CLASE 8 - viernes 23/04/2021

La clase pasada vimos:

Nanoestructuras: cables y puntos cuánticos

Aproximación de masa efectiva en heteroestructuras

Dopaje remoto

Esta clase veremos:

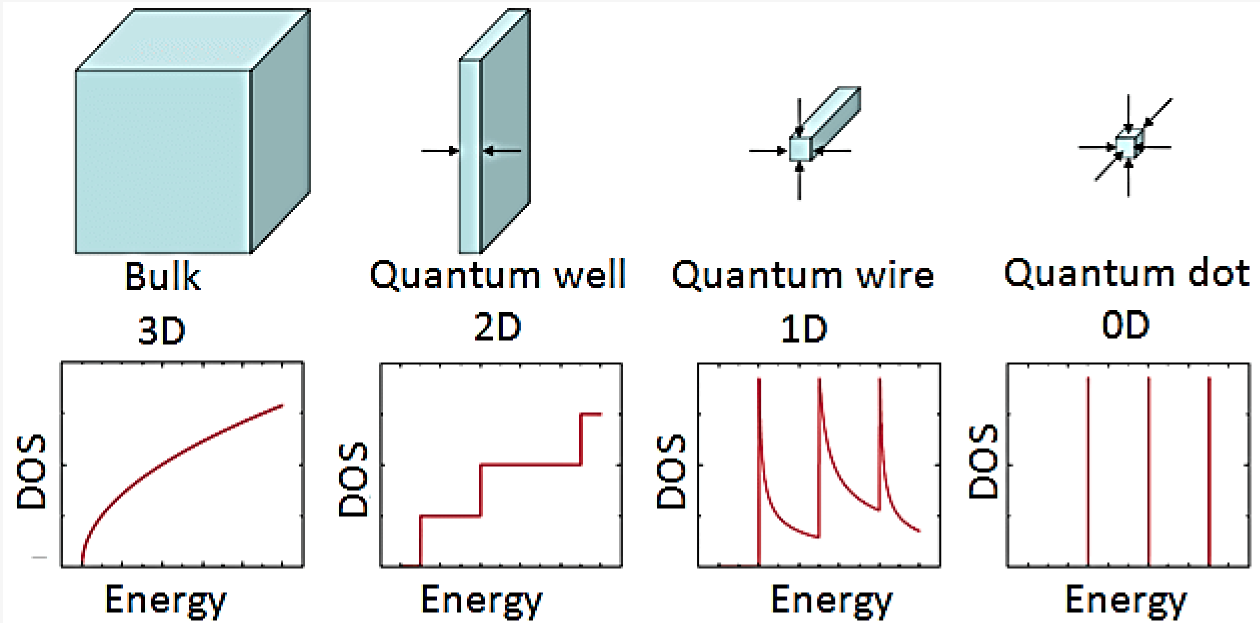
Autoestados y autoenergías en sistemas cuasi-2D

Ocupación de subbandas

Sistemas cuasi-1D

REPASO

Nanoestructuras: cables y puntos cuánticos



Quantum Wires

To make the transition from a 2D electron gas (quantum well) to a 1D electron gas (quantum wire), the electrons should be confined in two directions and only 1 degree of freedom remain. The **x direction** remains the **only one** for **free-electron propagation**.

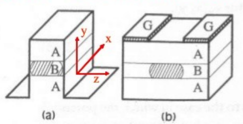
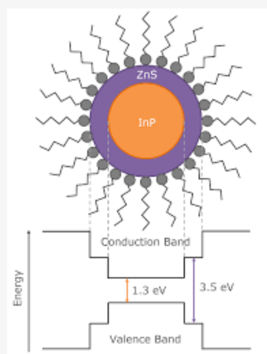
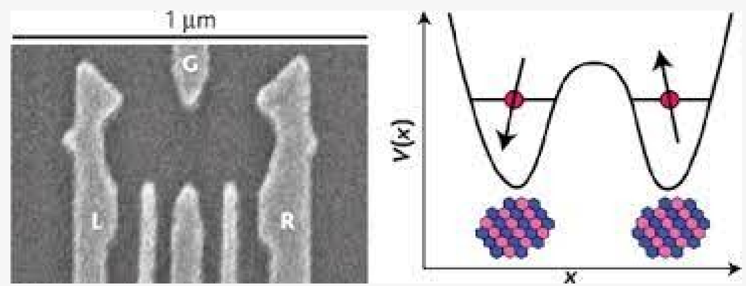


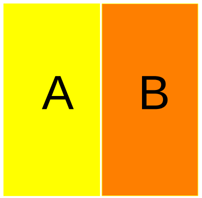
Figure 3.11. Two of the simplest examples of structures providing electron confinement in two dimensions: case (a) uses an etching technique, while case (b) is based on the split-gate technique.





REPASO

Aproximación de masa efectiva en heteroestructuras



$$\chi(0_A) = \chi(0_B), \qquad \frac{1}{m_A}\frac{d\chi(z)}{dz}\bigg|_{z=0_A} = \frac{1}{m_B}\frac{d\chi(z)}{dz}\bigg|_{z=0_B}$$

$$-\frac{\hbar^2}{2m_0}\frac{d}{dz}\left[\frac{1}{m(z)}\frac{d\chi}{dz}\right] + V(z)\chi(z) = E\chi(z)$$

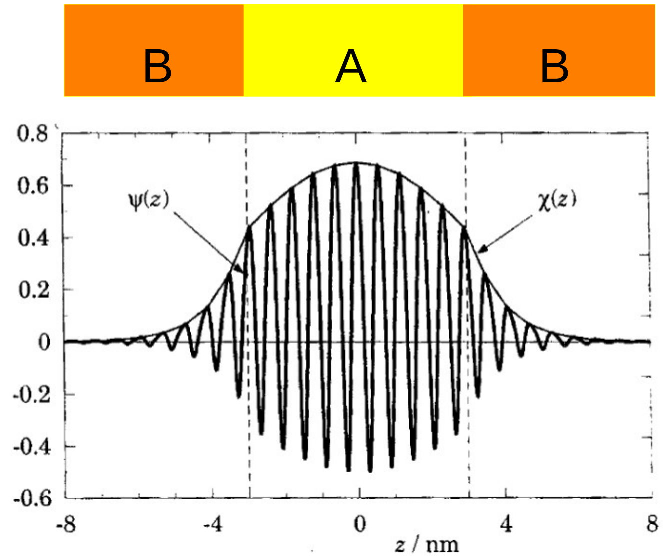


FIGURE 3.22. Wave function for the lowest state in a 6 nm quantum well in a heterostructure, including the Bloch functions. The thin curve is an approximate envelope function joining the peaks of the full wave function. [Redrawn from Burt (1994).]

REPASO

Dopaje remoto – *modulation doping*

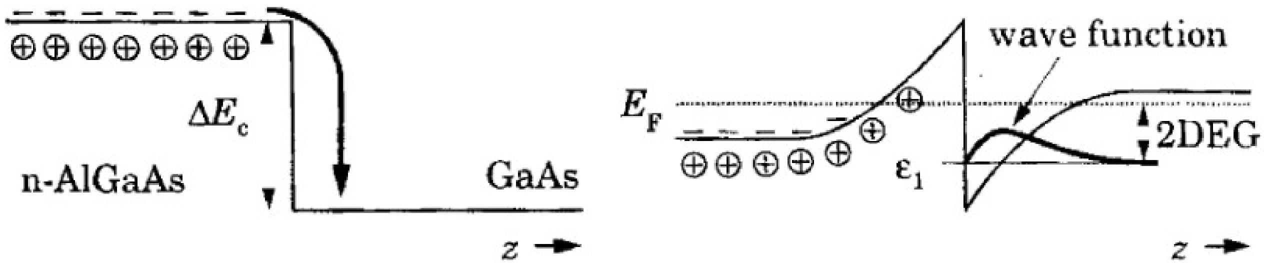


FIGURE 3.9. Conduction band around a heterojunction between n-AlGaAs and undoped GaAs, showing how electrons are separated from their donors to form a two-dimensional electron gas.

Los electrones migran de una región a otra y evitan las colisiones con los iones (cargados atractivamente)

Autoestados y autoenergías en sistemas de baja dimensionalidad con Hamiltoniano separable

THE PHYSICS OF LOW-DIMENSIONAL SEMICONDUCTORS

AN INTRODUCTION

JOHN H. DAVIES

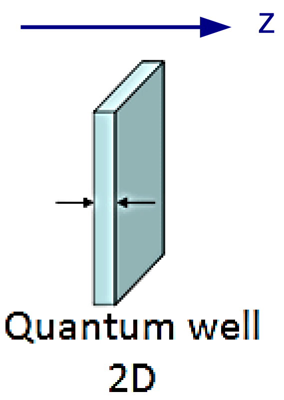
Glasgow University

4 QUANTUM WELLS AND LOW-DIMENSIONAL SYSTEMS

Ejemplos pozos 1D

Pozos cuánticos: Hamiltoniano separable cuasi-2D

Pozos cuánticos: Hamiltoniano separable cuasi-2D



$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{R})\right]\psi(\mathbf{R}) = E\,\psi(\mathbf{R})$$

$$V(\mathbf{R}) = V(z)$$

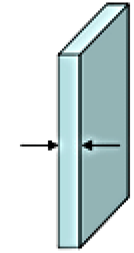
Confinamiento cuasi-2D
electrones libres en plano xy

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + V(z)\right]\psi(x, y, z) = E\,\psi(x, y, z)$$

Tenemos un Hamiltoniano separable: $H(\mathbf{r}) = H_x(x) + H_y(y) + H_z(z)$

Buscamos autoestados factorizados: $\psi(x, y, z) = \exp(ik_x x)\,\exp(ik_y y)\,u(z)$

Pozos cuánticos: Hamiltoniano separable cuasi-2D



$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+V(z)\right]\exp(ik_x x)\,\exp(ik_y y)\,u(z)$$

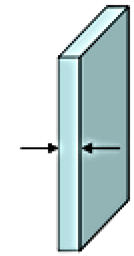
$$=\left[\frac{\hbar^2 k_x^2}{2m}+\frac{\hbar^2 k_y^2}{2m}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+V(z)\right]\exp(ik_x x)\,\exp(ik_y y)\,u(z)$$

$$=E\,\exp(ik_x x)\,\exp(ik_y y)\,u(z).$$

Llegamos a un problema de autovalores unidimensional:

$$\left[\frac{\hbar^2 k_x^2}{2m}+\frac{\hbar^2 k_y^2}{2m}-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}+V(z)\right]u(z)=E\,u(z)$$

Pozos cuánticos: Hamiltoniano separable cuasi-2D



Quantum well
2D

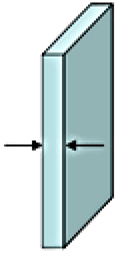
Pasamos la energía cinética en el plano al otro lado:

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}+V(z)\right]u(z)=\left[E-\frac{\hbar^2k_x^2}{2m}-\frac{\hbar^2k_y^2}{2m}\right]u(z)$$

$$\varepsilon = E-\frac{\hbar^2k_x^2}{2m}-\frac{\hbar^2k_y^2}{2m}$$

$$\longrightarrow \quad \boxed{\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}+V(z)\right]u(z)=\varepsilon\, u(z)}$$

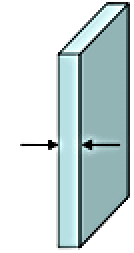
Pozos cuánticos: Hamiltoniano separable cuasi-2D



$$\psi_{k_x,k_y,n}(x,y,z) = \exp(ik_x x)\exp(ik_y y)\,u_n(z)$$

$$E_n(k_x,k_y) = \varepsilon_n + \frac{\hbar^2 k_x^2}{2m} + \frac{\hbar^2 k_y^2}{2m}.$$

Pozos cuánticos: Hamiltoniano separable cuasi-2D



Quantum well
2D

$$\mathbf{r} = (x, y) \qquad \mathbf{k} = (k_x, k_y)$$

$$\begin{cases} \psi_{\mathbf{k},n}(\mathbf{r}, z) = \exp(i\mathbf{k}\cdot\mathbf{r})\, u_n(z) \\ E_n(\mathbf{k}) = \varepsilon_n + \dfrac{\hbar^2 \mathbf{k}^2}{2m} \end{cases}$$

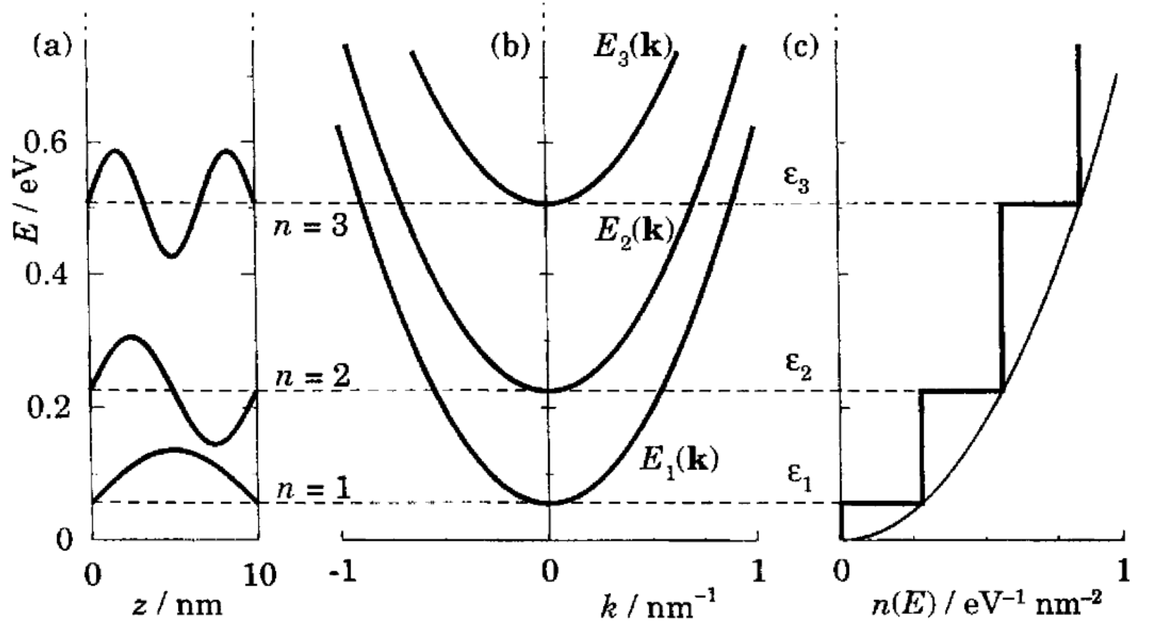
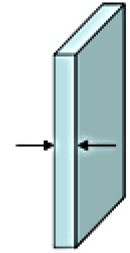
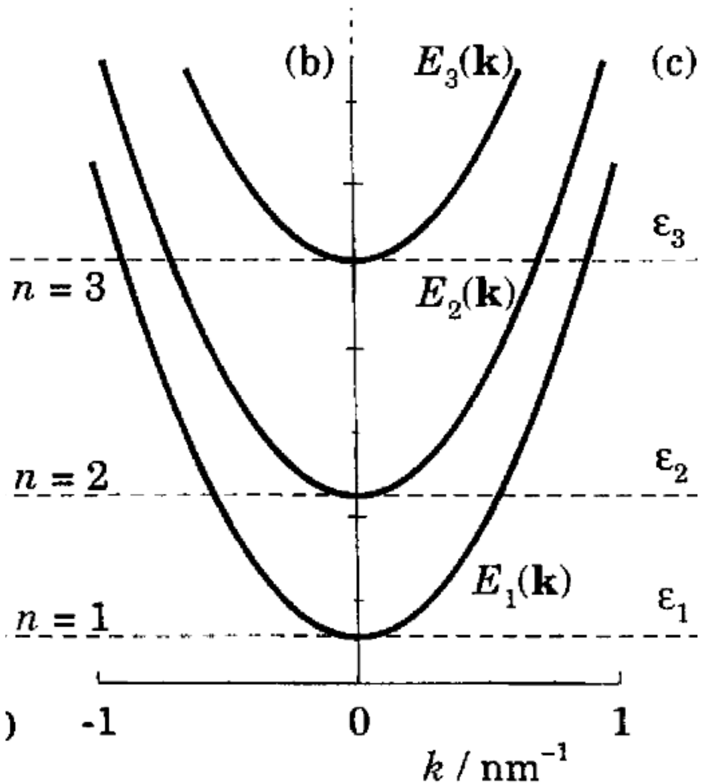


FIGURE 4.7. (a) Potential well with energy levels, (b) total energy including the transverse kinetic energy for each subband, and (c) steplike density of states of a quasi-two-dimensional system. The example is an infinitely deep square well in GaAs of width 10 nm. The thin curve in (c) is the parabolic density of states for unconfined three-dimensional electrons.

Ocupación de subbandas

Pozos cuánticos: ocupación de subbandas





Datos necesarios:

- Densidad de electrones, por unidad de área: n_{2D}
- Temperatura: T

$$n_{2D} = \int_{-\infty}^{\infty} n(E)\, f(E, E_F)\, dE$$

$$n_{2D} = \sum_j n_j$$

Pozos cuánticos: ocupación de subbandas

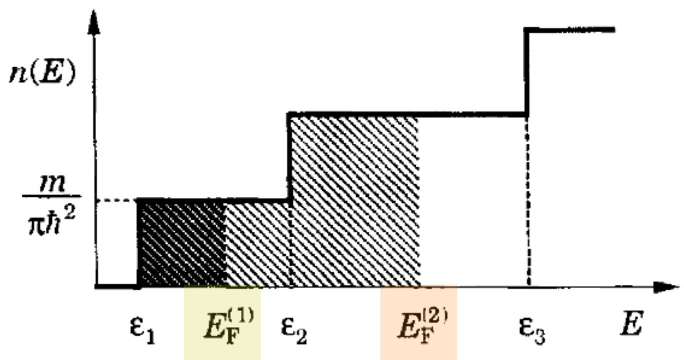
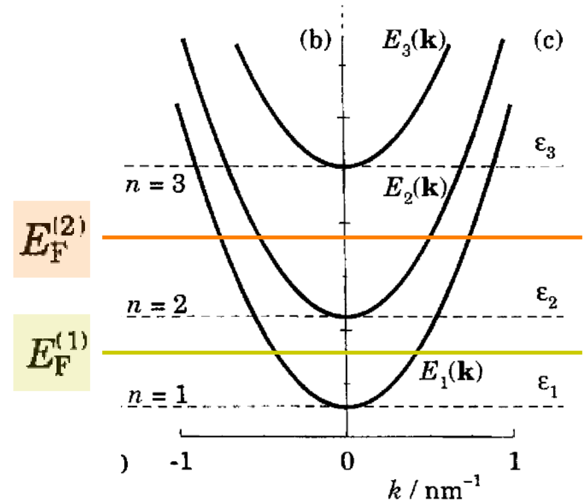


FIGURE 4.8. Occupation of steplike density of states for a quasi-two-dimensional system. Only one subband is occupied if the Fermi energy takes the lower value $E_F^{(1)}$, but two are occupied at the higher value $E_F^{(2)}$.



$$n_{2D} = \int_{-\infty}^{\infty} n(E)\, f(E, E_F)\, dE$$

$$n_j = \frac{m}{\pi\hbar^2}\int_{\varepsilon_j}^{\infty} f(E, E_F)\, dE = \frac{m k_B T}{\pi\hbar^2}\ln\left[1 + \exp\left(\frac{E_F - \varepsilon_j}{k_B T}\right)\right]$$

$$n_{2D} = \sum_j n_j$$

Límite de T = 0

$$n_{2D} = \sum_j n_j = \frac{m}{\pi\hbar^2}\sum_j (E_F - \varepsilon_j)\,\Theta(E_F - \varepsilon_j)$$

Pozos cuánticos: ocupación de subbandas

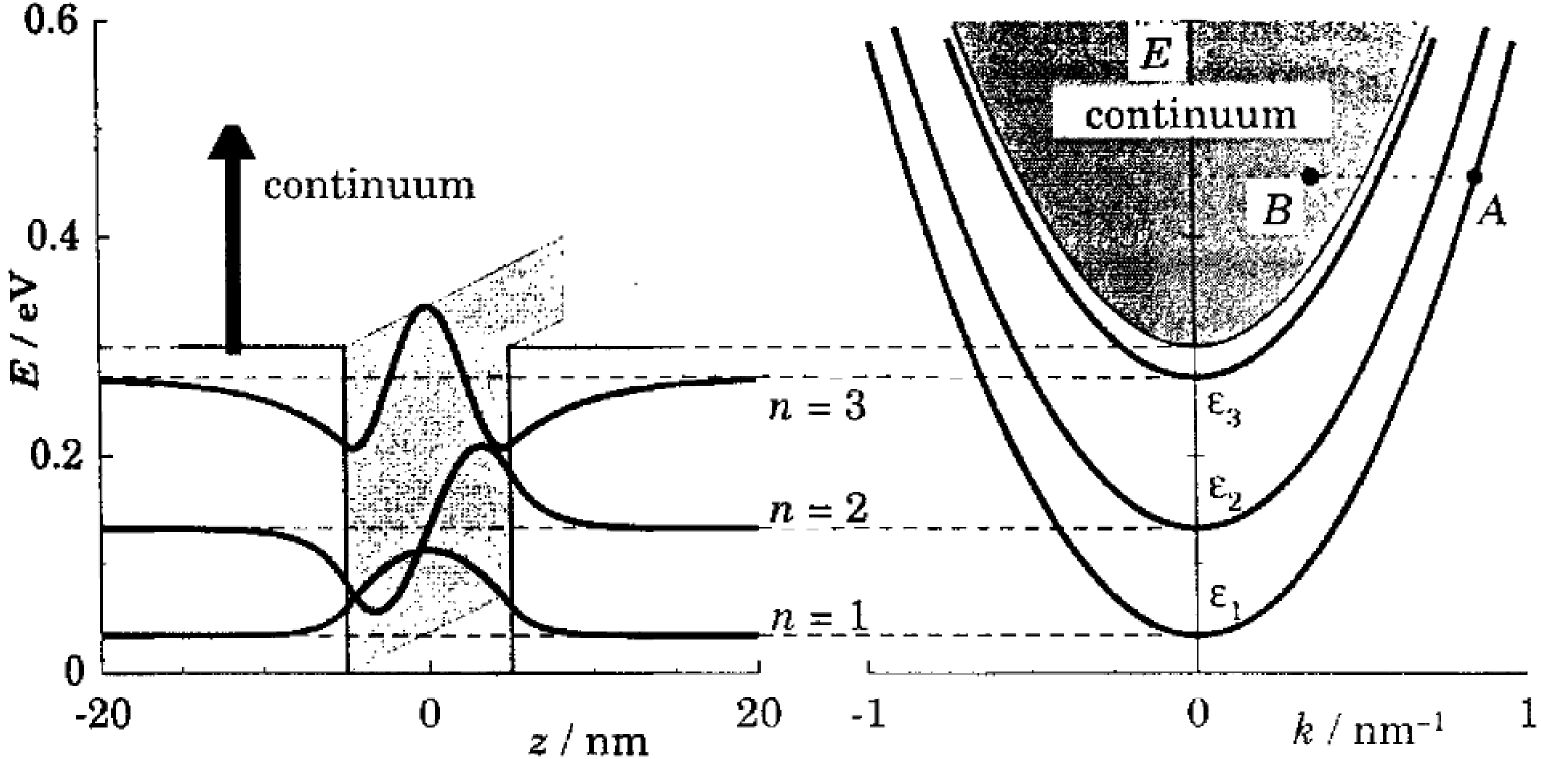
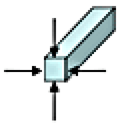


FIGURE 4.9. Quasi-two-dimensional system in a potential well of finite depth. Electrons with the same total energy can be bound in the well (*A*) or free (*B*).

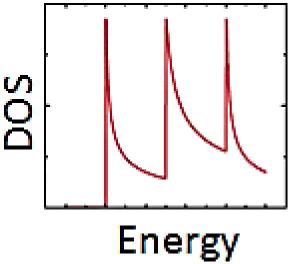
Cables cuánticos: Hamiltoniano separable cuasi-1D

Cables cuánticos: Hamiltoniano separable cuasi-1D

$$\mathbf{r} = (x\,,\, y)$$



Quantum wire
1D



$$\left[-\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + V(\mathbf{r}) \right] u_{m,n}(\mathbf{r}) = \varepsilon_{m,n} u_{m,n}(\mathbf{r})$$

$$\psi_{m,n,k_z}(\mathbf{R}) = u_{m,n}(\mathbf{r}) \exp(i k_z z),$$

$$E_{m,n}(k_z) = \varepsilon_{m,n} + \frac{\hbar^2 k_z^2}{2m}.$$

$$n(E) = \sum_{m,n} \frac{1}{\pi \hbar} \sqrt{\frac{2m}{E - \varepsilon_{m,n}}} \Theta(E - \varepsilon_{m,n})$$

Densidad de estados
(por unidad de longitud)

Cables cuánticos: Hamiltoniano separable cuasi-1D

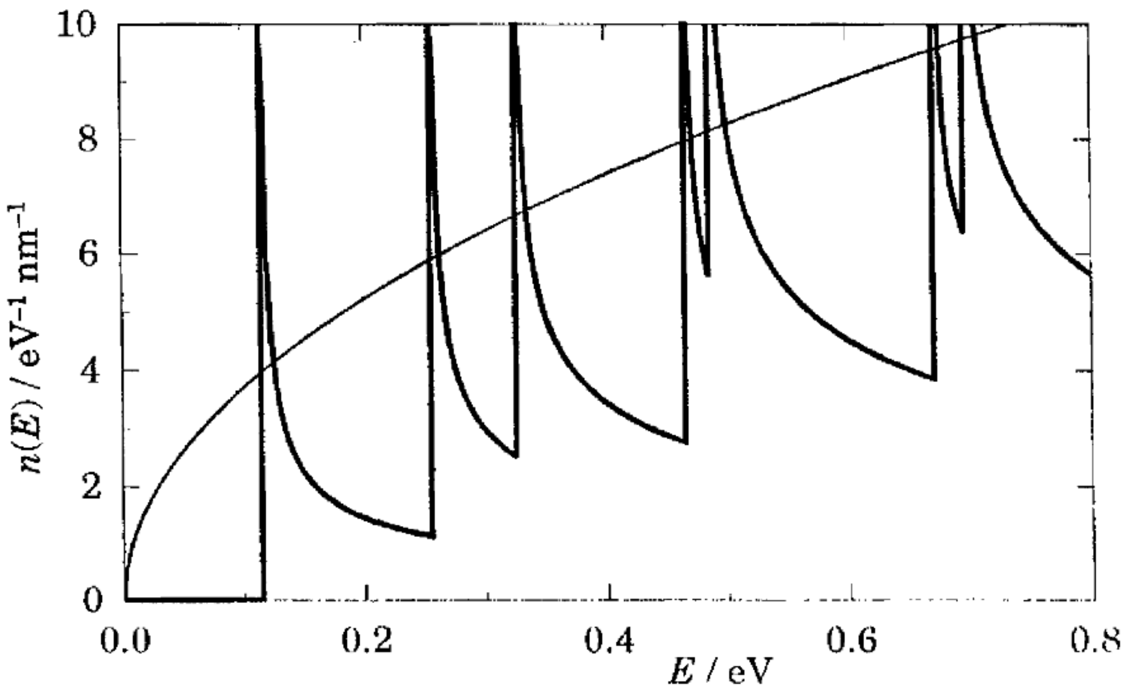
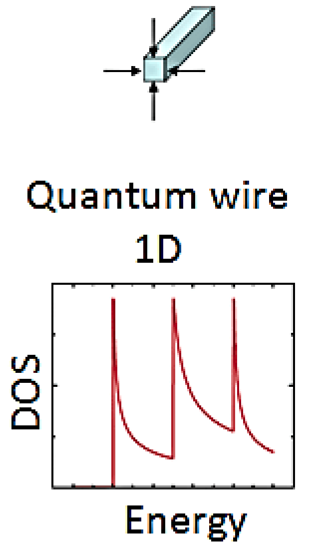


FIGURE 4.11. Density of states of a quasi-one-dimensional system. The curve was calculated for electrons in a 9 × 11 nm infinitely deep well in GaAs. The thin parabola is the density of states for unconfined three-dimensional electrons.



Resumen de la clase 8

Autoestados y autoenergías en sistemas cuasi-2D

Ocupación de subbandas

Sistemas cuasi-1D