{\psi}_s^\dagger(\mathbf{r}, t) \hat{\psi}_{s'}(\mathbf{r}', t) \rangle$$

## Polarización interbanda

$$\left. \begin{aligned}
 \mathbf{P}(t) &= \sum_s \int d^3r \langle \hat{\psi}_s^\dagger(\mathbf{r}, t) e\mathbf{r} \hat{\psi}_s(\mathbf{r}, t) \rangle \\
 \rho_{ss'}(\mathbf{r}, \mathbf{r}', t) &= \langle \hat{\psi}_s^\dagger(\mathbf{r}, t) \hat{\psi}_{s'}(\mathbf{r}', t) \rangle
 \end{aligned} \right\} \mathbf{P}(t) = \sum_s \int d^3r \rho_{ss'}(\mathbf{r}, \mathbf{r}', t)|_{r=r', s=s'} e\mathbf{r}$$

Ahora expresamos los operadores de campo en términos de operadores de creación y destrucción en la base de **estados de Bloch** del sólido:

$$\mathcal{H}_0|\lambda\mathbf{k}\rangle = E_\lambda(\mathbf{k})|\lambda\mathbf{k}\rangle = \hbar\epsilon_{\lambda,\mathbf{k}}|\lambda\mathbf{k}\rangle \quad \longrightarrow \quad \psi_\lambda(\mathbf{k}, \mathbf{r}) = \langle \mathbf{r}|\lambda\mathbf{k}\rangle$$

$$\psi_\lambda(\mathbf{k}, \mathbf{r}) = \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{L^{3/2}} u_\lambda(\mathbf{k}, \mathbf{r})$$

$$\longrightarrow \hat{\psi}_s(\mathbf{r}, t) = \sum_{\lambda, \mathbf{k}} a_{\lambda, \mathbf{k}, s}(t) \psi_\lambda(\mathbf{k}, \mathbf{r})$$

## Polarización interbanda

$$\left. \begin{aligned} \mathbf{P}(t) &= \sum_s \int d^3r \langle \hat{\psi}_s^\dagger(\mathbf{r}, t) e\mathbf{r} \hat{\psi}_s(\mathbf{r}, t) \rangle \\ \hat{\psi}_s(\mathbf{r}, t) &= \sum_{\lambda, \mathbf{k}} a_{\lambda, \mathbf{k}, s}(t) \psi_\lambda(\mathbf{k}, \mathbf{r}) \end{aligned} \right\} \longrightarrow$$
  
$$\longrightarrow \mathbf{P}(t) = \sum_{s, \lambda, \lambda', \mathbf{k}, \mathbf{k}'} \langle a_{\lambda, \mathbf{k}, s}^\dagger a_{\lambda', \mathbf{k}', s} \rangle \int d^3r \psi_{\lambda, \mathbf{k}}^*(\mathbf{r}) e\mathbf{r} \psi_{\lambda', \mathbf{k}'}(\mathbf{r})$$

The integral has been evaluated in Chap. 5, Eqs. (5.11) – (5.20).

$$\int d^3r \psi_{\lambda, \mathbf{k}}^*(\mathbf{r}) e\mathbf{r} \psi_{\lambda', \mathbf{k}'}(\mathbf{r}) \simeq \delta_{\mathbf{k}, \mathbf{k}'} \mathbf{d}_{\lambda \lambda'} \quad \lambda \neq \lambda'$$

## Polarización interbanda

$$\mathbf{P}(t) = \sum_{s,\lambda,\lambda',\mathbf{k},\mathbf{k}'} \langle a_{\lambda,\mathbf{k},s}^\dagger a_{\lambda',\mathbf{k}',s} \rangle \int d^3r \psi_{\lambda,\mathbf{k}}^*(\mathbf{r}) e\mathbf{r} \psi_{\lambda',\mathbf{k}'}(\mathbf{r})$$

$$\int d^3r \psi_{\lambda,\mathbf{k}}^*(\mathbf{r}) e\mathbf{r} \psi_{\lambda',\mathbf{k}'}(\mathbf{r}) \simeq \delta_{\mathbf{k},\mathbf{k}'} \mathbf{d}_{\lambda\lambda'}$$



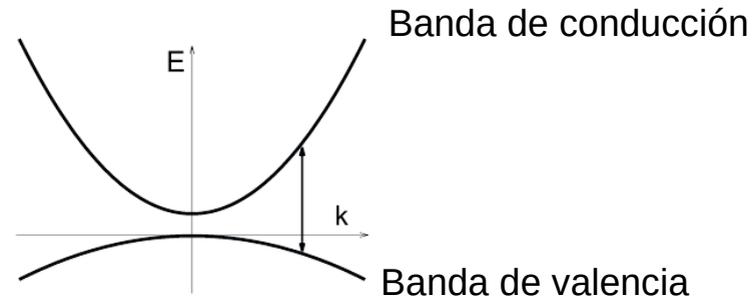
$$\mathbf{P}(t) = \sum_{\mathbf{k},s,\lambda,\lambda'} \langle a_{\lambda,\mathbf{k},s}^\dagger a_{\lambda',\mathbf{k},s}(t) \rangle \mathbf{d}_{\lambda\lambda'} = \sum_{\mathbf{k},s,\lambda,\lambda'} P_{\lambda\lambda',\mathbf{k},s}(t) \mathbf{d}_{\lambda\lambda'}$$

Donde definimos:

$$P_{\lambda\lambda',\mathbf{k},s}(t) = \langle a_{\lambda,\mathbf{k},s}^\dagger a_{\lambda',\mathbf{k},s}(t) \rangle$$

Modelo de dos bandas

# Modelo de dos bandas



Dos bandas:  $\lambda = v$      $\lambda' = c$ ,

Incluimos el índice de espín  $s$  en  $\mathbf{k}$   
(factor 2 por sumas sobre espín)

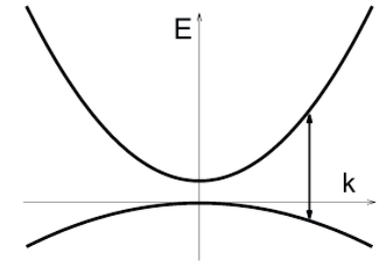
$$P_{\lambda\lambda',\mathbf{k},s}(t) = \langle a_{\lambda,\mathbf{k},s}^\dagger a_{\lambda',\mathbf{k},s}(t) \rangle \quad \longrightarrow \quad P_{vc,\mathbf{k}}(t) = \langle a_{v,\mathbf{k}}^\dagger a_{c,\mathbf{k}}(t) \rangle$$

Término no diagonal de la  
matriz densidad

Es cero en equilibrio si no hay una polarización  
permanente, en ausencia de campo eléctrico.

Hamiltoniano de interacción luz-electrones:

$$\mathcal{H}_I(t) = -ex\mathcal{E}(t) = -d\mathcal{E}(t) \quad \longrightarrow$$



$$\mathcal{H}_I = \int d^3r \hat{\psi}^\dagger(\mathbf{r})(-e\mathbf{r}) \cdot \mathcal{E}(\mathbf{r}, t)\hat{\psi}(\mathbf{r})$$

$$\mathcal{E}(\mathbf{r}, t) = \mathcal{E}(t)\frac{1}{2}(e^{i\mathbf{q}\cdot\mathbf{r}} + c.c.)$$

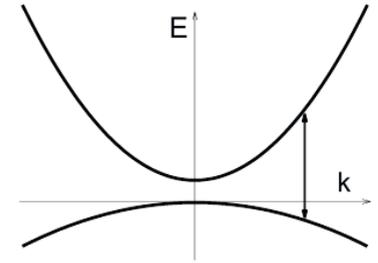
$$\hat{\psi}_s(\mathbf{r}, t) = \sum_{\lambda, \mathbf{k}} a_{\lambda, \mathbf{k}, s}(t)\psi_\lambda(\mathbf{k}, \mathbf{r})$$

limit  $q \rightarrow 0$  (dipole approximation)

$$\mathcal{H}_I \simeq - \sum_{\mathbf{k}} \mathcal{E}(t)(a_{c, \mathbf{k}}^\dagger a_{v, \mathbf{k}} d_{cv} + h.c.)$$

# Hamiltoniano

$$\mathcal{H}_I \simeq - \sum_{\mathbf{k}} \mathcal{E}(t) (a_{c,\mathbf{k}}^\dagger a_{v,\mathbf{k}} d_{cv} + h.c.)$$



$$\mathcal{H}_{el} = \sum_{\lambda, \mathbf{k}} E_{\lambda, \mathbf{k}} a_{\lambda, \mathbf{k}}^\dagger a_{\lambda, \mathbf{k}} + \frac{1}{2} \sum_{\substack{\mathbf{k}, \mathbf{k}' \\ \mathbf{q} \neq \mathbf{0} \\ \lambda, \lambda'}} V_{\mathbf{q}} a_{\lambda, \mathbf{k} + \mathbf{q}}^\dagger a_{\lambda', \mathbf{k}' - \mathbf{q}}^\dagger a_{\lambda', \mathbf{k}'} a_{\lambda, \mathbf{k}}$$

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## Coulomb quantum kinetics of degenerate resonant femtosecond four-wave mixing

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PACS. 78.47+p – Time-resolved optical spectroscopies and other ultrafast optical measurements in condensed matter.

PACS. 72.20Dp – General theory, scattering mechanisms.

**Abstract.** – Resonant time-resolved four-wave-mixing spectra are calculated for direct-gap semiconductors. For pulses shorter than the build-up time of the Coulomb screening (given by the period of a typical plasma oscillation) the dephasing and relaxation of the excited carriers are dominated by the bare Coulomb potential scattering. The quantum kinetics of this regime results in an unusual decay law  $\propto \exp[-an\tau^3]$  for the integrated four-wave-mixing signal with the delay time  $\tau$  and the carrier density  $n$ . Due to the wave properties contained in the early time regime of quantum kinetics a sideband appears in the spectrum of the time-resolved four-wave mixing with a carrier-density-dependent frequency separation from the main band. The conditions under which the predicted new features can be measured are discussed.

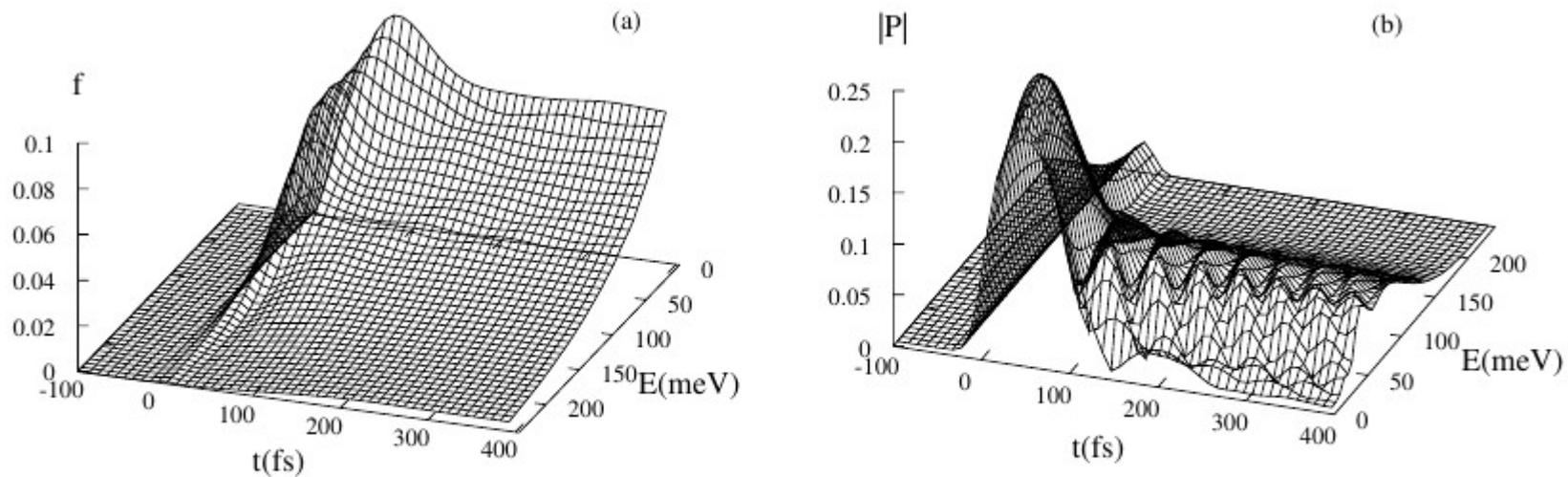


Fig. 1. – (a) Electron distribution  $f_k(t)$  and (b) absolute value of the interband polarization  $|P_k(t)|$  vs. pair energy and time for an excited pair density of  $n = 0.9 \times 10^{17} \text{ cm}^{-3}$ .

# Resumen de la clase 20

Introducción a Excitones

Polarización interbanda

Modelo de dos bandas